**Wavefronts**

**Rays**

**Conjugate Caustic Surfaces**

But **Critical Curves** are the directions we would need to look from Earth in order to see lensed images of objects which lie on Conjugate Caustic Surfaces.

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**Closed Box Models**

(and friends and relatives)

Gas $\rightarrow$ stars $\rightarrow$ enriched gas

- $S =$ mass of stars
- $M =$ mass of metals (heavy elements) in ISM
- $G =$ total mass of gas in ISM

Assume instantaneous recycling from massive stars.

From a new generation of stars:

- $dS =$ mass of low mass stars added to $S$
- $p \ dS =$ mass of heavy elements added to $M$ from massive stars in this generation.  
  where $p =$ yield.

- $dM = p \ dS - Z \ dS$
- $dG = -dS$  since $dG = -dS$

But

- $dZ = d(M/G) = (1/G) \ dM - (M/G^2) \ dG$
- $dZ = -p \ ln [(G(t)/G(0))$ or $G(t)=G(0) e^{-Z(t)/p}$

**G dwarf problem**

$S[Z<Z(t)] = S(t) = G(0) - G(t)$

$= G(0) \ {1 - e^{-Z(t)/p}}$

$Z(t) =$ gas metallicity at time $t$

Compare to case when gas had some arbitrary fraction $\alpha$ of that metallicity:

$S[Z<\alpha Z(t)] = 1 - \alpha$

$S[Z<Z(t)] = 1 - X$

where $X = \frac{G(t)}{G(0)} - 0.1 - 0.2$

Predicts broad distribution in metallicity of stars.

- $S[Z<1/4 Z_\odot] = 0.4 \ S[Z<Z_\odot]$

Very different than what is observed in solar neighborhood:

$S[Z<1/4 Z_\odot] = 0.02 \ S[Z<Z_\odot]$

Also... Leaky box (gas driven out by stars).

Accreting box models.
Closed Box Models
(and friends and relatives)

Gas → stars → enriched gas

\[ S = \text{mass of stars} \]
\[ M = \text{mass of metals (heavy elements) in ISM} \]
\[ G = \text{total mass of gas in ISM} \]

Assume instantaneous recycling from massive stars.

From a new generation of stars:

\[ dS = \text{mass of low mass stars added to } S \]
\[ p \ dS = \text{mass of heavy elements added to } M \]

\[ dM = p \ dS - Z \ dS \]
\[ = -p \ dG + Z \ dG \]

But

\[ dZ = \frac{dM}{M(G)} = \frac{(1/G) \ dM - (M/G^2) \ dG}{-p \ dG/G} \]

\[ Z(t) = -p \ln \left( \frac{G(t)}{G(0)} \right) \]

\[ Z = \frac{M}{G} \]

\[ Z_0 = 0.02 \]

G dwarf problem

\[ S[Z<Z(t)] = S(t) = G(0) - G(t) = G(0) \left( 1 - e^{-Z(t)/p} \right) \]

\[ Z(t) = \text{gas metallicity at time } t \]

Compare to case when gas had some arbitrary fraction \( \alpha \) of that metallicity:

\[ S\left[Z<\alpha Z(t)\right] = 1 - \alpha \]

\[ S\left[Z<Z(t)\right] = 1 - X \]

where

\[ X = \sim 0.1 - 0.2 \]

Predicts broad distribution in metallicity of stars.

\[ S[Z<1/4 Z] = 0.4 \]

\[ S[Z<Z] = 0.02 \]

Very different than what is observed in solar neighborhood:

\[ S[Z<1/4 Z_0] = 0.02 \]

Also… Leaky box (gas driven out by stars),

Accreting box models.

The Initial Mass Function (IMF)

- \( dN = N \xi(M) \ dM \) = number of stars born with masses in range \( M, M+dM \)
- Salpeter (1955) IMF: \( \xi(M) \propto M^{-2.35} \)
- Scalo (1986) IMF:
  
  \[ \xi(M) \propto M^{-2.45} \text{ for } M > 10M_\odot \]
  
  \[ \xi(M) \propto M^{-3.27} \text{ for } 1 < M < 10M_\odot \]
  
  \[ \xi(M) \propto M^{-1.33} \text{ for } 0.2 < M < 1M_\odot \]

- Others as well.

- Star Formation rate = \( \psi(t) \)

- Stellar birthrate function

\[ B(M,t) = \psi(t) \xi(M) \ dM \ dt \]

\[ = \text{number of stars born per unit volume with masses in range } M, M+dM \text{ in time interval } t, t+dt \]

[CO Fig. 26.18]

[CO eqn. 26.4]
Modeling chemical enrichment

- One zone, accreting box model.
  - Start with pure H, He mix.
  - Further H, He falls in at specified rate.
- Follow evolution of individual elements H, He, C, N, O, Ne, Mg, Si, S, Ar, Ca and Fe.
- Subdivide stellar population into three classes of stars:
  - < 1M⊙ nothing recycled
  - 1.0 – 8.0 M⊙ fraction give Type Ia supernovae
  - > 8M⊙ Type Ib, Ic or II supernovae.
- Assume that each class of stars spews specified % of its mass back into ISM in the form of each element at end of a specified lifetime.
- Must provide IMF to specify mix of star masses.
- Two extreme models:
  - "Solar neighborhood": conventional IMF, slow stellar birthrate, slow infall (15% gas at 10 Gyr).
  - "Giant Elliptical": flatter IMF, 100x higher birthrate, fast infall (15% gas at 0.5 Gyr).

Gradual processes in Interiors of Stars
- H burning (4H → He)
- α process (C,O,Ne,Mg,Si,S...)
- s process
  - slow neutron capture, relative to beta-decay timescale

Supernovae
- e process
  - nuclear statistical equilibrium
- r process
  - rapid neutron capture

B²FH (1957)
Formation of the Chemical Elements
Burbidge, Burbidge, Fowler & Hoyle. Reviews of Modern Physics, 29, 547.
The First Stars

• Population III
  – Metallicity $Z = 0$
  – Expected to be very massive

• short-lived
• lots of UV photons $\Rightarrow$ reionization of IGM
• supernovae/hypernovae $\Rightarrow$ 1st round of chemical enrichment of ISM
  – Gamma ray bursts?
• No low-mass survivors found yet

• The Metal-Poorest Pop II stars
  – Down to $[\text{Fe/H}] = -5.4$ (Frebel et al. 2005, Nature, 434, 871)
    • 3 stars with $[\text{Fe/H}] < -4.0$ (Norris et al. 2007, ApJ, 670, 774)
  – Aim is to trace details of element synthesis from Pop III and very first Pop II stars
    • Heavy-element abundance pattern in observed star may come from single Pop III supernova/hypernova event.
    • Sensitive tracer of mass of Pop III star
  – Abundance patterns change from star to star due to as yet unmixed space distribution of Pop III and/or first Pop II predecessors.