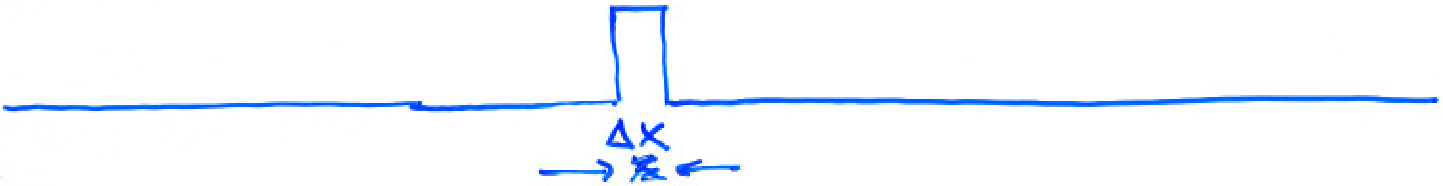
on the other hand...

Here's a particle, well-localized in space...



what's its momentum?

MERELY ATTEMPTING TO DETERMINE WHICH SLIT

A PARTICLE CHARACTERISTIC

DESTROYED THE INTERFERENCE

- one of the weirdnesses:
  - ask a wave question

    undisturbed double slit
  - get a wave answer

    interference
  - ask a particle question

    which slit
  - get a particle answer

    no interference

THIS WAS ALL DERIVED BY HEISENBERG...

I've been waving (get it?) my hands.

The real statement between uncertainty in position and uncertainty in momentum is:

$$\Delta x \, \Delta p_x \geq \frac{\hbar}{2}$$

Heisenberg Uncertainty Principle

try to pin down the position...

lose precision — KNOWLEDGE — of momentum

try to pin down the momentum...

lose precision — KNOWLEDGE — of position

THE UNCERTAINTY PRINCIPLE

IS NOT:

- a statement that measurement disturbs a system
- a statement that measuring instruments don't work well enough.

IT IS A STATEMENT ABOUT NATURE

OBJECTS WITH PERFECTLY PRECISE POSITION AND MOMENTUM

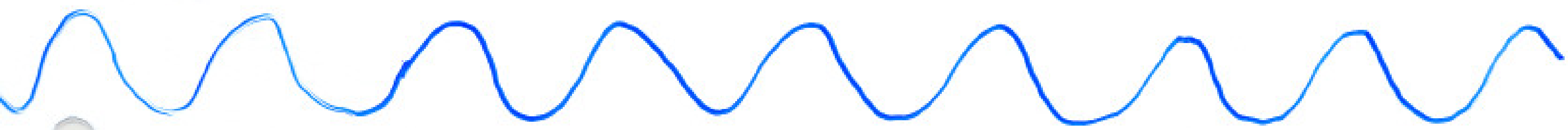
DON'T EXIST.

Think about it this way...

What's the most precise momentum state for a quantum object?

$$p = \frac{h}{\lambda}$$

one value for p $\Rightarrow$ one value for λ $\Rightarrow$ one frequency



What's the position of this quantum object?

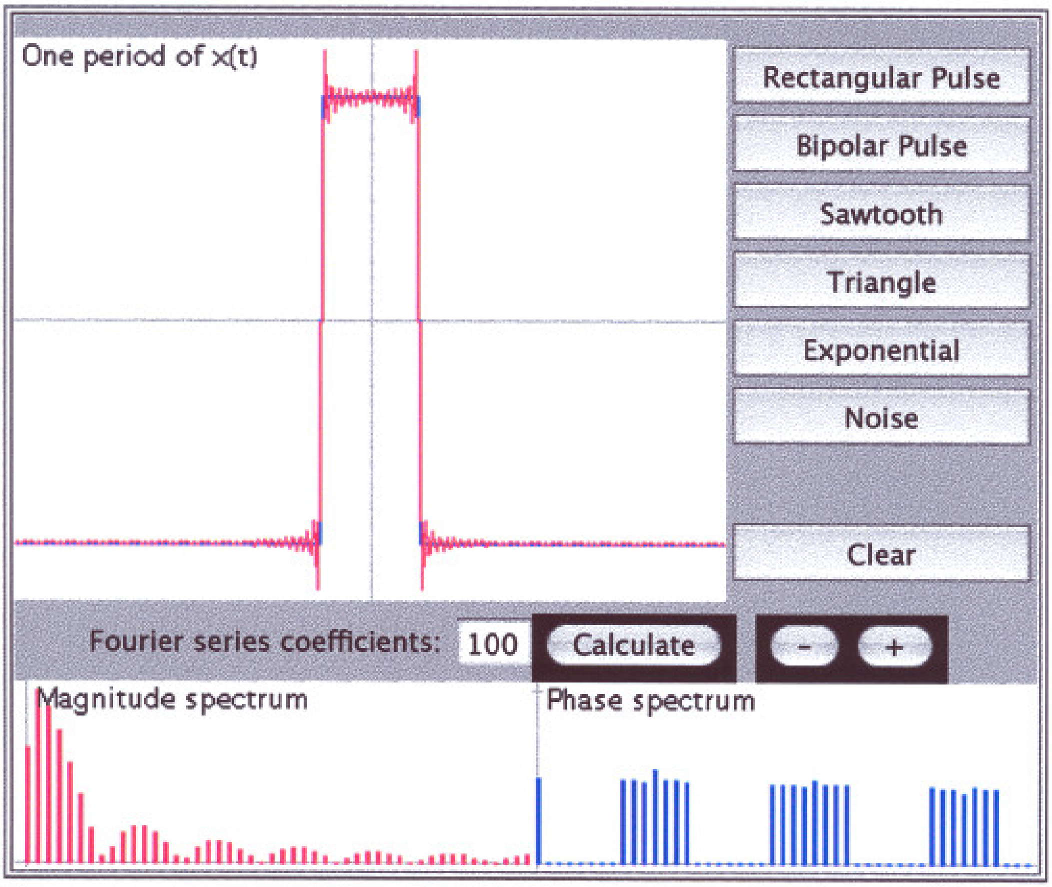
it's everywhere!

