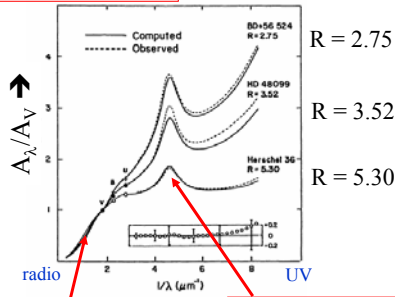


Particle size ~ 0.01 - 0.1 μm



~ 1/λ dependence in IR-optical.

2175 Å feature ⇒ graphite

$$I_\lambda = I_{0,\lambda} e^{-\tau_\lambda}$$

in magnitudes

$$\begin{aligned} A_\lambda &= -2.5 \log_{10}(I_\lambda / I_{0,\lambda}) \\ &= -2.5 \log_{10} e^{-\tau_\lambda} \\ &= 1.08 \tau_\lambda \end{aligned}$$

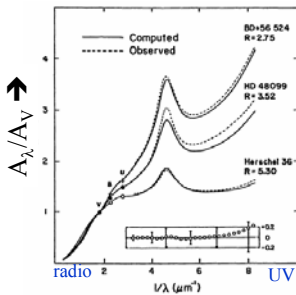
Determine  $\tau_\lambda$  from change in color

$$A_V = R_V \cdot E(B-V)$$

$$E(B-V) = (B-V) - (B-V)_0$$

$R_V \sim 3$  but different in different places.

### The reddening curve:



Shape of reddening curve depends only on dust properties:

$$\frac{A_\lambda}{A_V} = \frac{1.08 \tau_\lambda}{1.08 \tau_V} = \frac{1.08 \alpha_\lambda \int n ds}{1.08 \alpha_V \int n ds}$$

Optical depth:

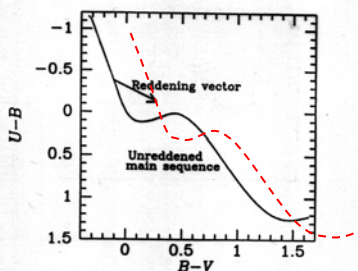
$$\tau_v = \int \alpha_v n ds$$

Column density:

(atoms/cm<sup>2</sup> along line of sight)

$$N = \int n ds$$

### The color-color diagram:



Slope of reddening vector depends only on shape of extinction curve:

$$\frac{\Delta(U-B)}{\Delta(B-V)} = \frac{A_U - A_B}{A_B - A_V} = \frac{\alpha_U - \alpha_B \int n ds}{\alpha_B - \alpha_V \int n ds}$$

But distance stars slide along that slope depend also on dust column density:

$$\Delta(B-V) = A_B - A_V = (\alpha_B - \alpha_V) \int n ds$$

# Star Counts

$n_M(M, S, \Omega, r) dM$  = number of stars per unit volume at distance  $r$ , in solid angle  $\Omega$ , in abs. Mag range  $M, M+dM$ , with other attribute  $S$ .

Relate to observable quantities:

- Total number of stars in abs mag range  $M, M+dM$  out to distance  $d$  (Integrated star count):

[Eq. 24.3] 
$$N_M(M, S, \Omega, d) dM = \left[ \int_0^d n_M(M, S, \Omega, r) \Omega r^2 dr \right] dM$$

- Integrated star count to limiting apparent magnitude  $m$ :

Use  $d = 10^{(m-M-a)/5}$  in [24.3] to find

a = Extinction

$$\bar{N}_M(M, S, \Omega, m) dM$$

- Differential star count in apparent mag range  $m, m+dm$

[Eq. 24.4] 
$$A_m(M, S, \Omega, m) dM dm \equiv \frac{d\bar{N}_M}{dm} dM dm$$

[BS]

1980, ApJ Suppl. 44, 73.

