
PHY481: Electromagnetism

Homework 5

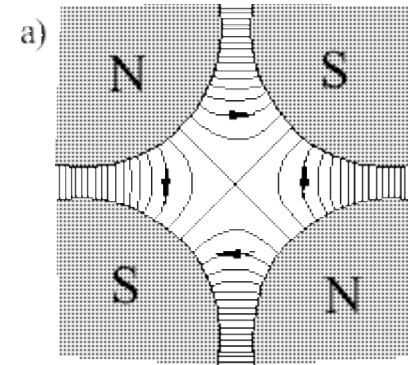
What do I need to study for the final exam?

HW 8

Problem 8.4

Positive particles headed out of slide, on the left or right, are **bent inward (focused)**.

Positive particles headed out of slide, on the top or bottom, are **bent outward (defocused)**.



$$\mathbf{F} = q\mathbf{v}_0 \times \mathbf{B}; \quad \mathbf{B} = b(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$$

$$\mathbf{v}_0 = v_0\hat{\mathbf{k}} = v_3\hat{\mathbf{k}}$$

Newton's 3rd law: $m\ddot{\mathbf{x}} = \mathbf{F}; \quad m\ddot{x}_i = q\epsilon_{i3k}v_3B_k$

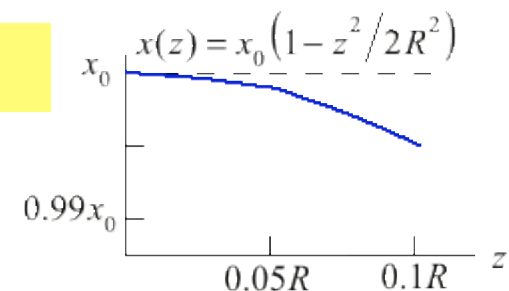
$$\ddot{x} = -\frac{q}{m}v_0B_y = -\frac{qv_0b}{m}x$$

Solution: $x(t) = C\cos(\omega t + \phi); \quad \omega^2 = qv_0b/m$

Boundary conditions: $x(0) = x_0 = C; \quad \dot{x}(0) = 0 = -\omega C\sin\phi \rightarrow \phi = 0$

With $\omega t \ll 1$ $t \approx z/v_0$ $x(t) = C\cos(\omega t) \approx C(1 - \omega^2 t^2/2)$

Trajectory: $x(z) \approx x_0(1 - z^2/2R^2)$ $R = \sqrt{mv_0/qb}$



HW 8 (cont'd)

Problem 8.10

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{x}') \times \hat{\mathbf{r}}}{r^2} d^3x' = \frac{\mu_0 I}{4\pi} \int \frac{d\boldsymbol{\ell} \times \hat{\mathbf{r}}}{r^2} \quad d\ell = a d\phi \hat{\boldsymbol{\phi}}$$

$$= \frac{\mu_0 I a}{4\pi (a^2 + z^2)} \int d\phi \hat{\boldsymbol{\phi}} \times \hat{\mathbf{r}}$$

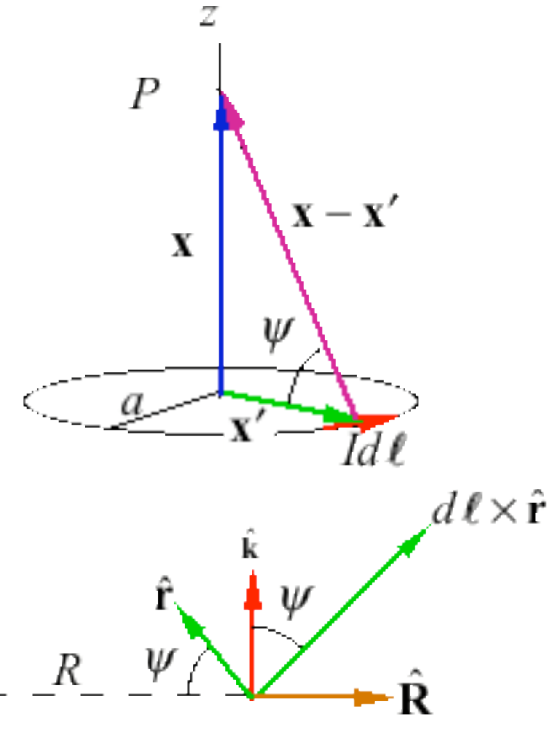
$$= \frac{\mu_0 I a}{4\pi (a^2 + z^2)} \int d\phi (\hat{\mathbf{k}} \cos \psi + \hat{\mathbf{R}} \sin \psi)$$