fourier series approximation

For a continuous-time, T-periodic signal $x(t)$, the N-harmonic Fourier series approximation can be written as

$$x(t) = a_0 + a_1 \cos (w_o t + q_1) + a_2 \cos (2w_o t + q_2)$$
$$+ \ldots + a_N \cos (Nw_o t + q_N)$$

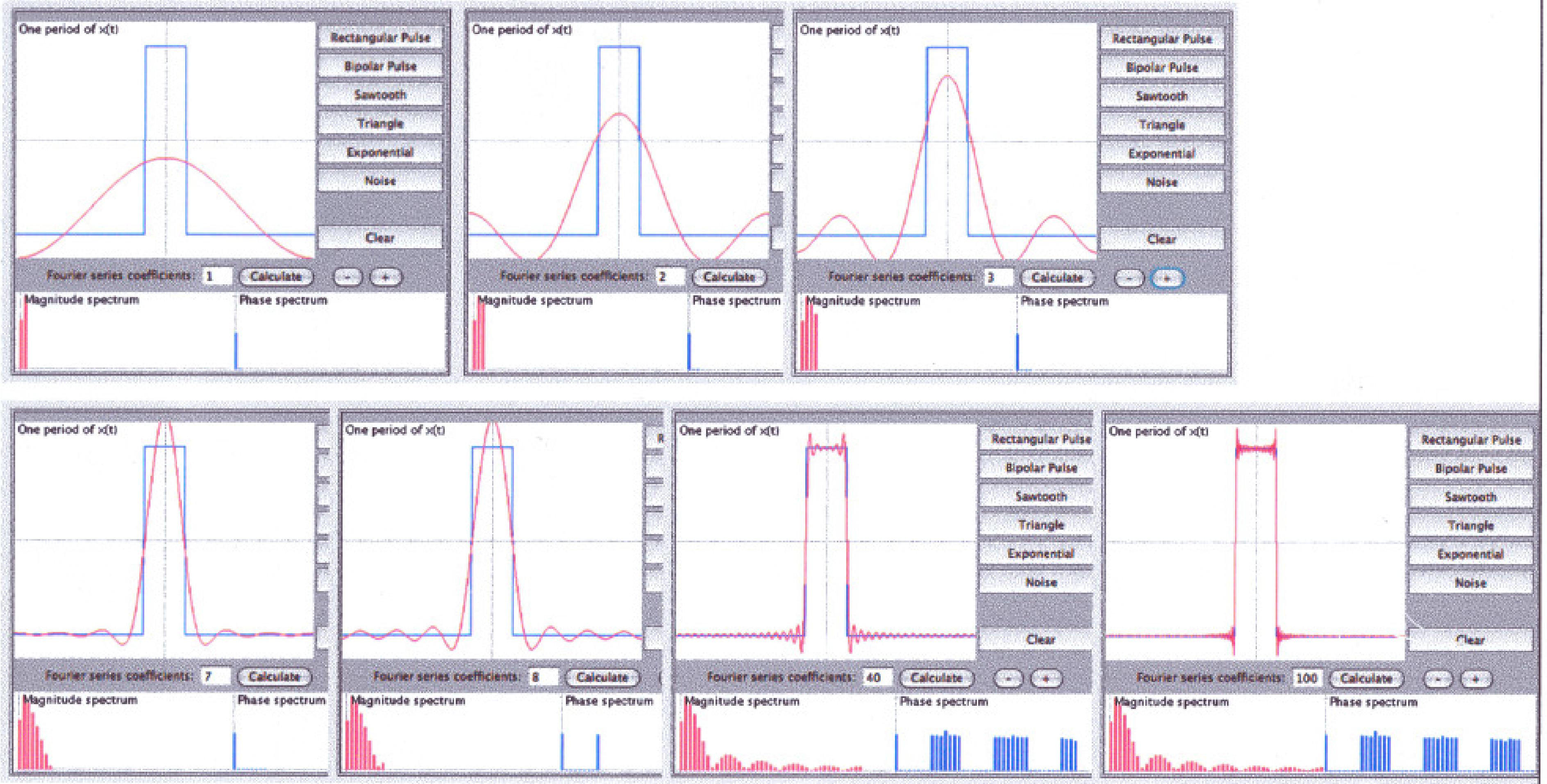
where the fundamental frequency w_o is $2p /T$ rad/sec, the amplitude coefficients $a_1, \ldots, a_N$ are non-negative, and the radian phase angles satisfy 0 £ $q_1 , \ldots, q_N < 2p$. To explore the Fourier series approximation, select a labeled signal, use the mouse to sketch one period of a signal, or use the mouse to modify a selected signal. Specify the number of harmonics, N, and click "Calculate." The approximation will be shown in red. In addition, the magnitude spectrum (a plot of a_n vs. n) and phase spectrum (a plot of q_n vs. n) are shown. (If the dc-component is negative, $a_0 < 0$, then $|a_0|$ is shown in the magnitude spectrum and an angle of p radians is shown in the phase spectrum.) To see a table of the coefficients, click "Table."



