Physics 321 – Spring 2006

Homework #7, Due at beginning of class Wednesday Mar 15.

1. [8 pts] A "triangle wave" can be defined by $F(t) = 1 - 2\omega|t|/\pi$ for $-\pi/\omega < t < +\pi/\omega$, with $F(t)$ defined at all other values of the time t by the property of having period $2\pi/\omega$.

   (a) Find the Fourier series representation of the $F(t)$. Express your answer BOTH in exponential form and in the form of sines and/or cosines. But do the exponential form first (as usual, it's easier), and then get the sines+cosines form from that.

   (b) Solve the driven damped oscillator equation $\ddot{x} + 2\beta\dot{x} + x = F(t)$ in the form of an infinite series. (Note that for convenience, units of time have been chosen such that the natural frequency of the oscillator is equal to 1.) You will probably find the exponential form of your answer to part (a) more convenient.

   (c) Plot the solution $x(t)$ over a time interval of two periods: $0 < t < 4\pi/\omega$ for the case $\beta = 0.1$, with $\omega = 1/3$, 1, 2. (Make three separate plots—one for each choice of ω.)

   In doing this problem, I would prefer that you keep the entire infinite series in your answers. But it would be acceptable to keep only the frequencies up through $5\,\omega$. You can neglect the solutions to the homogeneous equation, which contain two arbitrary constants, because those "transient" effects go away like $e^{-\beta t}$ if you wait long enough.

2. [4 pts] Marion & Thornton, problem 4-3 (same in 4th edition). Draw the phase space diagram ($\dot{x}$ vs. x) right below the potential energy plot, as is done in Figure 4-5 in the book, so it is easy to see the correspondence between the two diagrams.

3. [4 pts] Marion & Thornton, problem 4-6 (same in 4th edition). Use conservation of energy to calculate $\dot{\theta}$ as a function of θ.

4. [4 pts] Marion & Thornton, problem 4-8 (same in 4th edition).

(Last updated 3/02/2006.)