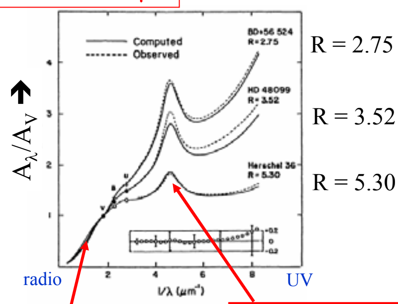
Particle size ~ 0.01 – 0.1 μm



2175Å feature ⇒graphite

~ 1/λ dependence in IR-optical.

$$I_\lambda = I_{0,\lambda} e^{-\tau_\lambda}$$

in magnitudes

$$A_\lambda = -2.5 \log_{10}(I_\lambda / I_{0,\lambda})$$

$$= -2.5 \log_{10} e^{-\tau_\lambda}$$

$$= 1.08\, \tau_\lambda$$

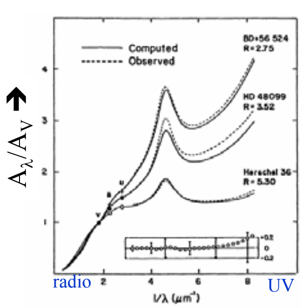
Determine τ_λ from change in color

$$A_V = R_V \cdot E(B-V)$$

$$E(B-V) = (B-V) - (B-V)_0$$

$R_V \sim 3$ but different in different places.

The reddening curve:



Shape of reddening curve depends only on dust properties:

$$\frac{A_\lambda}{A_V} = \frac{1.08\,\tau_\lambda}{1.08\,\tau_V} = \frac{1.08\,\alpha_\lambda \int n ds}{1.08\,\alpha_V \int n ds}$$

Optical depth:

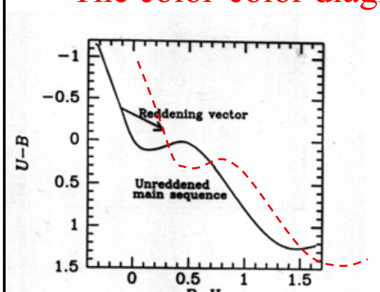
$$\tau_\nu = \int \alpha_\upsilon\, n\, ds$$

Column density:

(atoms/cm² along line of sight)

$$N = \int n\, ds$$

The color-color diagram:



Slope of reddening vector depends only on shape of extinction curve:

$$\frac{\Delta(U-B)}{\Delta(B-V)} = \frac{A_U - A_B}{A_B - A_V} = \frac{\alpha_U - \alpha_B}{\alpha_B - \alpha_V}\frac{\int nds}{\int nds}$$

But distance stars slide along that slope depend also on dust column density:

$$\Delta(B-V) = A_B - A_V = (\alpha_B - \alpha_V)\int nds$$

Star Counts

$n_M\,(M,S,\Omega,r)\,dM$ = number of stars per unit volume at distance r, in solid angle Ω, in abs. Mag range M, M+dM, with other attribute S.

Relate to observable quantities:

- Total number of stars in abs mag range M, M+dM out to distance d (Integrated star count):

[Eq. 24.3] $$N_M(M,S,\Omega,d)\,dM = \left[\int_0^d n_M(M,S,\Omega,r)\,\Omega\, r^2 dr\right] dM$$

- Integrated star count to limiting apparent magnitude m:

Use $d = 10^{(m-M-a+5)/5}$ in [24.3] to find

a = Extinction

$$\bar{N}_M(M,S,\Omega,m)\,dM$$

- Differential star count in apparent mag range m, m+dm

[Eq. 24.4] $$A_M(M,S,\Omega,m)\,dM\,dm \equiv \frac{d\bar{N}_M}{dm}\,dM\,dm$$

[BS]

1980, ApJ Suppl. 44, 73.

