PHY481: Electromagnetism

Current & Resistance

Current and current densities

No charge can accumulate in a uniform wire carrying current. Charge going in = charge going out.

$\rho_c(\mathbf{x}) = \text{charge density}$ $\qquad \dfrac{\partial \rho_c}{\partial t} = 0$

Beware: ρ is also used for resistivity of a material.

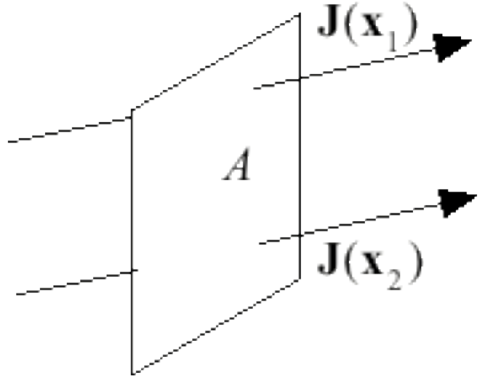
Volume current density $\mathbf{J}(\mathbf{x}) = n_V q \mathbf{v}(\mathbf{x})$ (3 D)

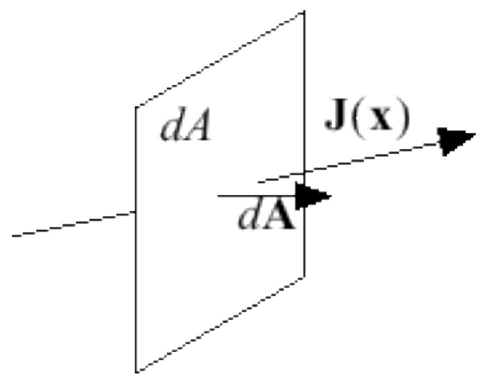
$\mathbf{K}(\mathbf{x}) = n_S q \mathbf{v}(\mathbf{x})$ (2 D)

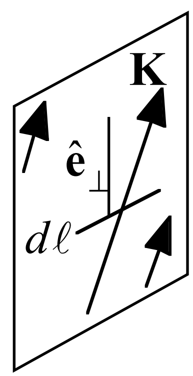
Current density **J** is position dependent

Local current density **J** for a differential area element

Surface current **K**







Current is charge through A per unit time

$I = dQ/dt$ (1 D)

Current through *dA*

$dI = \mathbf{J}(\mathbf{x}) \cdot d\mathbf{A}$ (1 D)

Current crossing $d\ell$

$dI = \mathbf{K} \cdot \hat{\mathbf{e}}_\perp d\ell$ (1 D)

Continuity equation -- conservation of charge

Continuity equation: $$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_c}{\partial t}$$

Remember that ρ_c and **J** are functions of 3D position

$$\rho_c(\mathbf{x}) \;\; \& \;\; \mathbf{J}(\mathbf{x})$$

Integrate over volume V, and use Gauss's theorem

$$\int_V \nabla \cdot \mathbf{J}\, d^3x = -\int_V \frac{\partial \rho_c}{\partial t}\, d^3x$$

Rate of change of charge inside of V

Continuity equation, integral form.

Flux of **J** through surface S

$$\int_S \mathbf{J} \cdot d\mathbf{A} = -\frac{d}{dt}\int_V \rho_c d^3x$$

Rate of change of charge inside of V

Boundary condition on **J** at a surface

Discontinuity in normal component of **J** at a surface

$$J_{2n} - J_{1n} = -\frac{\partial \sigma_c}{\partial t}$$

Rate of change of surface charge density

$\frac{\partial \rho_c}{\partial t} = 0$ and $\frac{\partial \sigma_c}{\partial t} = 0$ with constant currents

Reminiscent of $$E_{2n} - E_{1n} = \frac{\sigma_c}{\varepsilon_0}$$

Good conductors & poor conductors (resistors)

Macroscopic | Microscopic - Local forms of Ohm's law

Resistance

$$I = \frac{1}{R}V$$

Resistivity ρ

$$\mathbf{J}(\mathbf{x}) = \frac{1}{\rho_R}\mathbf{E}(\mathbf{x})$$

Conductivity σ

$$\mathbf{J}(\mathbf{x}) = \sigma_R\,\mathbf{E}(\mathbf{x})$$

$$\sigma_R = \frac{1}{\rho_R}$$

ρ and σ are an intrinsic property of a material

"Wire" with area A, and resistivity ρ_R

$$R = \int dR = \frac{1}{A}\int_0^z \rho_R(z')dz'$$

With constant resistivity ρ_R

$$R = \frac{\rho_R}{A}z$$

Resistance linear in z (length) and ρ .

