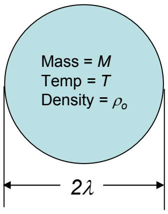
# The Simplest Picture of Galaxy Formation and Why It Fails

(chapter title from Longair, “Galaxy Formation”)

Read [CO 30.2]

## Will a condensation collapse?
## The Jeans criterion:

(see [CO Sect. 12.2 and pg. 1250]



Unstable to collapse if

$$2K < -U$$

$$\frac{3MkT}{\mu m_H} < \frac{3}{5}\frac{GM^2}{\lambda}$$

$$M = \frac{4}{3}\pi\lambda^3\rho_o > \underbrace{\left(\frac{5kT}{G\mu m_H}\right)^{3/2}\left(\frac{3}{4\pi\rho_o}\right)^{1/2}}_{\text{Jeans mass } M_J}$$

## How fast will it collapse?

In a static medium (e.g. star formation):

Perturbation analysis shows

$M < M_J$

$\delta\rho/\rho \propto \exp(-ir/\lambda - i\omega t)$ ➔ Oscillations

$M > M_J$

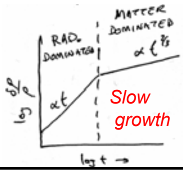
$\delta\rho/\rho \propto \exp(-ir/\lambda + Kt)$ ➔ Exponential growth

In an expanding medium (e.g. the universe):

Outside the perturbation (flat universe): $H^2R^2 - \frac{8}{3}\pi G\rho R^2 = 0$

Inside the perturbation (closed mini-universe): $H^2R^2 - \frac{8}{3}\pi G\rho' R^2 = -kc^2$

$$\frac{\delta\rho}{\rho} = \frac{\rho' - \rho}{\rho} = \frac{3kc^2}{8\pi G\rho R^2}$$



$$\frac{\delta\rho}{\rho} = \left(\frac{\delta\rho}{\rho}\right)_i\left(\frac{t}{t_i}\right)$$ Radiation era

$$\frac{\delta\rho}{\rho} = \left(\frac{\delta\rho}{\rho}\right)_i\left(\frac{t}{t_i}\right)^{2/3}$$ Matter era

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# The Simplest Picture of Galaxy Formation and Why It Fails

## Will a condensation collapse?
## The Jeans criterion Version 2:

*Collapse if* $2K < -U$

$$2\left(\frac{1}{2}M_T v_s^2\right) < \frac{3}{5}\frac{GM_T^2}{\lambda}$$

$$v_s^2 < \frac{3}{5}\frac{GM_T}{\lambda} = \frac{3G}{5}\frac{(4/3)\pi\lambda^3\rho_T}{\lambda} = \frac{4}{5}\pi G\lambda^2\rho_T$$

$$\left(\frac{3M_b}{4\pi\rho_b}\right)^{2/3} = \lambda^2 > \frac{5v_s^2}{4\pi G\rho_T}$$

$$M_b > \left(\frac{5}{4}\right)^{3/2}\frac{4\pi\rho_b v_s^3}{3\pi^{1/2}(G\rho_T)^{3/2}}$$ = [CO eq. 20.7] (almost)

Radiation era

$$v_s = \frac{c}{\sqrt{3}}$$

$$\rho_b \propto R(t)^{-3} \propto T^3$$

$$\rho_T \propto R(t)^{-4} \propto T^4$$

$$M_{J,b} \propto T^{-3}$$

After decoupling

$$v_s = \sqrt{\frac{5kT}{3\mu m_H}}$$

$$\rho_b \propto T^0$$

$$\rho_T \propto T^0$$

$$M_{J,b} \propto T^{3/2}$$

Before decoupling

- Particle Horizon

  $d_h = 2ct \propto R(t)^2 \propto T^{-2}$ (radiation era)

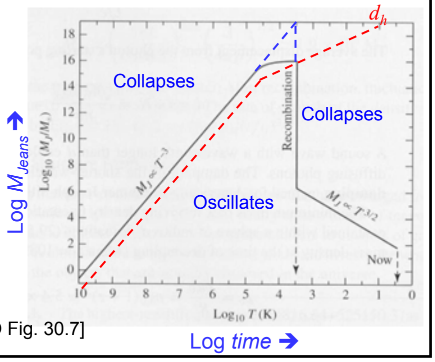
- Proper distance containing mass $M$

  $l = (M/\rho)^{1/3} \propto M^{1/3}R(t) \propto M^{1/3}T^{-1}$

- Mass for which $l = d_h$

  $M \propto T^{-3} \propto t^{3/2}$ (radiation era)

  $M \propto T^{-3/2}$ (matter era)



[CO Fig. 30.7]

# The Simplest Picture of Galaxy Formation and Why It Fails

- Cosmic Microwave Background is smooth to a few parts in $10^5$

  $\delta\rho/\rho \sim 10^{-4}$

- Yet high contrast structures (QSOs, galaxies) by $z \sim 6$.

