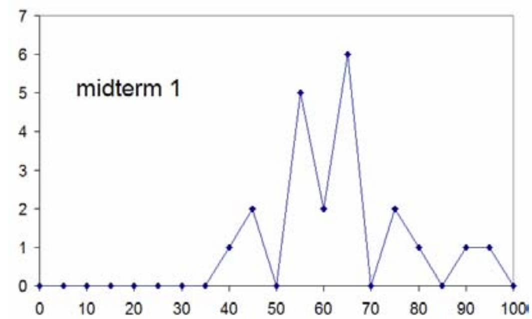
Midterm scores

95
88
77
75
74
64
64
63
62
62
61
59
58
55
54
53
51
51
42
41
40

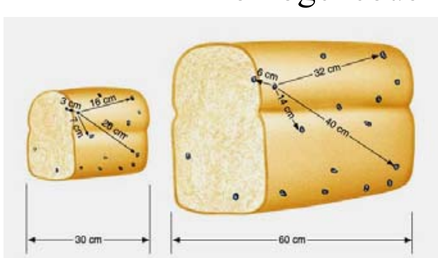
More Homework

Add to assignment due NEXT Wed. (Oct. 17):
[CO 29.3] = [27.4 in 1st ed]
[CO 29.7] = [27.9 in 1st ed]

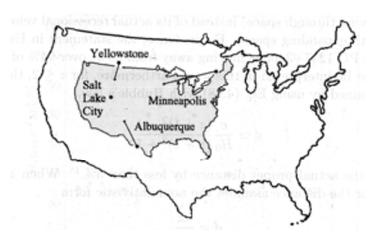
Average = 61.4



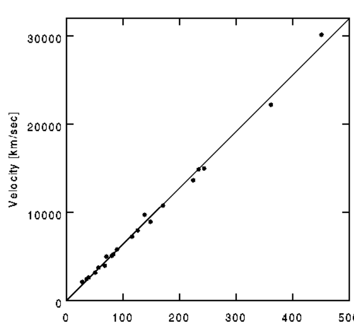
The Expanding Universe
Homogeneous * Isotropic



As seen from a Wonder Bread raisen



As seen from Utah



The Cosmological Principle:
At any given time, the universe is the same everywhere.

Cosmological Principle: Universe is homogeneous & isotropic

Newtonian Cosmology

- Energy:

Kinetic + Potential = Total

[29.1]

$$\frac{1}{2}mv^2 - \frac{GM_r m}{r} = -\frac{1}{2}mkc^2\varpi^2$$

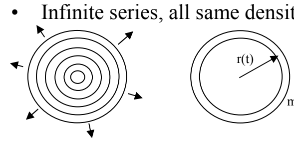
$$\frac{1}{2}mv^2 - \frac{G\frac{4}{3}\pi r^3 \rho m}{r} = -\frac{1}{2}mkc^2\varpi^2$$

$$v^2 - \frac{8}{3}\pi G\rho r^2 = -kc^2\varpi^2$$ [29.2]

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$ [29.10]

Friedman eq'n:

- Nested, expanding shells
  - Infinite series, all same density ρ(t)



  - Follow single shell, mass *m* [29.3]

- **r(t) = (Scale factor) x (co-moving coordinate)**

  r(t) = R(t) ϖ

Define: Total Energy $= -\frac{1}{2}mkc^2\varpi^2$

Why???

Cosmological principle ➔

For bound universe, each nested shell must simultaneously have KE➔0

$$E = -G\frac{4}{3}\pi r^2 \rho m \propto m\varpi^2$$

Other forms of the Friedman Equation:

Kinetic + Potential = Total

[29.10]

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$

[29.11]

$$\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G\rho_o}{3R} = -kc^2$$

[29.9]

$$\left(H^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$

- Define: $R(t_o) = 1$
- Conservation of mass:

$R^3(t)\rho(t) = R^3(t_o)\rho(t_o) = \rho_o$ [29.5]

- Hubble's law: [29.7]

  v(t) = H(t) r(t)

But also: *r(t) = R(t)* ϖ

$$\frac{dr(t)}{dt} = v(t) = \frac{dR(t)}{dt}\varpi$$

$$H(t) = \frac{v(t)}{r(t)} = \frac{\frac{dR(t)}{dt}\varpi}{R(t)\varpi} = \frac{1}{R}\frac{dR(t)}{dt}$$ [29.8]

The Critical Density

Kinetic + Potential = Total

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2 \qquad [29.10]$$

$$\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G\rho_o}{3R} = -kc^2 \qquad [29.11]$$

$$\left(H^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2 \qquad [29.9]$$

$$E = -\frac{1}{2}mkc^2\varpi^2$$

$$\text{Energy per unit mass} = -\frac{1}{2}kc^2\varpi^2$$

$k > 0$ ➔ negative E, shells will collapse back
$k = 0$ ➔ E = 0, each shell has exactly escape velocity.
$k < 0$ ➔ positive E, shells expand forever

Critical density

$$k = 0 \;\; ➔ \;\; H^2 = \frac{8}{3}\pi G\rho$$

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G} \qquad [29.15]$$

$$\rho_{c,o} = 1.88 \times 10^{-26}\, h^2 \text{ kg m}^{-3} \qquad [27.15]$$

$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)} \qquad \Omega_o = \frac{\rho_o}{\rho_{c,o}} \qquad [29.18]\ [29.19]$$