THE UNIVERSE AT FAINT MAGNITUDES. I. MODELS FOR THE GALAXY AND THE PREDICTED STAR COUNTS

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Received 1979 August 6; accepted 1980 March 19

CONTENTS

[Table 24.1]

Disks	Neutral Gas	Thin Disk	Thick Disk
M (10^{10} $M_\odot$)	0.5[a]	6	0.2 to 0.4
L_B (10^{10} $L_\odot$)[b]	—	1.8	0.02
M/L_B ($M_\odot/L_\odot$)	—	3	—
Radius (kpc)	25	25	25
Form	e^{-z/h_z}	e^{-z/h_z}	e^{-z/h_z}
Scale height (kpc)	< 0.1	0.35	1
σ_w (km s^{-1})	5	16	35
[Fe/H]	> +0.1	−0.5 to +0.3	−2.2 to −0.5
Age (Gyr)	$\lesssim$ 10	8[c]	10[d]

Spheroids	Central Bulge[e]	Stellar Halo	Dark-Matter Halo
M (10^{10} $M_\odot$)	1	0.3	190^{+360}_{-170}[f]
L_B (10^{10} $L_\odot$)[b]	0.3	0.1	0
M/L_B ($M_\odot/L_\odot$)	3	~ 1	—
Radius (kpc)	4	> 100	> 230
Form	boxy with bar	$r^{-3.5}$	$(r/a)^{-1}(1+r/a)^{-2}$
Scale height (kpc)	0.1 to 0.5[g]	3	170
σ_w (km s^{-1})	55 to 130[h]	95	—
[Fe/H]	−2 to 0.5	< −5.4 to −0.5	—
Age (Gyr)	< 0.2 to 10	11 to 13	~ 13.5

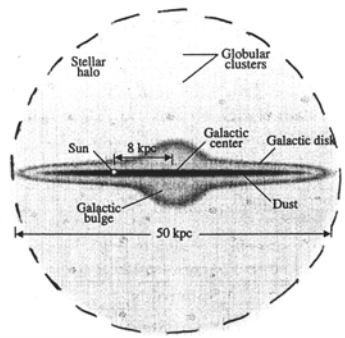
Morphology of Milky Way

Exponential disk

$$n(z,R) = n_0\left(e^{-z/z_{thin}} + 0.02\, e^{-z/z_{thick}}\right)e^{-R/h_R}$$

+ spheroids with various "forms"

$$n = n_0 \times \text{form}$$



Usually see deVaucouleurs' $r^{1/4}$ surface brightness law:

$$\log_{10}\left[\frac{I(r)}{I_e}\right] = -3.3307\left[\left(\frac{r}{r_e}\right)^{1/4} - 1\right]$$

Stellar Populations

- Abundances
- Kinematics
- Ages

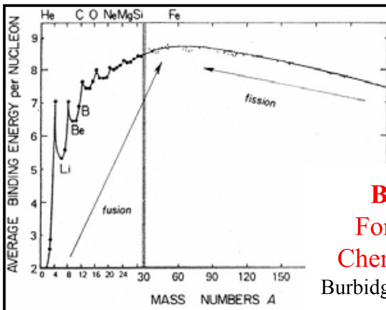
X,Y,Z = mass fractions
X ~ 0.73
Y ~ 0.25

- Pop I : Metal rich (Z ~ 0.02), disk, younger
  - Disk field stars (up to 10-12 Gyr old)
  - Open clusters
  - Gas
  - Star formation regions

Baade (1944)

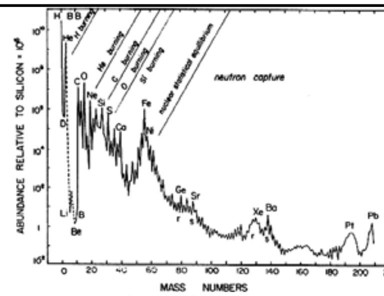
- Pop II: Metal poor (Z ~ 0.001), halo, older
  - Globular clusters (12-15 Gyr)
  - Halo field stars
  - Bulge??? ….but includes Super Metal Rich (SMR) stars.
- Abundance Determinations
  - Stellar spectroscopy
    - [Fe/H], etc. ➔ $\log(N_{Fe}/N_H) - \log(\text{solar})$
    - Iron ejected by Sne Ia after about 10^9 yrs.
  - Stellar colors
  - HII regions

