The reddening curve:

Shape of reddening curve depends only on dust properties:

\[ \frac{A_v}{1.08 \tau_v} = 1.08 \int n ds \]

The color-color diagram:

Slope of reddening vector depends only on shape of extinction curve:

\[ \Delta(U - B) = \frac{A_u - A_B}{A_B - A_V} = \frac{\int n ds}{\int n ds} \]

But distance stars slide along that slope depend also on dust column density:

\[ \Delta(B - V) = A_B - A_V = (\alpha_B - \alpha_V) \int n ds \]
Star Counts

\( n_m(M, S, \Omega, r) \, dM = \text{number of stars per unit volume at distance } r, \text{ in solid angle } \Omega, \text{ in abs. Mag range } M, M+dM, \text{ with other attribute } S. \)

Relate to observable quantities:

- Total number of stars in abs mag range \( M, M+dM \) out to distance \( d \) (Integrated star count):

\[
N_m(M, S, \Omega, r) \, dM = \left[ \int_0^d n_M(M, S, \Omega, r) \, \Omega \, r \, dr \right] \, dM
\]

- Integrated star count to limiting apparent magnitude \( m \):

Use \( d = 10^{m_m - M + a + 5 \log \text{sys}} \) in \[24.3\] to find

\[
\overline{N}_m(M, S, \Omega, m) \, dM
\]

- Differential star count in apparent mag range \( m, m+dm \):

\[
\frac{dN_m}{dm} \, dm = \frac{dN_m}{dm} \, dm
\]

---


THE UNIVERSE AT FAINT MAGNITUDES. I. MODELS FOR THE GALAXY AND THE PREDICTED STAR COUNTS

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Usually see deVaucouleurs’ $r^{1/4}$ surface brightness law:

### Stellar Populations

- Abundances
- Kinematics
- Ages
- Pop I: Metal rich ($Z \sim 0.02$), disk, younger
  - Disk field stars (up to 10-12 Gyr old)
  - Open clusters
  - Gas
  - Star formation regions
- Pop II: Metal poor ($Z \sim 0.001$), halo, older
  - Globular clusters (12-15 Gyr)
  - Halo field stars
  - Bulge?? …but includes Super Metal Rich (SMR) stars.
- Abundance Determinations
  - Stellar spectroscopy
    - $[\text{Fe/H}]$, etc. $\log(N_{\text{Fe}}/N_{\text{H}}) - \log($solar$)$
    - Iron ejected by Sne Ia after about $10^8$ yrs.
  - Stellar colors
  - HII regions

### Table 24.1

<table>
<thead>
<tr>
<th>Disk</th>
<th>Neutral Gas</th>
<th>Thin Disk</th>
<th>Thick Disk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M \left(10^9 M_\odot\right)$</td>
<td>0.5</td>
<td>6</td>
<td>0.2 to 0.4</td>
</tr>
<tr>
<td>$L_B \left(10^8 L_\odot\right)$</td>
<td>—</td>
<td>1.8</td>
<td>0.02</td>
</tr>
<tr>
<td>$M/L_B (M_\odot/L_\odot)$</td>
<td>—</td>
<td>3</td>
<td>—</td>
</tr>
<tr>
<td>Radius (kpc)</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>Form $e^{a(r)}$</td>
<td>$e^{-a(r)}$</td>
<td>$e^{a(r)}$</td>
<td>$e^{-a(r)}$</td>
</tr>
<tr>
<td>Scale height (kpc)</td>
<td>5</td>
<td>16</td>
<td>35</td>
</tr>
<tr>
<td>$[\text{Fe/H}]$</td>
<td>$&gt; +0.1$</td>
<td>$-0.5$ to $+0.3$</td>
<td>$-2.2$ to $-0.5$</td>
</tr>
<tr>
<td>Age (Gyr)</td>
<td>$\lt 10$</td>
<td>8</td>
<td>$10^9$</td>
</tr>
</tbody>
</table>

### Spheroids

<table>
<thead>
<tr>
<th>Central Bulge</th>
<th>Stellar Halo</th>
<th>Dark-Matter Halo</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M \left(10^9 M_\odot\right)$</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>$L_B \left(10^8 L_\odot\right)$</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>$M/L_B (M_\odot/L_\odot)$</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Radius (kpc)</td>
<td>4</td>
<td>$&gt; 100$</td>
</tr>
<tr>
<td>Form</td>
<td>boxy with bar</td>
<td>$r^{-3.6}$</td>
</tr>
<tr>
<td>Scale height (kpc)</td>
<td>0.1 to 0.5</td>
<td>3</td>
</tr>
<tr>
<td>$\sigma_v$ (km sec$^{-1}$)</td>
<td>55 to 130</td>
<td>95</td>
</tr>
<tr>
<td>$[\text{Fe/H}]$</td>
<td>$-2$ to $-0.5$</td>
<td>$&lt; -5.4$ to $-0.5$</td>
</tr>
<tr>
<td>Age (Gyr)</td>
<td>$&lt; 0.2$ to 10</td>
<td>11 to 13</td>
</tr>
</tbody>
</table>

