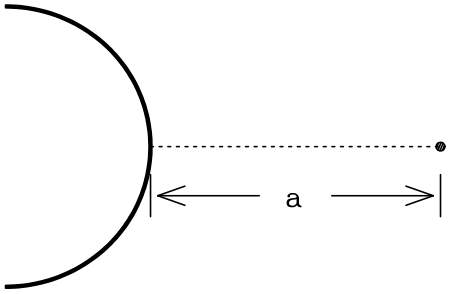
PHYSICS 321 EXAM 3 April 18, 2005 NAME _____

1. [5 pts] Imagine a peculiar cloud of cosmic dust whose mass distribution is spherically symmetric with a density (mass per unit volume) given by $\rho(r) = k\,r^2$ where r is the distance from the center and k is a constant. Find the gravitational force it produces on a "test mass" m at radius R which lies inside the dust cloud.

2. [5 pts] A particle of mass m moves in a circular orbit of radius R under the influence of a potential $U(r) = k\,r^3$ where $r = \sqrt{x^2 + y^2}$ and k is a positive constant. Find its kinetic energy. (Hint: you can use the Effective Potential, or you can find the force and use the methods of first year physics – your choice.)

3. [5 pts] Find the gravitational potential energy of a point mass m that is located a distance **a** from a half-circle of wire mass M as shown. The radius of the half-circle is R. You may leave your answer in the form of a well-defined integral—you do not need to evaluate the integral.



4. [5 pts] Derive the conditions under which $\langle \dot{S} \rangle = 0$, where $S = \vec{p} \cdot \vec{r}$. (We used this in connection with the Virial Theorem.)

5. [5 pts] The equation for an elliptical orbit can be written in the form

$$\frac{1}{r} = C_1 + C_2 \cos\theta \,.$$

Express this equation in terms of Cartesian (x and y) coordinates and use your result to derive the condition on C_1 and C_2 for the orbit to be elliptical.

6. [5 pts] A satellite of small mass m is in a circular orbit of radius R about a planet of large mass M. At a certain instant of time, a powerful rocket motor on the satellite is fired briefly, so as to increase the velocity without changing its direction. As a result, the satellite goes into an elliptical orbit as shown. Find the total energy (kinetic + potential) in the elliptical orbit in terms of G, m, M, and R.