

## Angular Diameters

- RW metric:
 
$$(ds)^2 = (cdt)^2 - R^2(t) \left[ \left( \frac{d\tilde{\omega}}{\sqrt{1-k\tilde{\omega}^2}} \right)^2 + (\tilde{\omega} d\theta)^2 + (\tilde{\omega} \sin\theta d\phi)^2 \right]$$
- What is angular size of galaxy at distance  $\tilde{\omega}$  ?
 
$$dt = d\tilde{\omega} = d\phi = 0$$

Galaxy's diameter is proper ~~distance~~ linear diameter:

$$D = \int \sqrt{-(ds)^2} = R(t_e) \tilde{\omega}_e \theta$$

Using  $\varpi$  coordinate  $\rightarrow$  Looks like Euclidean result, regardless of curvature of space.

$$\theta = \frac{D}{R(t_e) \tilde{\omega}} \quad \text{but must use } R(t_e)$$

$$= \frac{D(1+z)}{\tilde{\omega}} \quad \text{using } 1+z = \frac{1}{R(t_e)}$$

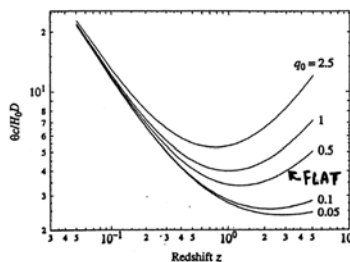
$$\theta = \frac{D(1+z)^2}{d_L}$$

## More angular diameter

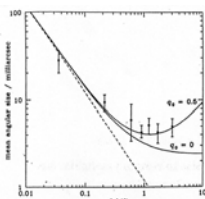
$$\theta = \frac{D(1+z)^2}{d_L}$$

$$\theta = \frac{H_0 D}{c} \frac{z_0^2 (1+z)^2}{z_0 z - (1-z_0)(\sqrt{1+z z_0^2} - 1)}$$

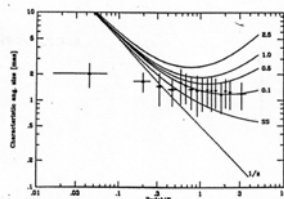
- Surprise!
  - Even for flat universe,  $\theta$  first decreases but then increases with increasing  $z$ .
  - Two competing effects:
    - $\theta = 1/\text{distance}$
    - Universe expands under photons while they are in transit.



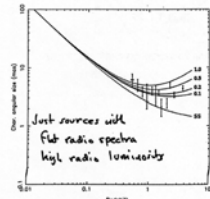
VLBI measurements of compact radio sources:



Kellerman (1993)



Gurvits (1994)



Gurvits, Kellerman & Frey (1999)

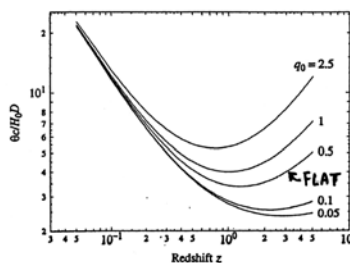
Authors say "consistent with"  $q_0 = 0.5$ , no evolution.

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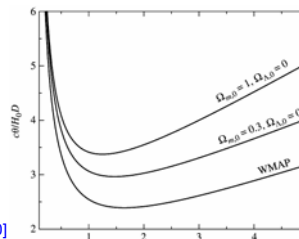
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In practice  
(because of that  $\Omega_{\Lambda}$  cosmological constant)

$$\frac{c\theta}{H_0 D} = \frac{(1+z)}{S(z)}$$

29.136



[CO Fig. 29.30]