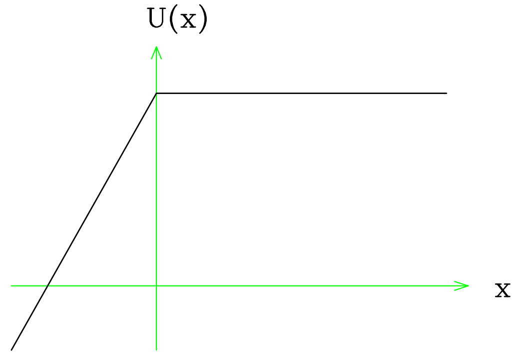
PHYSICS 321   EXAM 2   Mar 21, 2005   NAME ______________________________

1. [6 pts] A particle of mass $M = 1$ moves in one dimension in the potential $U(x)$, where

$$U(x) = \begin{cases} 3 & \text{if} \quad x > 0 \\ 3 + 2\,x & \text{if} \quad x < 0 \end{cases} .$$

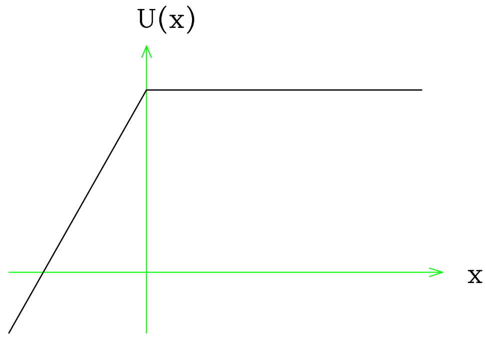


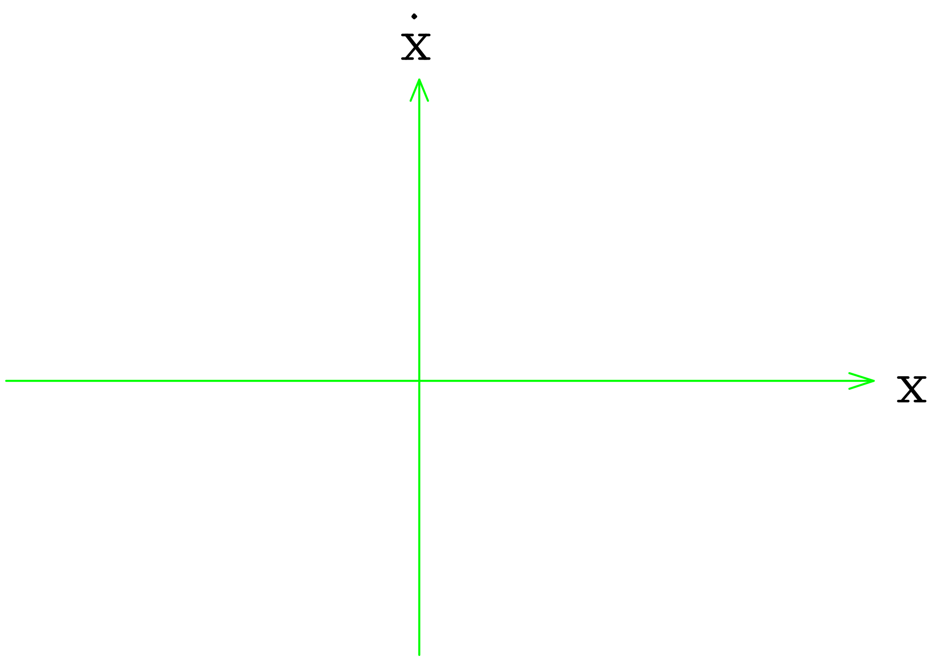
(Units have been chosen to keep things simple, so don't worry about the dimensions.) Solve for the motion $x(t)$ for the case that the total energy is $E = 2$. Make sure that your answer includes the appropriate number of arbitrary constants.

2. [6 pts] A particle of mass $M = 1$ moves in one dimension in the potential $U(x)$, where

$$U(x) = \begin{cases} 3 & \text{if} \quad x > 0 \\ 3 + 2x & \text{if} \quad x < 0 \end{cases} .$$

(This is the same $U(x)$ as in the preceeding problem.) Sketch the motion in phase space — i.e., in the $(x, \dot{x})$ plane — for several values of the total energy, using enough different values to show all of the different types of motion that are possible. Include arrows in your sketch to show the direction of the motion.

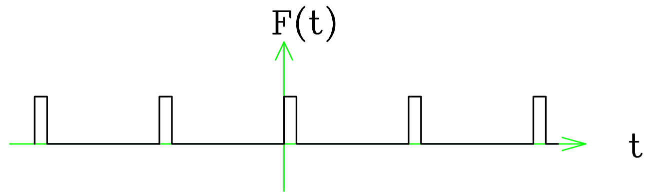




3. [6 pts] Find the most general solution to the equation $\ddot{x}+x = \sin(\omega t)$ where ω is a constant.

4. [6 pts] The function $F(t)$ is periodic with period $T = \frac{2\pi}{\omega}$, i.e., $F(t+T) = F(t)$ for all t. $F(t)$ is defined by

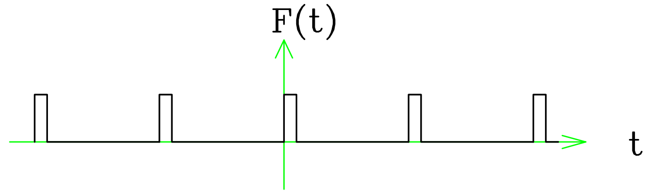
$$F(t) = \begin{cases} A & \text{if} \quad 0 < t < 0.1\,T \\ 0 & \text{if} \quad 0.1\,T < t < T \end{cases}$$



where A is a constant. Express $F(t)$ in the form of a Fourier series. (You may find either the exponential form or the sine + cosine form. As usual, I recommend the exponential form since it is easier.)

5. [6 pts] The function $F(t)$ is periodic with period $T = \frac{2\pi}{\omega}$, i.e., $F(t+T) = F(t)$ for all t, where $F(t)$ is defined by

$$F(t) = \begin{cases} A & \text{if} \quad 0 < t < 0.1\,T \\ 0 & \text{if} \quad 0.1\,T < t < T \end{cases}$$



where A is a constant. (This $F(t)$ is the same as in the preceeding problem.) What values of T will make the solution to the equation

$$\ddot{x} + 0.02\,\dot{x} + x = F(t)$$

oscillate strongly? Explain your answers, but no detailed calculations are necessary. You do not need to use your detailed answer to the previous question.