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}} \hat{\mathbf{k}}$$

$$\hat{\mathbf{R}} = \hat{\mathbf{i}} \cos \phi + \hat{\mathbf{j}} \sin \phi$$

$$\cos \psi = a / (a^2 + z^2)^{1/2}$$

$$\int_0^{2\pi} d\phi \hat{\mathbf{R}} = 0$$



HW 8 (cont'd)

Problem 8.10 (cont'd)

$$B_z(z) = \frac{\mu_0 I a^2}{2[a^2 + (z + s/2)^2]^{3/2}} + \frac{\mu_0 I a^2}{2[a^2 + (z - s/2)^2]^{3/2}}$$

$$\frac{dB_z}{dz} = -\frac{3\mu_0 I a^2}{2} \left\{ \frac{(z + s/2)}{[a^2 + (z + s/2)^2]^{5/2}} + \frac{(z - s/2)}{[a^2 + (z - s/2)^2]^{5/2}} \right\}$$

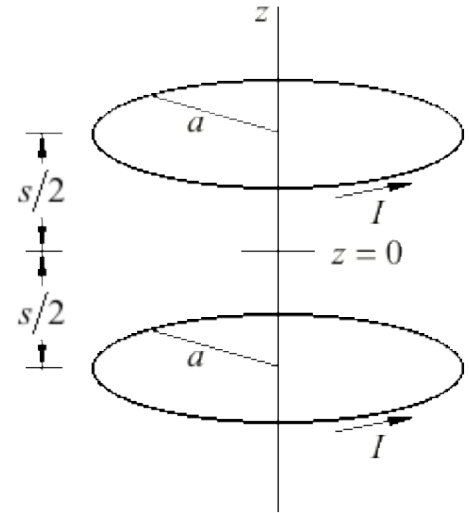
$$\frac{dB_z(0)}{dz} = 0$$

$$\frac{d^2 B_z}{dz^2} = -\frac{3\mu_0 I a^2}{2} \left\{ \frac{1}{[a^2 + (z + s/2)^2]^{5/2}} + \frac{1}{[a^2 + (z - s/2)^2]^{5/2}} - \frac{5(z + s/2)^2}{[a^2 + (z + s/2)^2]^{7/2}} - \frac{5(z - s/2)^2}{[a^2 + (z - s/2)^2]^{7/2}} \right\}$$

$$\frac{d^2 B_z(0)}{dz^2} = -3\mu_0 I a^2 \left\{ \frac{1}{[a^2 + (s/2)^2]^{5/2}} - \frac{5(s/2)^2}{[a^2 + (s/2)^2]^{7/2}} \right\} = 0$$

$$0 = 1 - \frac{5(s/2)^2}{[a^2 + (s/2)^2]}$$

$$s = a$$



HW 8 (cont'd)

Problem 8.14

$$\mathbf{B} = axy\hat{\mathbf{i}} + by^2\hat{\mathbf{j}}$$

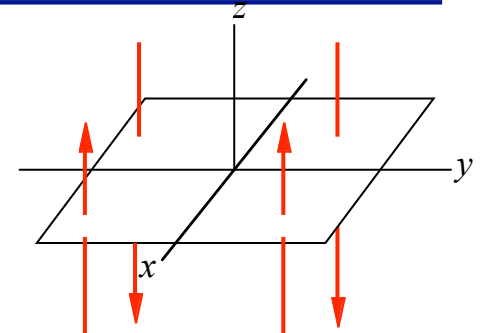
$$\nabla \cdot \mathbf{B} = 0 = ay + 2by$$