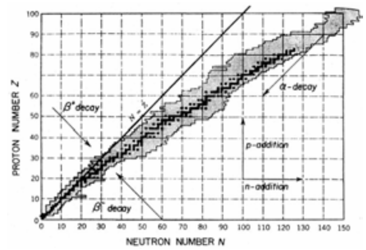
	[Fe/H]
Thin Disk	-0.5➔ +0.3
Thick Disk	-0.6➔ -0.4
Halo	-2.5➔ -0.8
Bulge	-1.0➔+1.0



B^2FH (1957)
Formation of the Chemical Elements

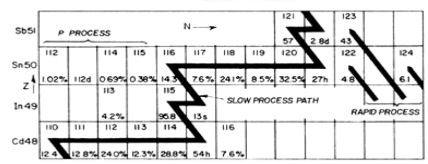
Burbidge, Burbidge, Fowler & Hoyle.
Reviews of Modern Physics, 29, 547.





Gradual processes in Interiors of Stars

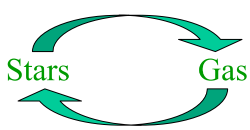
- H burning (4H ➔ He)
- α process (C,O,Ne,Mg,Si,S…)
- s process
  - slow neutron capture, relative to beta-decay timescale

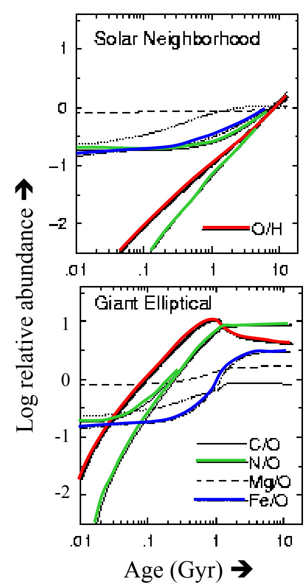


Supernovae

- e process
  - nuclear statistical equilibrium
  - iron peak elements
- r process
  - rapid neutron capture

Chemical Enrichment





Hamann & Ferland 1992

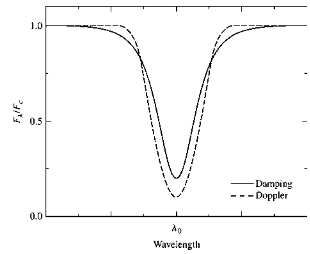
Chemical Enrichment Models

- Simulate what is going on in a volume of space.
- Recipes for how many of which elements created by stars of different masses.
- Include lifetimes of stars, fraction of mass returned to ISM.
- More about this in a few weeks.

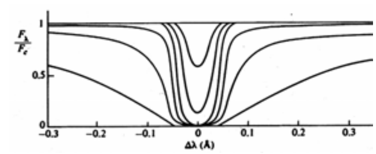
Measuring abundances from absorption lines

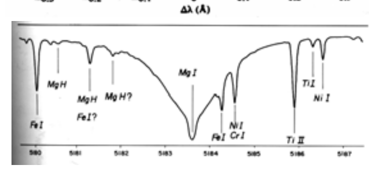
(see [9.5] for gory details)

- Lorentz profile
  - Natural profile of stationary absorber.
    - wings due to finite lifetime of excited state in QM model..
    - Or to "damping" in classical oscillator model



- Voigt profile
  - Lorentz profile convolved with Gaussian velocity distribution.
  - Line shape increases in funny way.





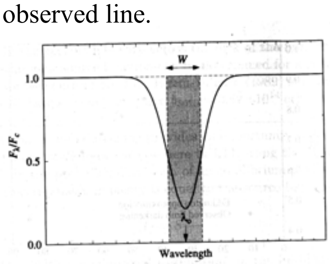
EQUIVALENT WIDTH

- Often, wavelength resolution and/or signal:noise too low to measure details of line profile.
- Can still measure fraction of continuum light that is absorbed
- then convert to *column density* of absorbing atoms.

$$W_\lambda = \int \left[1 - \frac{I_\nu}{I_\nu(0)}\right] d\lambda = \frac{\lambda^2}{c} \int \left[1 - e^{-\tau_\nu}\right] d\nu$$

since $d\lambda = (\lambda^2/c)d\nu$

- in units of Å
- same as width of square profile going to zero and having same W_λ as observed line.



