PHYSICS 321 FINAL EXAM May 4, 2005 NAME ____________________

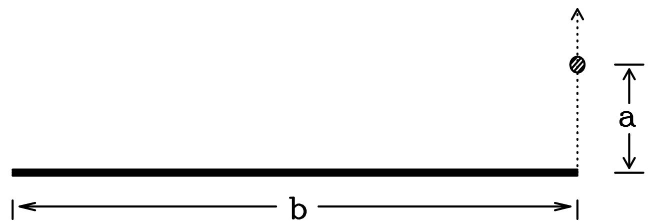
1. [5 pts] A particle of mass M moves in one dimension in the potential $V(x) = a + bx$ where a and b are constants. Find all possible motions $x(t)$.

2. [5 pts] The motion of a system with two degrees of freedom is described by the two coordinates x and y. The kinetic energy and potential energies are given by

$$T = \frac{1}{2}[\, \dot{x}^2 + \dot{x}\,\dot{y}\,\cos(x) + (1+t^2)\dot{y}^2\,] \qquad\qquad V = x^2 + t\,y^2$$

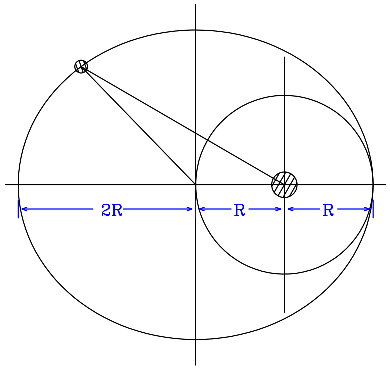
Write the Lagrangian equations of motion. (But don't try to solve them—someone might get hurt!)

3. [5 pts] A point mass **m** is located at a distance **a** from one end of a uniform wire of mass **M** and length **b** as shown. Find the gravitational potential energy of the mass. You may leave your answer in the form of a *well-defined* integral—you do not have to evaluate the integral.

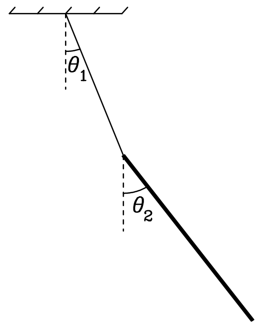


4. [5 pts] A uniform disk of mass M and radius R is rotated about its diameter—like a coin spinning on its edge. Calculate the moment of inertia.

5. [5 pts] A satellite of small mass m is in a circular orbit of radius R about a planet of large mass M. At a certain instant of time, a powerful rocket motor on the satellite is fired briefly, so as to increase the velocity without changing its direction. As a result, the satellite goes into an elliptical orbit as shown. Find the new kinetic energy in terms of G, m, M, and R. Hint: $E = \frac{1}{2}\, m\, \dot{r}^2 \,+\, \frac{\ell^2}{2\, m\, r^2} \,-\, \frac{G\, m\, M}{r}$.



6. A uniform stick of length b and mass M hangs from the ceiling by a massless flexible string of length a. Use the coordinates θ_1 and θ_2.



(a) [5 pts] Calculate the part of the kinetic energy due to the motion of the center of mass of the stick.

(b) [3 pts] Calculate the part of the kinetic energy due to the rotation of the stick about its center of mass. (Hint: The moment of inertia about the center of mass is $Mb^2/12$.)

(c) [3 pts] Calculate the gravitational potential energy.

7. [5 pts] The motion of a particle is given by $x = \cos(t)$ in units where $M = g = L = 1$, where M is the mass, g is the acceleration due to gravity, and L has the dimensions of angular momentum. (You can get the dimensions of L from $\vec{L} = \vec{r} \times \vec{p} = M\vec{r} \times \vec{v}$.) Write $x(t)$ with the units M, g, L properly restored.

8. [5 pts] Write the most general solution to the equation $\ddot{x} + x = \sin(3\,t)$.

9. [5 pts] The coordinates of a particle of mass M as a function of time are

$$\begin{aligned} x(t) &= e^t - 1 \\ y(t) &= t^2 + t \end{aligned}$$

Find the magnitude of the force.

10. [5 pts] The force on a particle of mass $M = 1$ is equal to $p - 1$, where p is the momentum. Find the position as a function of time (including any arbitrary constants that may be possible.)

Enjoy the summer and all that follows!