$$a = -2b; \quad \mathbf{B} = b(-2xy\hat{\mathbf{i}} + y^2\hat{\mathbf{j}})$$

$$\nabla \times \mathbf{B} = 2bx\hat{\mathbf{k}} = \mu_0 \mathbf{J}$$

$$\mathbf{J} = 2bx / \mu_0 \hat{\mathbf{k}}$$

y independent, changes sign $\pm x$
zero in yz-plane



Problem 8.18

$$\int \mathbf{B} \cdot d\ell = \mu_0 I_{encl} = \mu_0 \int \mathbf{J} \cdot d\mathbf{A}$$

$$2\pi r B_1 = \mu_0 \frac{I}{\pi a^2} \pi r^2$$

$$B_1 = \frac{\mu_0 I r}{2\pi a^2}$$

$$2\pi r B_2 = \mu_0 I$$

$$B_2 = \frac{\mu_0 I}{2\pi r}$$

$$2\pi r B_3 = \mu_0 \left(I - \frac{I(r^2 - b^2)}{(c^2 - b^2)} \right)$$

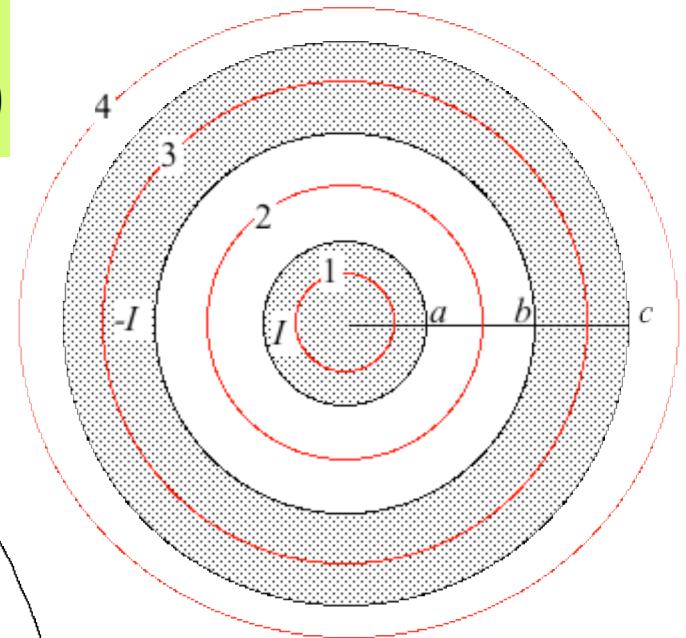
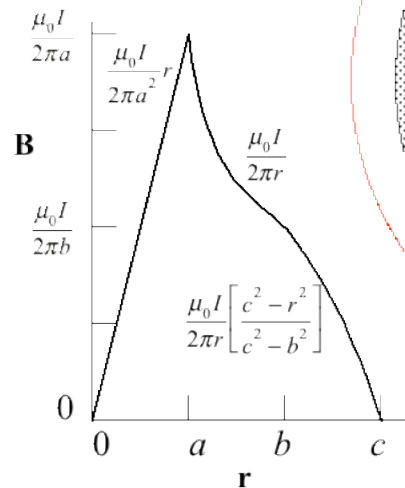
$$B_3 = \frac{\mu_0 I}{2\pi r} \left[\frac{c^2 - r^2}{c^2 - b^2} \right]$$

$$2\pi r B_4 = 0$$

$$B_4 = 0$$

$$J_1 = I / \pi a^2$$

$$J_3 = -I / \pi (c^2 - b^2)$$



HW 8 (cont'd)