What is still true about **E** ?

$\mathbf{E} = -\nabla V$ field <--> potential relationship, e.g., $\mathbf{E} = E_z\hat{\mathbf{k}}; \quad V = -E_z z$

$\nabla \times \mathbf{E} = 0$ violation requires changing magnetic fields

In good conductors, e.g., Cu, Ag, etc., the charge density is negligible

In resistors carrying constant current

$$\nabla \cdot \mathbf{E} = \frac{\rho_c}{\varepsilon_0} \Rightarrow \frac{\rho_{free}}{\varepsilon}$$

Charge density doesn't change $\frac{\partial \rho_c}{\partial t} = 0$ but $\rho_c \neq 0$

must specify "free charge density" and permittivity, ε .

Free charge and resistivity

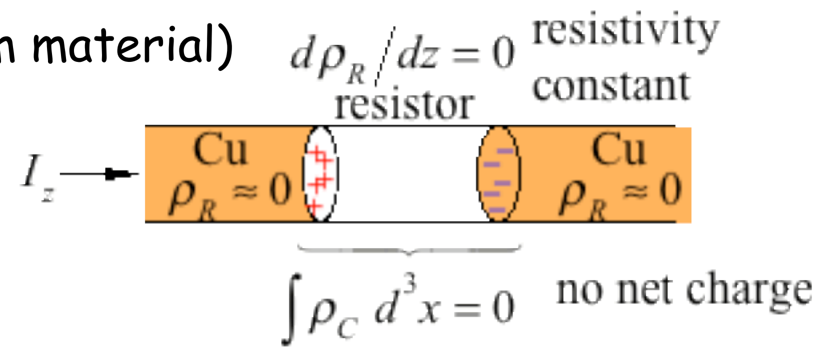
What is free charge, ρ_{free} ?

Charge carriers move w.r.t. atomic background, but resistance causes charge to (instantaneously) collect locally and then remain constant.

$\rho_{free} = 0$ with *constant* resistivity (uniform material)

No free charge in resistor volume

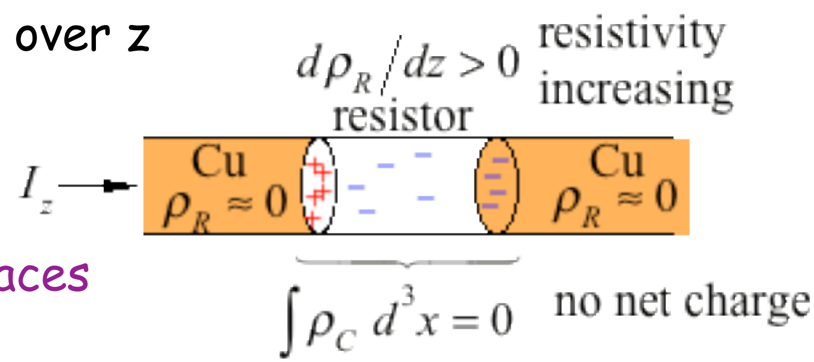
Free charge on up- & down-stream surfaces, due to resistivity discontinuity



$\rho_{free} = \frac{\varepsilon I}{A}\left(\frac{d\rho_R}{dz}\right)$ with changing resistivity over z

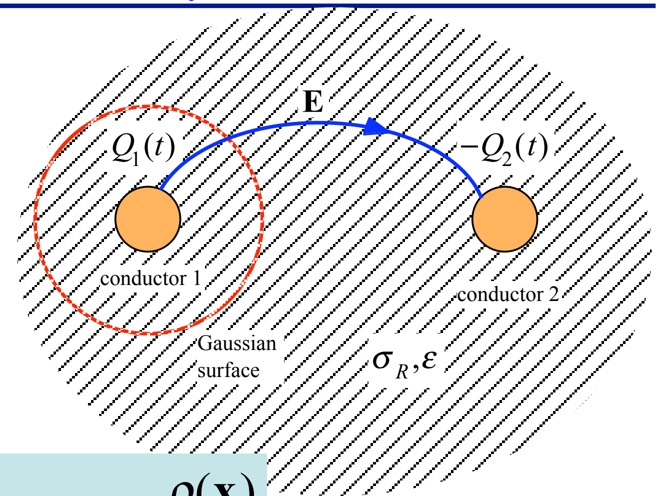
Free charge in resistor volume, and

free charge on up- & down-stream surfaces



Voltage and charge decay

Two conductors embedded in a resistive matrix
Charge each with a battery and disconnect.