## THE UNIVERSE AT FAINT MAGNITUDES. I. MODELS FOR THE GALAXY AND THE PREDICTED STAR COUNTS

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[Table 24.1]

	Disks		
	Neutral Gas	Thin Disk	Thick Disk
$M$ ( $10^{10} M_{\odot}$ )	$0.5^a$	6	0.2 to 0.4
$L_B$ ( $10^{10} L_{\odot}$ ) <sup>b</sup>	—	1.8	0.02
$M/L_B$ ( $M_{\odot}/L_{\odot}$ )	—	3	—
Radius (kpc)	25	25	25
Form	$e^{-z/h_z}$	$e^{-z/h_z}$	$e^{-z/h_z}$
Scale height (kpc)	< 0.1	0.35	1
$\sigma_w$ ( $\text{km s}^{-1}$ )	5	16	35
[Fe/H]	> +0.1	-0.5 to +0.3	-2.2 to -0.5
Age (Gyr)	$\lesssim 10$	8 <sup>c</sup>	10 <sup>d</sup>

	Spheroids		
	Central Bulge <sup>e</sup>	Stellar Halo	Dark-Matter Halo
$M$ ( $10^{10} M_{\odot}$ )	1	0.3	$190^{+360}_{-170}$
$L_B$ ( $10^{10} L_{\odot}$ ) <sup>b</sup>	0.3	0.1	0
$M/L_B$ ( $M_{\odot}/L_{\odot}$ )	3	$\sim 1$	—
Radius (kpc)	4	> 100	> 230
Form	boxy with bar	$r^{-3.5}$	$(r/a)^{-1} (1+r/a)^{-2}$
Scale height (kpc)	0.1 to 0.5 <sup>g</sup>	3	170
$\sigma_w$ ( $\text{km s}^{-1}$ )	55 to 130 <sup>h</sup>	95	—
[Fe/H]	-2 to 0.5	< -5.4 to -0.5	—
Age (Gyr)	< 0.2 to 10	11 to 13	$\sim 13.5$

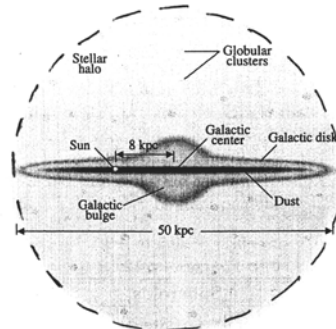
MORPHOLOGY OF MILKY WAY

Exponential disk

$$n(z, R) = n_0 \left( e^{-z/2h_{thin}} + 0.02 e^{-z/2h_{thick}} \right) e^{-R/h_r}$$

+ spheroids with various forms<sup>k</sup>

$$n = n_0 \times \text{form}$$



Usually see deVaucouleurs'  $r^{1/4}$  surface brightness law:

$$\log_{10} \left[ \frac{I(r)}{I_e} \right] = -3.3307 \left[ \left( \frac{r}{r_e} \right)^{1/4} - 1 \right]$$

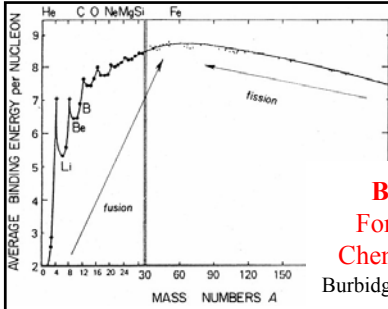
## Stellar Populations

- Abundances
- Kinematics
- Ages
- Pop I: Metal rich ( $Z \sim 0.02$ ), disk, younger
  - Disk field stars (up to 10-12 Gyr old)
  - Open clusters
  - Gas
  - Star formation regions
- Pop II: Metal poor ( $Z \sim 0.001$ ), halo, older
  - Globular clusters (12-15 Gyr)
  - Halo field stars
  - Bulge??? ...but includes Super Metal Rich (SMR) stars.
- Abundance Determinations
  - Stellar spectroscopy
    - [Fe/H], etc.  $\rightarrow \log(N_{Fe}/N_H) - \log(\text{solar})$
    - Iron ejected by Sne Ia after about  $10^9$  yrs.
  - Stellar colors
  - HII regions

X, Y, Z = mass fractions  
 X ~ 0.73  
 Y ~ 0.25

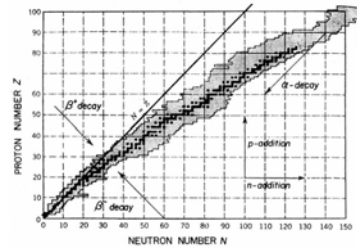
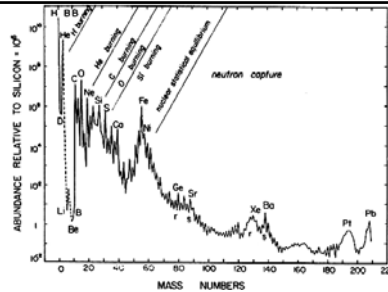
Baade (1944)