Problem 8.22

$$\mathbf{B}_{in} = \nabla \times \mathbf{A}_{in} = \frac{\mu_0 \sigma a \omega}{3} \nabla \times (\hat{\mathbf{k}} \times r \hat{\mathbf{r}})$$

$$\mathbf{B}_{in} = \frac{2\mu_0 \sigma a \omega}{3} \hat{\mathbf{k}}$$

$$\begin{aligned} \hat{\mathbf{k}} \times r \hat{\mathbf{r}} &= r(\hat{\mathbf{r}} \cos \theta - \hat{\boldsymbol{\theta}} \sin \theta) \times \hat{\mathbf{r}} = r \sin \theta \hat{\boldsymbol{\phi}} \\ \nabla \times (r \sin \theta \hat{\boldsymbol{\phi}}) &= \frac{\hat{\mathbf{r}}}{r \sin \theta} \frac{\partial}{\partial \theta} (r \sin^2 \theta) - \frac{\hat{\boldsymbol{\theta}}}{r} \frac{\partial}{\partial r} (r^2 \sin \theta) \\ &= 2(\hat{\mathbf{r}} \cos \theta - \hat{\boldsymbol{\theta}} \sin \theta) = 2\hat{\mathbf{k}} \end{aligned}$$

$$\begin{aligned} \mathbf{B}_{out} &= \nabla \times \mathbf{A}_{out} = \frac{\mu_0 \sigma a^4 \omega}{3} \nabla \times \left(\hat{\mathbf{k}} \times \frac{\hat{\mathbf{r}}}{r^2} \right) \\ &= \frac{\mu_0 \sigma a^4 \omega}{3} \nabla \times \left(\hat{\mathbf{k}} \times \frac{\hat{\mathbf{r}}}{r^2} \right) = \frac{\mu_0 \sigma a^4 \omega}{3} \nabla \times \left(\frac{\sin \theta}{r^2} \hat{\boldsymbol{\phi}} \right) \end{aligned}$$

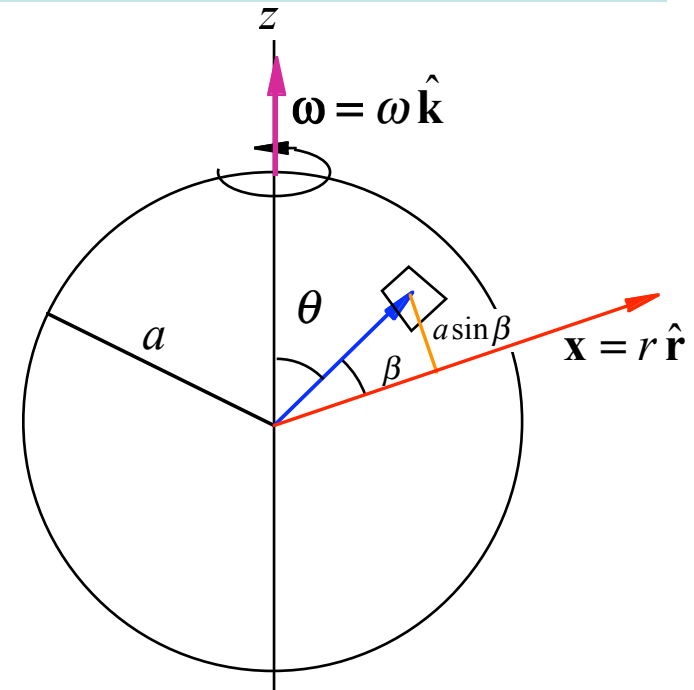
$$\mathbf{B}_{out} = \frac{\mu_0 \sigma a^4 \omega}{3r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}})$$

$$\mathbf{B}_{out} = \frac{\mu_0 \sigma a^4 \omega}{3r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}})$$

$$\mathbf{m} = m \hat{\mathbf{k}}$$

$$\mathbf{B}_{dipole} = \frac{\mu_0 m}{4\pi r^3} (2 \cos \theta \hat{\mathbf{r}} + \sin \theta \hat{\boldsymbol{\theta}})$$

$$\mathbf{m} = \frac{4\pi a^4 \sigma \omega}{3} \hat{\mathbf{k}}$$



HW 8 (cont'd)