Relate Total current I to material

Total current: $I = \int_S \mathbf{J}(\mathbf{x}) \cdot \hat{\mathbf{n}}\, dA$ $\quad \mathbf{J}(x) = \sigma_R \mathbf{E}(\mathbf{x})$

Gauss's Law: $I = \sigma_R \int_S \mathbf{E}(\mathbf{x}) \cdot \hat{\mathbf{n}}\, dA I = \sigma_R \int_V \nabla \cdot \mathbf{E}(\mathbf{x}) d^3x$ $\quad \nabla \cdot \mathbf{E}(\mathbf{x}) = \frac{\rho(\mathbf{x})}{\varepsilon}$

$I = \frac{\sigma_R}{\varepsilon} \int_V \rho(\mathbf{x}) d^3x = \frac{\sigma_R}{\varepsilon} Q$ $\quad Q = CV$ $\quad \frac{V}{R} = \frac{\sigma_R}{\varepsilon} CV \longrightarrow RC = \frac{\varepsilon}{\sigma_R}$

constants with
dimensions of time

$\frac{dQ(t)}{dt} = \frac{\sigma_R}{\varepsilon} Q(t) = \frac{1}{RC} Q(t)$ $\quad Q_1(t) = Q_0 e^{-t/RC}$

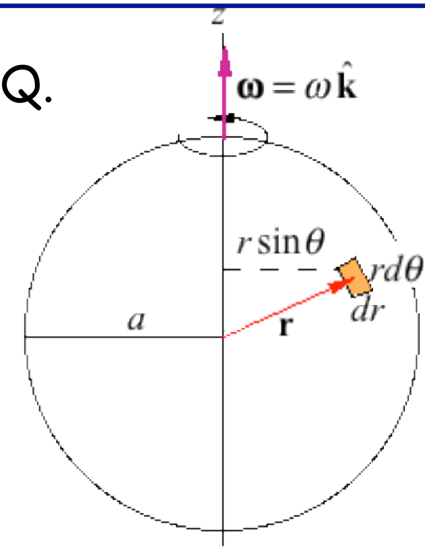
Exponential decay
of the charge

HW

7.3 Solid sphere radius a, uniformly distributed Q.

Constant angular velocity. Calculate **J**(**x**) .

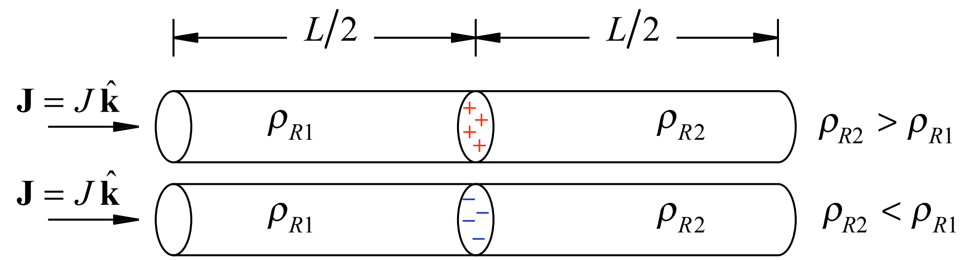
$$\int_S \mathbf{J}(r)\cdot\hat{\mathbf{n}}\,dA = \frac{d}{dt}\int_V \rho_c d^3x$$



$$d\phi = \omega dt$$

7.11 Find surface charge density at discontinuity in resistivity.

$$E_{2n} - E_{1n} = \frac{\sigma_c}{\varepsilon_0}$$



HW

7.9 Cylinders length L , resistive media between radius a and $3a$. What is the resistance? What is the charge density on the interface at $2a$?

Use general solution for cylidrical symmetry twice

$$V_1(r) = A\ln r + B$$

$$V_2(r) = C\ln r + D$$