	[Fe/H]
Thin Disk	-0.5 $\rightarrow$ +0.3
Thick Disk	-0.6 $\rightarrow$ -0.4
Halo	-2.5 $\rightarrow$ -0.8
Bulge	-1.0 $\rightarrow$ +1.0



**B<sup>2</sup>FH (1957)**  
**Formation of the**  
**Chemical Elements**

Burbidge, Burbidge, Fowler & Hoyle.  
 Reviews of Modern Physics, 29, 547.

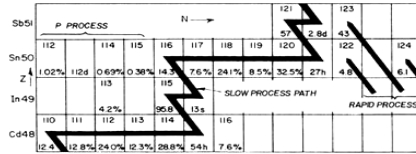


**Gradual processes in Interiors of Stars**

- H burning ( $4H \rightarrow He$ )
- $\alpha$  process (C,O,Ne,Mg,Si,S...)
- s process
  - slow neutron capture, relative to beta-decay timescale

**Supernovae**

- e process
  - nuclear statistical equilibrium
  - iron peak elements
- r process
  - rapid neutron capture

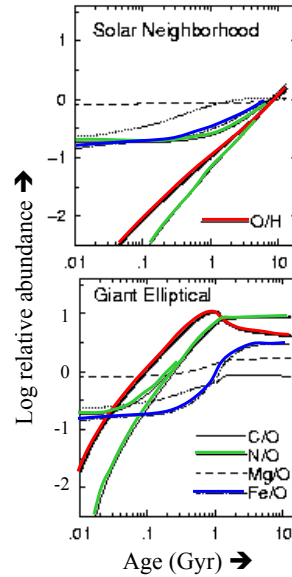


**Chemical Enrichment**



**Chemical Enrichment Models**

- Simulate what is going on in a volume of space.
- Recipes for how many of which elements created by stars of different masses.
- Include lifetimes of stars, fraction of mass returned to ISM.
- More about this in a few weeks.

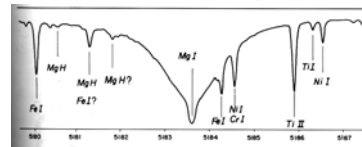
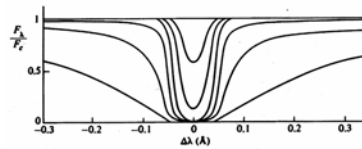
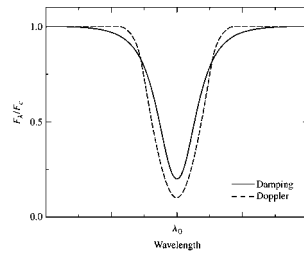


Hamann & Ferland 1992

# Measuring abundances from absorption lines

(see [9.5] for gory details)

- Lorentz profile
  - Natural profile of stationary absorber.
    - wings due to finite lifetime of excited state in QM model..
    - Or to “damping” in classical oscillator model
- Voigt profile
  - Lorentz profile convolved with Gaussian velocity distribution.
  - Line shape increases in funny way.



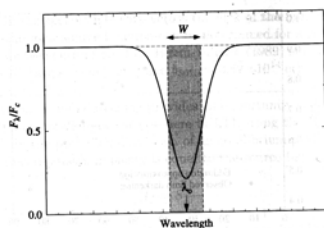
## EQUIVALENT WIDTH

- Often, wavelength resolution and/or signal:noise too low to measure details of line profile.
- Can still measure fraction of continuum light that is absorbed
- then convert to *column density* of absorbing atoms.

$$W_\lambda = \int \left[ 1 - \frac{I_v}{I_v(0)} \right] d\lambda = \frac{\lambda^2}{c} \int [1 - e^{-\tau_v}] dv$$

since  $d\lambda = (\lambda^2/c) dv$

- in units of Å
- same as width of square profile going to zero and having same  $W_\lambda$  as observed line.