  $\delta\rho/\rho >> 1$

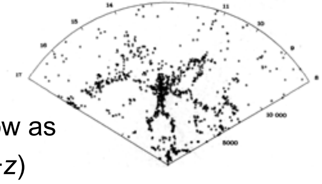
- Adiabatic perturbations grow as

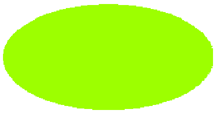
  $\delta\rho/\rho \propto t^{2/3} \propto R(t) \propto 1/(1+z)$

- Expect only

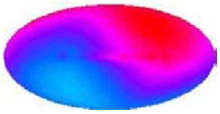
$$\left(\frac{\delta\rho}{\rho}\right)_{QSO} = \frac{(1+z)_{CMB}}{(1+z)_{QSO}}\left(\frac{\delta\rho}{\rho}\right)_{CMB} = \frac{1100}{7}\times 10^{-4} = 0.01$$

So where did galaxies and clusters come from?

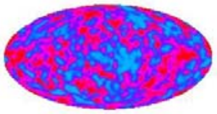




Blue = 0°K
Red = 4°K



Blue = 2.724°K
Red = 2.732°K
Dipole Anistropy
~ 1 part in 300



After removing dipole
Red – blue = 0.0002°K
~ 1 part in $10^5$

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# The Solution: Dark Matter

- Particle Horizon

  $d_h = 2ct \propto R(t)^2 \propto T^{-2}$

- Proper distance containing mass $M$

  $l = (M/\rho)^{1/3} \propto M^{1/3}R(t) \propto M^{1/3}T^{-1}$

- Mass for which $l = d_h$

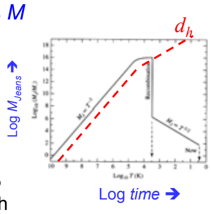
  $M \propto T^{-3} \propto t^{3/2}$

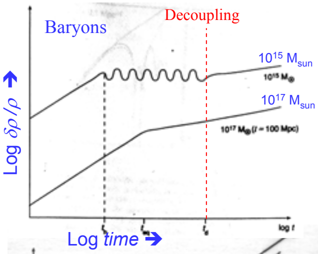
- At $t_{CMB}$ this mass was ~ $10^{16}\,M_\odot$
  - $M > 10^{16}\,M_\odot$ ➔ continued growth
  - $M < 10^{16}\,M_\odot$ ➔ oscillations once mass scale comes into particle horizon.

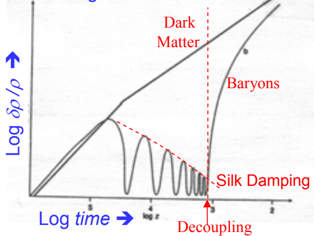
- + Silk damping
  - Photons drag baryons out of density condensations, up until time of decoupling.

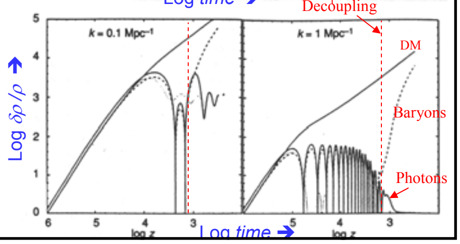
- But Dark Matter not subject to all this.
  - Just keeps collapsing…

- Baryons fall into Dark Matter potential wells as soon as decoupling removes photon pressure support.