Suggested Exercises:

1. *Sketch a signal that has a large fundamental frequency component, but small small dc-component and small higher harmonics.*

2. *Sketch a signal that has large dc and fundamental frequency components, but small higher harmonics.*



One period of x(t)
Rectangular Pulse
Bipolar Pulse
Sawtooth
Triangle
Exponential
Noise
Clear
Fourier series coefficients: 1
Calculate
Magnitude spectrum
Phase spectrum
Fourier series coefficients: 2
Fourier series coefficients: 3
Fourier series coefficients: 7
Fourier series coefficients: 8
Fourier series coefficients: 40
Fourier series coefficients: 100

Another thing about Uncertainty...

notion of "conjugate variables"

$x \leftrightarrow p_x$

$y \leftrightarrow p_y$

$z \leftrightarrow p_z$

$t \leftrightarrow E$

all have an uncertainty relation

$$\Delta x p_x \geq \hbar/2$$

$$\Delta t \, \Delta E \geq \hbar/2$$

etc.

remember in Relativity...

$$s^2 = x^2 + y^2 + z^2 - (ct)^2$$

$\updownarrow$ same form involving conjugate variables

$$m^2c^4 = p_x^2c^2 + p_y^2c^2 + p_z^2c^2 - E^2$$

DEEP, man.

THINGS YOU'RE USED TO KNOWING

YOU CAN'T

Think about "classical" dynamics & kinematics —

knowing $\vec{x}$ & $\vec{p}$ and forces you can predict the position of an object at any future time... where it is & how fast it's going.

not for electrons... or any such tiny object.

Can't know the initial conditions!

→ can't know the TRAJECTORY with precision

Yet, quantum mechanics is routinely precise to 8–12 decimal places... that's a story.

You want DEEP?

I got DEEP.

For light... the intensity is directly related to $|\vec{E}|^2$... electric field is what "waves"

For an electron (neutron, proton, nucleus, quark, etc)

what "waves"?

1. (total for problem: 13 pts) There is a sub-nuclear particle called a "kaon," K^0. (The "0" indicates that it is electrically neutral...it has charged partners as well.) It is unstable and decays most of the time into two neutral pions, π^0. If the kaon decays at rest:

a. (2 pts) How does the momentum of one pion compare to the other?

P_π $\quad\quad$ P_π

$\longleftarrow \; \bullet \; \longrightarrow$

A

B

b. (1 pt) Draw a "before" and an "after" picture from the kaon's rest frame.

$E_B = E_A$

$E_K = E_\pi$'s

$K_\pi = 114$ MeV

$m_K c^2 = 2 m_\pi c^2 + 2 K_\pi$

c. (5 pts) The rest energy of the kaon is 498 MeV and the rest energy of the pion is 135 MeV. What is the kinetic energy, in MeV, of one of the pions?

d. (5 pts) What is the velocity of that pion as a fraction of the speed of light...ie., what is β_π ?