Optical depth:

$$\tau_v = \int \alpha_v n ds$$

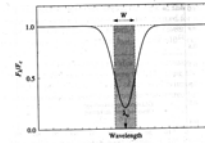
Column density:

(atoms/cm<sup>2</sup> along line of sight)

$$N = \int n ds$$

## CONVERTING $W_\lambda$ TO COLUMN DENSITY OF ABSORBING ATOMS:

$$W_\lambda = \int \left[ 1 - \frac{I_\nu}{I_\nu(0)} \right] d\lambda = \frac{\lambda^2}{c} \int [1 - e^{-\tau_\nu}] d\nu$$



**CURVE OF GROWTH** shows how  $W_\lambda$  depends on  $N$

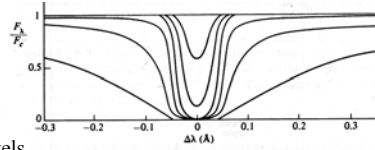
- **For small column density:**

$$W_\lambda = \lambda^2 \tau_\nu$$

$$\frac{W_\lambda}{\lambda} \propto N_j f_{jk} \lambda$$

where  $j, k$  are lower, upper levels,

$f_{jk}$  is oscillator strength = effective number of oscillators participating in transition.



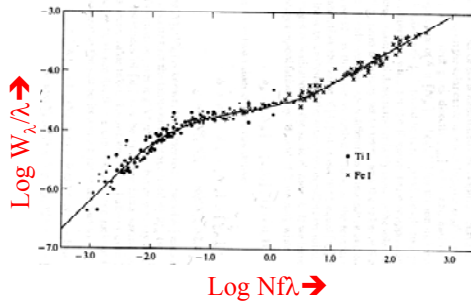
- **For intermediate column density:**

where  $b = \sqrt{v_o^2 + v_{\text{turbulent}}^2}$ :

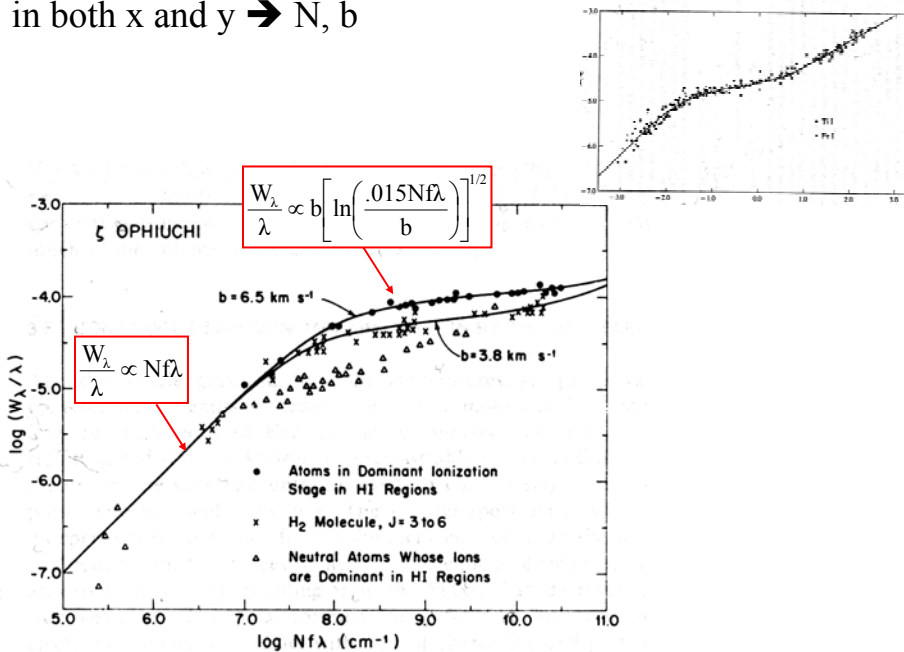
$$\frac{W_\lambda}{\lambda} \propto b \left[ \ln \left( \frac{.015 N f \lambda}{b} \right) \right]^{1/2}$$

- **For large column density:**

$$\frac{W_\lambda}{\lambda} \propto (\lambda^2 N f)^{1/2}$$



Sliding observed c.o.g. over theoretical c.o.g  
in both x and y  $\rightarrow N, b$



Sliding observed c.o.g. over theoretical c.o.g.  
 in both x and y  $\rightarrow$  N, b

