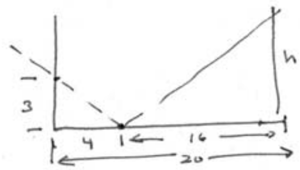
FINAL EXAM

Name:____

Student ID:________________________ Date:____________________

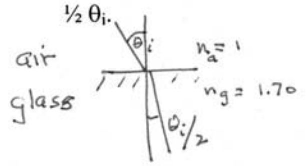
Show all work on these pages and circle your answers.

1. [10] You find yourself in a dark chamber deep beneath a pyramid. Once a year, a beam of sunlight enters the chamber via a hole in a wall located 3.0 m above the floor. The beam reflects from a gold mirror embedded in the floor 4.0 m from the wall and impinges on a large diamond in the crown of a statue of the deity Z'party which stands 20 m from the wall. What is the statue's height?



$\frac{3}{4} = \frac{h}{16}$ $\boxed{h = 12\text{ m}}$

2. [15] Light is incident from the air side of a planar air-glass (n_g= 1.70) interface at an angle θ_i. Find the incident angle (in degrees) such that the transmitted angle is ½ θ_i.



$n_a \sin\theta_i = n_g \sin(\theta_i/2)$

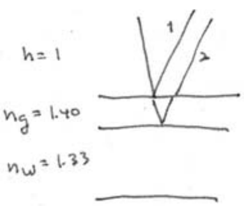
$\sin\theta_i = 1.70 \sin(\theta_i/2)$

$2\sin\frac{\theta_i}{2}\cos\frac{\theta_i}{2} = 1.70\sin\frac{\theta_i}{2}$

$\cos\frac{\theta_i}{2} = 0.85$; $\boxed{\theta_i = 31.8\times 2 = 63.6^\circ}$

3. [15]At noon on a bright sunny day, you look down to find yourself standing in a pool of water that looks uniformly green. You realize that some gasoline (n_g= 1.40) has leaked from your car's tank onto the water (n_w= 1.33). What is the minimum thickness of the gasoline film on the water?

λ_{green} ± 30% ok



let $\theta_i \to 0$

OPL difference $\equiv \delta = k_2\delta r_2 - k_1\delta r_1$

$\delta = \frac{2\pi}{\lambda_0} n_g \cdot 2t - \pi = 2\pi m$ $m = 0, 1, 2 \ldots$

$\delta_{min} \Rightarrow m = 0$

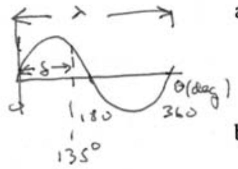
$\lambda_0 = 4 n_g t$

or $t = \frac{\lambda_0}{4n_g} = \frac{530\text{nm}}{5.6} = 95\text{nm}$

I will accept 95nm ± 30% !

4. [15]A light wave of frequency 5 x 10^{14} Hz propagates in vacuum.

$\lambda = \frac{c}{f} = \frac{3\times10^8\, ms^{-1}}{5\times10^{14}\, Hz} = 600nm$



a. What is the shortest distance (along the wave) that corresponds to a phase difference of 135 degrees?

Since a full cycle is 360° corresponding to 1 λ
$135/360 \times 600nm = 225nm$

More formally, $\delta = \frac{3\pi}{4} = kd = \frac{3\pi}{\lambda} d$ rad ; $d = \frac{3}{8}\lambda = \frac{3}{8}\times 600nm = 225nm$

b. What is the total phase shift in radians that occurs (at a point in space) over a time period of 10^{-10} seconds?

$\delta = \omega\tau = 2\pi f \tau$; $\tau = 10^{-10}s$

$\delta = 2\pi \cdot 5\times10^{14} \times 10^{-10} = \pi \cdot 10^5$ rad

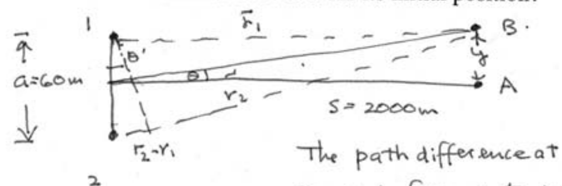
c. The electric field associated with the wave is expressed as $\vec{E} = E_0 \hat{y} \exp[i(\omega t - kx)]$. What is the magnitude of k and ω ? Which way does the wave propagate, in the +x, -x, +y, or −y direction (circle one)?

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{6\times10^{-7} m} = \frac{\pi}{3}\times10^{+7} m^{-1}$; $\omega = 2\pi f = 2\pi\cdot 5\times10^{14} Hz = \pi\cdot 10^{15} s^{-1}$

$\vec{E} \perp$ to propagation direction (transverse wave) which is +x.

Consider $|E|_{max} \Rightarrow (\omega t - kx) = 0$; As t increases, x must increase to maintain ( ) = 0.