$E = m_\pi c^2 + K = m \gamma c^2$

$\vdots$

$\beta = 0.84$

2. (3 pts) What was the experimental contradiction that the Michelson-Morley experiment suggested?

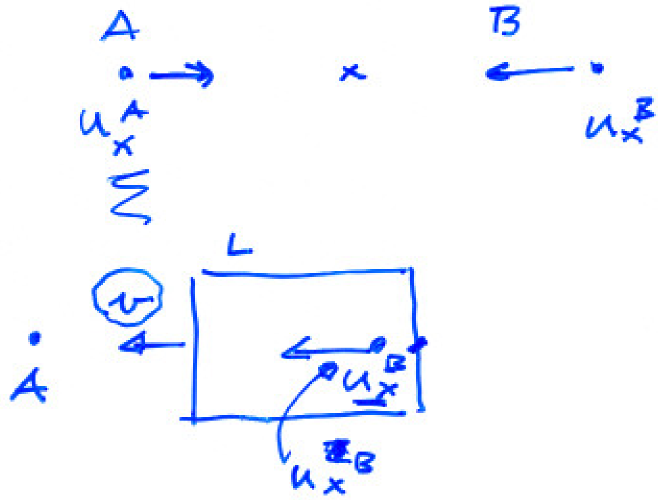
3. (3 pts) A line of charges at rest create a static electric field in the line's rest frame. If I am moving with constant velocity parallel to that line of charge, in words, generally what field configuration might I observe?

4. (3 pts) Two events occur in an inertial system K at the same time but 4 km apart. What is the time difference measured in a system K' moving between these two events when the distance of separation of the events is measured to be 5 km?

$$\underline{\underline{\Delta x}}^2 - (\overset{0}{\cancel{c\Delta t}})^2 = \underline{\underline{\Delta x'}}^2 - (c\Delta t')^2$$

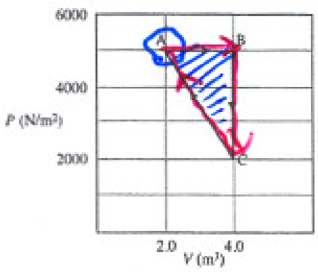
$$\Delta t' = 10^{-5}\ \mathrm{s}$$

5. (5 pts) A proton and an antiproton are moving toward each other in a head-on collision. If each has a speed of $0.8c$ with respect to the collision point, how fast are they moving with respect to one another? Express your answer as a fraction of c, ie. calculate β.



$$u_x^B = \frac{u_x'^B + v}{1 + \frac{v\, u_x^B}{c^2}}$$

$u_x^B = 0.98c$



6. (total for problem: 12 pts) One mole of an ideal gas is taken through the cyclic process ABCA as shown in the figure.

a. (3 pts) What is the temperature at point a?

$$T_A = \frac{P_A V_A}{nR} = 1203 \text{ K}$$

b. (3 pts) What is the work done by the gas in the cycle?

$W_{net} = 3000$ J

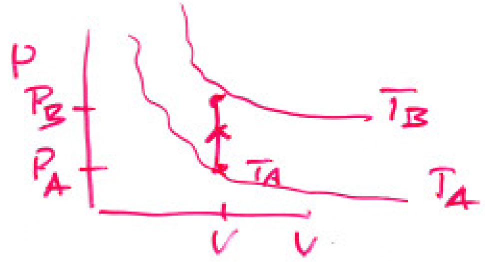
c. (3pts) What is the internal energy change in the cycle?

ϕ

d. (3 pts) What is the net amount of heat added to the gas during a cycle?

$$\Delta Q = W + \cancel{\Delta U}$$
$$= 3000 \text{ J}$$

7. (total for problem: 11 pts) A monotonic, ideal gas undergoes an isochoric (at constant volume) transition from lower pressure P_A to higher pressure, P_B.
a. (2 pts) Draw the P-V diagram for the process.



b. (2 pts) If the initial temperature is T_A and the final temperature is T_B, which is the higher temperature?

c. (4 pts) In terms of R, and temperatures, what is the expression for the change of entropy?

$$\Delta S = \frac{\Delta Q}{T} \qquad C_v = \left(\frac{dQ}{dT}\right)_v = T\left(\frac{dS}{dT}\right)_v$$

$$T\,dS = dQ \,.$$

$$\Delta S = \int dS = \int C_v \frac{dT}{T}$$

$$= C_v \ln T_B/T_A = 3/2\, R \ln T_B/T_A$$

d. (3 pts) Does the entropy increase or decrease?