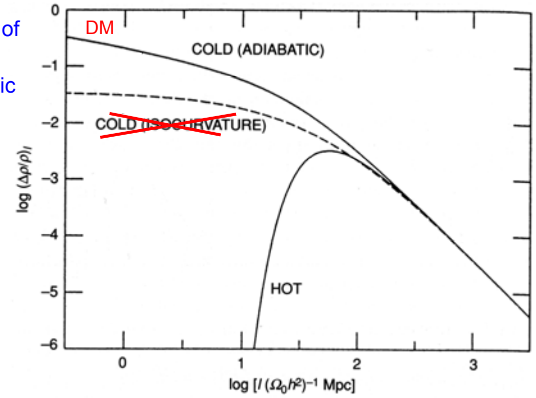






# Hot vs. Cold Dark Matter Perturbations

- Power spectrum of fluctuations
  - $P(k) \propto k^{n}$ where $P = |\delta\rho/\rho|^2$ and $k$ = wave number = $2\pi/l$
- Inflation predicts $n = 1$
  - "scale invariant":
    - $\delta\rho/\rho$ always has same value when perturbation enters horizon.
  - Predicts $\delta\rho/\rho \propto M^{-2/3} \propto l^{-2}$
- HDM perturbations with wavelengths shorter than horizon are lost
  - relativistic particles free stream out of smaller condensations
  - until particles become non-relativistic at $T \sim 10^5$K
  - smallest condensations have ~ $10^{13}$ $M_{sun}$
- CDM perturbations survive at all scales
  - some attenuation at shorter wavelengths
  - but most power still at shorter wavelength



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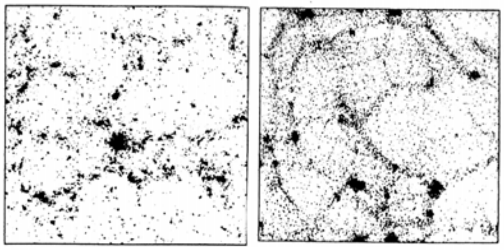
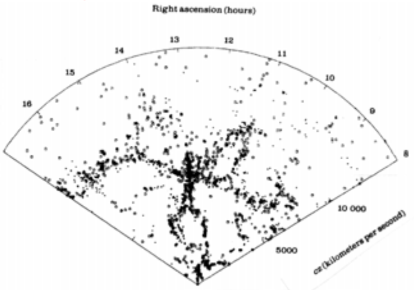
# N-body simulations ➔ CDM

Standard CDM = SDCM, replaced by $\Lambda$CDM model

- Start with perturbation spectrum at time of decoupling
- Follow perturbations into highly non-linear regime.



CDM HDM



- HDM models become too highly clustered over observed lifetime of galaxies

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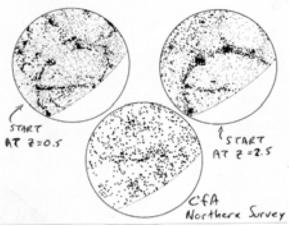
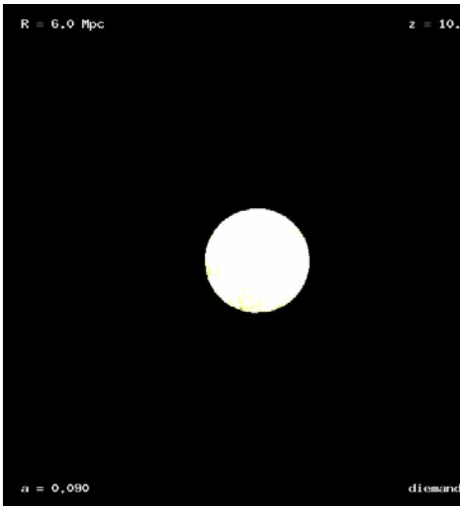
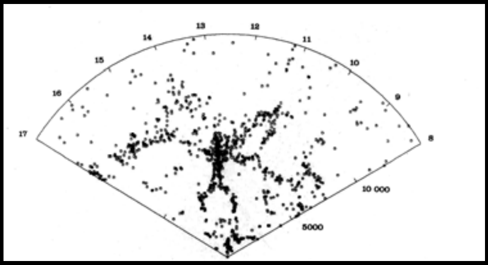
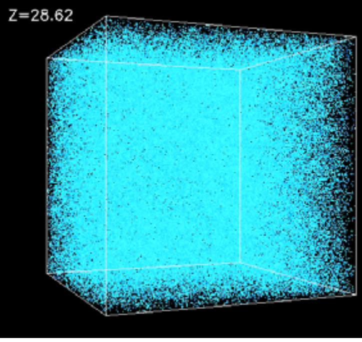