### Galaxy Model

$log_{10} \left[ \frac{\Sigma_e}{L/e} \right] = -35.307 \left[ \frac{\Sigma_e}{L/e} \right]^{1/3} - 1$

### Equation

$X, Y, Z = \text{mass fractions}$

$X \sim 0.73$

$Y \sim 0.25$

### Baade (1944)

<table>
<thead>
<tr>
<th>[Fe/H]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thin Disk</td>
</tr>
<tr>
<td>Thick Disk</td>
</tr>
<tr>
<td>Halo</td>
</tr>
<tr>
<td>Bulge</td>
</tr>
</tbody>
</table>
Gradual processes in Interiors of Stars
- H burning (4H → He)
- α process (C, O, Ne, Mg, Si, S…)
- s process
  - slow neutron capture, relative to beta-decay timescale

Supernovae
- e process
  - nuclear statistical equilibrium
- r process
  - rapid neutron capture

B²FH (1957)
Formation of the Chemical Elements
Burbidge, Burbidge, Fowler & Hoyle.
Reviews of Modern Physics, 29, 547.

Formation of the Chemical Elements
Burbidge, Burbidge, Fowler & Hoyle.
Reviews of Modern Physics, 29, 547.

Chemical Enrichment

Chemical Enrichment Models
- Simulate what is going on in a volume of space.
- Recipes for how many of which elements created by stars of different masses.
- Include lifetimes of stars, fraction of mass returned to ISM.
- More about this in a few weeks.
Measuring abundances from absorption lines
(see [9.5] for gory details)

- Lorentz profile
  - Natural profile of stationary absorber.
    - wings due to finite lifetime of excited state in QM model.
    - Or to “damping” in classical oscillator model

- Voigt profile
  - Lorentz profile convolved with Gaussian velocity distribution.
  - Line shape increases in funny way.

EQUIVALENT WIDTH

- Often, wavelength resolution and/or signal:noise too low to measure details of line profile.
- Can still measure fraction of continuum light that is absorbed
- then convert to column density of absorbing atoms.

\[
W_\lambda = \int \left[ 1 - \frac{I_\nu}{I_\nu(0)} \right] \, d\lambda = \frac{\lambda^2}{c} \int \left[ 1 - e^{-\tau_\nu} \right] \, d\nu
\]

since \( d\lambda = (\lambda^2/c) \, d\nu \)

- in units of Å
- same as width of square profile going to zero and having same \( W_\lambda \) as observed line.

Optical depth:
\[ \tau_\nu = \int \alpha_\lambda \, n \, ds \]

Column density:
\( \text{(atoms/cm}^2 \text{ along line of sight)} \)
\[ N = \int n \, ds \]
CONVERTING $W_\lambda$ TO COLUMN DENSITY OF ABSORBING ATOMS:

$$W_\lambda = \int \left[ 1 - \frac{1}{I_\lambda(0)} \right] \, d\lambda = \frac{\lambda^2}{c} \int \left[ 1 - e^{-\tau} \right] \, dv$$

CURVE OF GROWTH shows how $W_\lambda$ depends on $N$

- For small column density:

  $$W_\lambda \propto \lambda^2 \tau_{\nu}$$
  $$\frac{W_\lambda}{\lambda} \propto N \int f_{jk} \lambda$$

  where $j,k$ are lower, upper levels,
  $f_{jk}$ is oscillator strength = effective number of oscillators participating in transition.

- For intermediate column density:

  where $b = \sqrt{v_c^2 + v_{turbulent}^2}$:

  $$\frac{W_\lambda}{\lambda} \propto \left[ \ln \left( \frac{0.15 N b}{b} \right) \right]^{1/2}$$

- For large column density:

  $$\frac{W_\lambda}{\lambda} \propto (N f)^{1/2}$$

Sliding observed c.o.g. over theoretical c.o.g in both $x$ and $y \Rightarrow N$, $b$
Sliding observed c.o.g. over theoretical c.o.g in both x and y \( \Rightarrow \) N, b

\[
\frac{W_\lambda}{\lambda} \propto b \left[ \ln \left( \frac{0.15N\Omega_0}{b} \right) \right]^{1/2}
\]

Go directly to AST 304. Do not pass go. Do not collect $200.