5. [20] Two radio antennas that emit 10 MHz waves in phase with one another are separated by 60 m along a north-south line. A radio receiver, placed 2 km to the east and equidistant from both antennas, picks up a strong signal. When the receiver is moved north, the signal fades, then becomes strong again. How far has the receiver moved from its initial position?



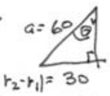
Find y, given that adjacent interference maxima occur at A + B.

The path difference at B is one wavelength $|r_2 - r_1| = 1\lambda$

Draw ⊥ from 1 to path r_2 ; the angle θ' is given by

$|r_2 - r_1| = a \sin\theta$. Also, $\tan\theta = \frac{y}{s}$

Since $\lambda = \frac{c}{f} = \frac{3\times10^8 ms^{-1}}{10^7 Hz} = 30m$, the Δ is



$\sin\theta' = \frac{1}{2}$, or $\theta' = 30°$

Since $\theta \simeq \theta'$, $y = s\tan\theta = 2000m \times 0.58 = 1160m$

In Young's expt, we usually assume that θ is very small i.e. $a \gg \lambda$, so that $\sin\theta \simeq \theta$, or $|r_2 - r_1| = a\theta$

In small angle limit $\lambda = a\theta$; $y = s\theta \Rightarrow y = \frac{s\lambda}{a}$

$= \frac{2000m \cdot 30m}{60m}$

or $y = 1000m$

I deducted only 2pts for this answer.

6. [25] A triplet lens system consists of a series of three thin lenses in close contact. The outer lenses, L1 and L3, are a plano-convex pair, whereas the center lens is bi-concave. All spherical surfaces have the same radius of curvature R = 100 cm. Through a manufacturing error, the lenses have different refractive indices: L1 with n_1=1.5, L2 with n_2=1.1n_1, and L3 with n_3=1.2n_1.

$$\frac{1}{f_{TOT}} = \sum_i \frac{1}{f_i} \; ; \; \frac{1}{f_i} = (n_i - 1)\left[\frac{1}{R_{i1}} - \frac{1}{R_{i2}}\right]$$

a. [15]Find the focal length of the lens assembly.

$i=1 \quad n_1 = n_1 = 1.5 \quad \frac{1}{R_{11}} = 0, \frac{1}{R_{12}} = -\frac{1}{R}, \frac{1}{f_1} = \frac{n_1 - 1}{R} = \frac{0.5}{R}, f = 200\,cm$

$i=2 \quad n_2 = 1.1n_1 = 1.65 \quad \frac{1}{R_{21}} = -\frac{1}{R}, \frac{1}{R_{22}} = \frac{1}{R}, \frac{1}{f_2} = (1.1n_1 - 1)\left(-\frac{2}{R}\right) = -\frac{2.2n_1}{R} + \frac{2}{R} = -\frac{1.3}{R} = -\frac{1}{76.9\,cm}$

$i=3 \quad n_3 = 1.2n_1 = 1.8 \quad \frac{1}{R_{31}} = \frac{1}{R}; \frac{1}{R_{32}} = 0, \quad \frac{1}{f_3} = \frac{n_3 - 1}{R} = \frac{1.2n_1}{R} - \frac{1}{R} = \frac{0.8}{R} = \frac{1}{125\,cm}$

$$\frac{1}{f_{TOT}} = \frac{n_1}{R} - \frac{1}{R} - \frac{2.2n_1}{R} + \frac{2}{R} + \frac{1.2n_1}{R} - \frac{1}{R} = \frac{n_1}{R}(1 - 2.2 + 1.2) = 0$$

b. [5] If an object is placed 100 cm in front of L1, where does the image appear?

$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} = 0 \; ; \; s_o = -s_i \; ; \; s_i = -100\,cm$

The middle (negative) lens compensates for the 2 positive lenses.

c. [5] If the object's vertical height is 3 cm, what is the height of the image?

$m_T = -\frac{s_i}{s_o} = +1 \; ; \; h' = 3\,cm$ image & object coincide

7. [10 extra credit] Consider two thin lenses in contact. Using the Gaussian lens formula, show algebraically that the focal length of the lens pair f is given by:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}.$$

Gaussian formula

$$\frac{1}{f_1} = \frac{1}{s_{o1}} + \frac{1}{s_{i1}} \; ; \; \frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}}$$

Since the inter lens spacing = 0 and the object for L2 is the image from L1

$s_{o2} = -s_{i1}$

Addenda:

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = 1 - 2\sin^2\alpha$

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{s_{o1}} + \underbrace{\frac{1}{s_{i1}} + \frac{1}{s_{o2}}}_{0} + \frac{1}{s_{i2}} = \frac{1}{s_{o1}} + \frac{1}{s_{i2}} = \frac{1}{f}$$

QED