*Fig. 4.* Equal area projections of the galaxy distributions on the northern sky and in artificial catalogues made from *N*-body simulations. The top two diagrams correspond to neutrino dominated universes in which galaxy formation began at a redshift 0.5 (top left) and 2.5 (top right). In both cases $\Omega = 1$, but $h = 0.8$ for the model at the left, and $h = 0.5$ for the model at the right. The circles represent the "galaxies" while the dots represent the neutrino distribution. The bottom diagram is the CfA northern survey. The outer circle represents galactic latitude +40, and the empty regions lie at declinations below 0°. Even the model with a completely unrealistic epoch of galaxy formation is more strongly clustered than the data. This disagreement persists for any combination of model parameters.





In “proper” coordinates



In
co-moving
coordinates

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# CDM structure-formation models

- Reproduce observed filamentary structure
  - Weinberg et al. astro-ph/9708213

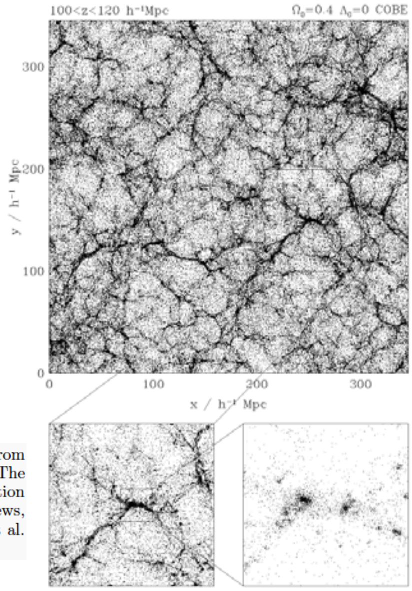
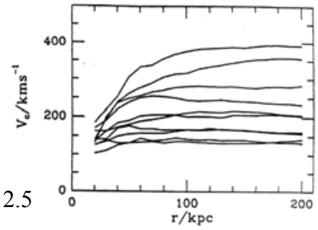


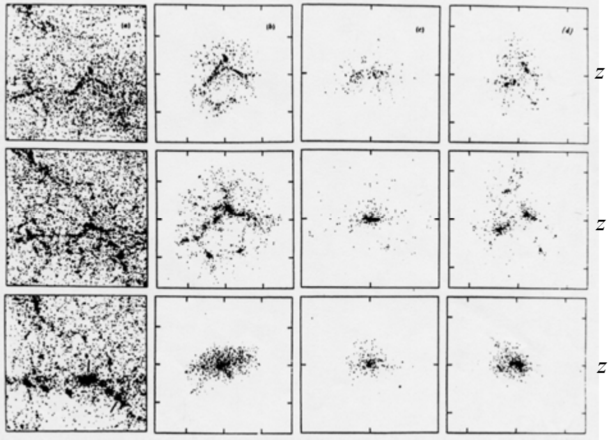
Figure 1. The particle distribution in a $20h^{-1}$ Mpc thick slice from an N-body simulation of an $\Omega_0 = 0.4$, open universe CDM model. The large panel shows the full cross section of the $360h^{-1}$ Mpc simulation box, while the two lower panels show successively expanded views, $100h^{-1}$ Mpc and $20h^{-1}$ Mpc on a side respectively. From Cole et al. (1997).

# More CDM Simulations

(Frenk 1991, Physica Scripta T36, 70)



flat rotation curves reproduced



**Large Cluster**
Violent assembly

**Disk Galaxy Halo**
Steady growth

**Small Group**
MW, M31

$z = 2.5$

$z = 1.0$

$z = 0.0$

*Fig. 8.* **Evolution of a (14 Mpc)$^3$ volume of a flat CDM universe and of selected galactic halos that formed in it. Time increases downwards in this figure. Each row corresponds to a different redshift as follows: $z = 2.5$ (top), $z = 1$ (middle) and $z = 0$ (bottom). The column labelled (a) shows the simulation as a whole, with positions plotted in comoving coordinates; the region shown at the top of this panel is thus 4 Mpc on a side. The three clumps marked with arrows at the bottom of (a) are shown in greater detail in (b)–(d). Physical, not comoving, coordinates are used in (b)–(d) and tickmarks represent 1-Mpc intervals. The three selected halos correspond to the two most massive clumps in the simulation and to a more isolated system. (From Ref. [63].)**