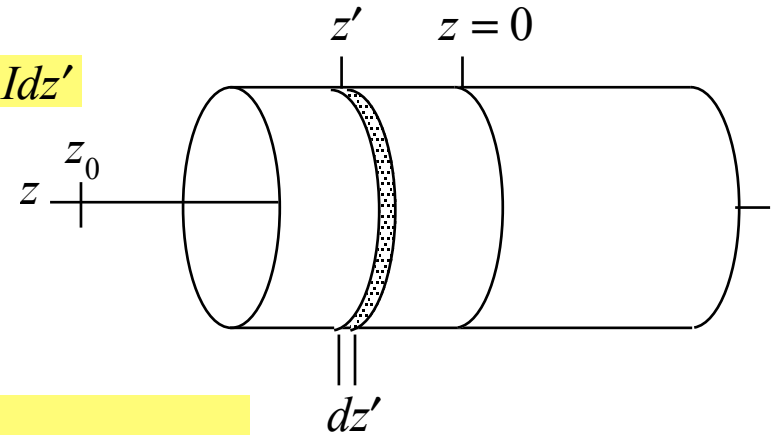
Problem 8.34

Using result of Problem 8.10 and $dI = nI dz'$

$$dB_z(0,0,z_0) = \frac{\mu_0 a^2 n I dz'}{2[a^2 + (z_0 - z')^2]^{3/2}}$$

$$\begin{aligned} B_z &= \frac{\mu_0 a^2 n I}{2} \int_{-\ell/2}^{\ell/2} \frac{dz'}{[a^2 + (z_0 - z')^2]^{3/2}} \quad x = z_0 - z', \quad dz' = -dx \\ &= \frac{\mu_0 a^2 n I}{2} \int_{z_0 - \ell/2}^{z_0 + \ell/2} \frac{dx}{[a^2 + x^2]^{3/2}} = \frac{\mu_0 a^2 n I}{2} \left[\frac{x}{a^2 \sqrt{a^2 + x^2}} \right]_{z_0 - \ell/2}^{z_0 + \ell/2} \end{aligned}$$

$$\mathbf{B} = \frac{\mu_0 a^2 n I \hat{\mathbf{k}}}{2} \left\{ \frac{\ell/2 - z_0}{[a^2 + (\ell/2 - z_0)^2]^{1/2}} + \frac{\ell/2 + z_0}{[a^2 + (\ell/2 + z_0)^2]^{1/2}} \right\}$$



HW 8 (cont'd)

Problem 8.37

$$\mathbf{A}(\mathbf{x}) = -\frac{A_0 a}{r} e^{-r^2/a^2} \hat{\phi}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = -\frac{2A_0}{a} e^{-r^2/a^2} \hat{\mathbf{k}}$$

$$\mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B} = -\frac{4A_0}{\mu_0 a^3} r e^{-r^2/a^2} \hat{\phi} \quad r < a$$

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{2A_0}{a^2} r e^{-r^2/a^2} \hat{\phi}$$

$$\mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B} = \frac{4A_0}{\mu_0 a^4} (a^2 - r^2) e^{-r^2/a^2} \hat{\mathbf{k}} \quad r > a$$

To plot

$$B_z = -\frac{2A_0}{a} e^{-x}, \quad x = r/a$$

$$J_\phi = -\frac{4A_0}{\mu_0 a^2} x e^{-x^2} \hat{\phi}$$

To plot

$$B_\phi = \frac{2A_0}{a} x e^{-x}$$

$$J_z = \frac{4A_0}{\mu_0 a^2} (1 - x^2) e^{-x^2} \hat{\mathbf{k}}$$

Final Exam

(Exam is on 12/12, 12:45-2:45, Rm 1415)

What do I need to study for the final exam?

Homework, Exams, and Text example type problems

- At this level in the study of E&M
 - Coordinate systems other than Cartesian cannot be avoided
 - Differential operators: gradient, divergence, curl, and Laplacian are part and parcel of E&M at this level
 - Expansions with electric and magnetic dipole (& occasionally higher) terms appear often in physical problems
 - Solving Laplace's equation is a useful skill
- Course failures are rare (unless there is evidence of giving up).
- Read the syllabus - missed HOMEWORK handed in before Exam.