Optical depth:

$\tau_\nu = \int \alpha_\upsilon n \, ds$

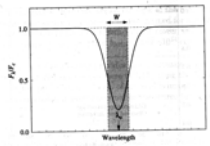
Column density:

(atoms/cm² along line of sight)

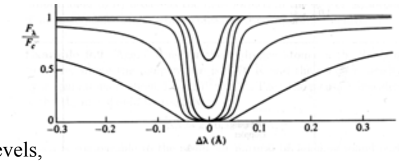
$N = \int n \, ds$

CONVERTING W_λ TO COLUMN DENSITY OF ABSORBING ATOMS:

$$W_\lambda = \int \left[1 - \frac{I_\nu}{I_\nu(0)}\right] d\lambda = \frac{\lambda^2}{c} \int \left[1 - e^{-\tau_\nu}\right] d\nu$$



CURVE OF GROWTH shows how W_λ depends on N



- **For small column density:**

$$W_\lambda = \lambda^2 \tau_\nu$$

$$\frac{W_\lambda}{\lambda} \propto N_j f_{jk} \lambda$$

where j,k are lower, upper levels,

f_{jk} is oscillator strength = effective number of oscillators participating in transition.

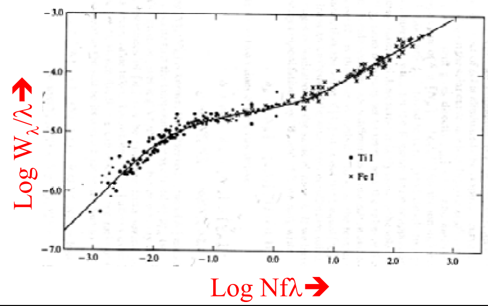
- **For intermediate column density:**

where b = sqrt($v_o^2 + v_{turbulent}^2$):

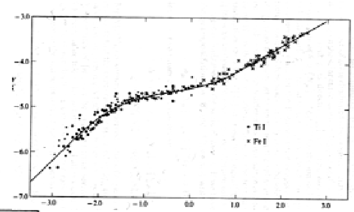
$$\frac{W_\lambda}{\lambda} \propto b\left[\ln\left(\frac{.015 N f \lambda}{b}\right)\right]^{1/2}$$

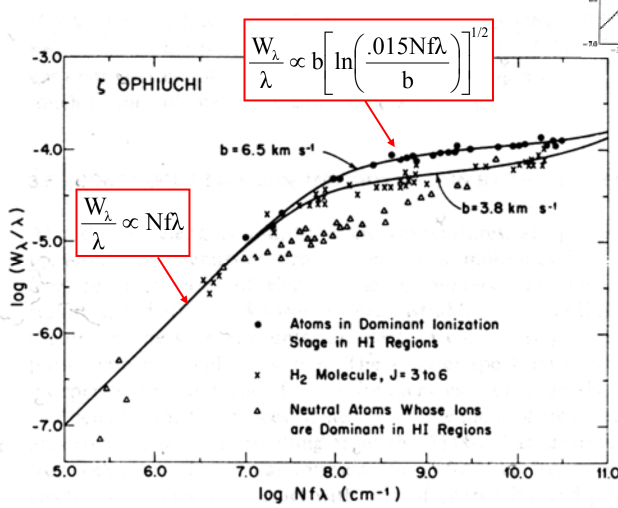
- **For large column density:**

$$\frac{W_\lambda}{\lambda} \propto \left(\lambda^2 N f\right)^{1/2}$$



Sliding observed c.o.g. over theoretical c.o.g in both x and y ➔ N, b





Sliding observed c.o.g. over theoretical c.o.g in both x and y ➔ N, b