The Critical Density

Kinetic + Potential = Total

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2 \qquad [29.10]$$

$$\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G\rho_o}{3R} = -kc^2 \qquad [29.11]$$

$$\left(\frac{dR}{dt}\right)^2 = \frac{8\pi G\rho_{c,0}}{3R}$$

$$\int_0^R \sqrt{R'}\, dR' = \sqrt{\frac{8\pi G\rho_{c,0}}{3}} \int_0^t dt'$$

$$R_{\text{flat}} = (6\pi G\rho_{c,0})^{1/3} t^{2/3}$$

$$= \left(\frac{3}{2}\right)^{2/3} \left(\frac{t}{t_H}\right)^{2/3}$$

$$E = -\frac{1}{2}mkc^2\varpi^2$$

$$\text{Energy per unit mass} = -\frac{1}{2}kc^2\varpi^2$$

$k > 0$ ➔ negative E, shells will collapse back
$k = 0$ ➔ E = 0, each shell has exactly escape velocity.
$k < 0$ ➔ positive E, shells expand forever

Critical density

$$k = 0 \;\; ➔ \;\; H^2 = \frac{8}{3}\pi G\rho$$

$$\frac{8\pi G\rho_{c,0}(t)}{3} = H_o^2 = \frac{1}{t_H^2}$$

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G} \qquad [29.15]$$

$$\rho_{c,o} = 1.88 \times 10^{-26}\, h^2 \text{ kg m}^{-3} \qquad [27.15]$$

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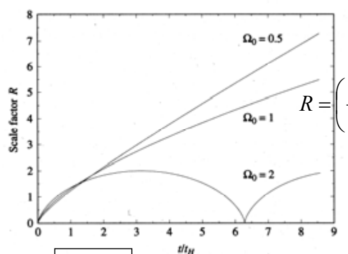
The Critical Density

Kinetic + Potential = Total

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$ [29.10]

$$\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G\rho_o}{3R} = -kc^2$$ [29.11]



$$R = \left(\frac{3}{2}\right)^{\frac{2}{3}}\left(\frac{t}{t_H}\right)^{\frac{2}{3}}$$

[Fig. 29.5]

Homework:
[CO 29.3] = [27.4 in 1st ed]
Due Oct. 17

For $\Omega \neq 1$, see parametric solutions in CO [29.32-29.39]

$$E = -\frac{1}{2}mkc^2\varpi^2$$

Energy per unit mass $= -\frac{1}{2}kc^2\varpi^2$

$k > 0$ ➔ negative E, shells will collapse back
$k = 0$ ➔ E = 0, each shell has exactly escape velocity.
$k < 0$ ➔ positive E, shells expand forever

Critical density

$k = 0$ ➔ $H^2 = \frac{8}{3}\pi G\rho$

$$\frac{8\pi G\rho_{c,0}(t)}{3} = H_o^2 = \frac{1}{t_H^2}$$

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$ [29.15]

$\rho_{c,o} = 1.88 \times 10^{-26}\ h^2$ kg m^{-3} [27.15]

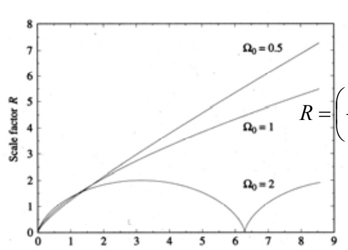
$$\Omega(t) = \frac{\rho(t)}{\rho_c(t)} \qquad \Omega_o = \frac{\rho_o}{\rho_{c,o}}$$ [29.18] [29.19]

A better way to plot R vs. t

Kinetic + Potential = Total

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$ [29.10]

$$\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G\rho_o}{3R} = -kc^2$$ [29.11]



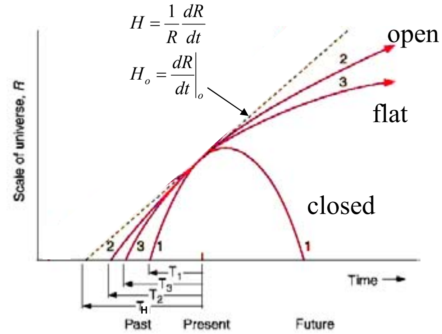
$$R = \left(\frac{3}{2}\right)^{\frac{2}{3}}\left(\frac{t}{t_H}\right)^{\frac{2}{3}}$$

[Fig. 29.5]

For $\Omega \neq 1$, see parametric solutions in CO [29.32-29.39]

$$H = \frac{1}{R}\frac{dR}{dt}$$

$$H_o = \left.\frac{dR}{dt}\right|_o$$



For flat universe:

$$\frac{t_1}{t_2} = \left(\frac{R_1}{R_2}\right)^{3/2}$$

$$\frac{t_o}{t_H} = \frac{2}{3} \qquad \text{since } R_o = 1$$

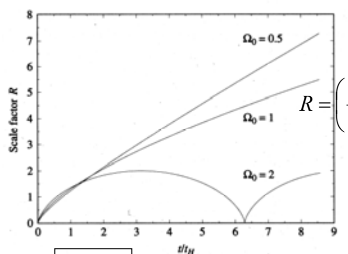
A better way t… R vs. t

SNEAK PREVIEW

Kinetic + Potential = Total

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$ [29.10]

$$\left(\frac{dR}{dt}\right)^2 - \frac{8\pi G\rho_o}{3R} = -kc^2$$ [29.11]



[Fig. 29.5]

$$R = \left(\frac{3}{2}\right)^{\frac{2}{3}}\left(\frac{t}{t_H}\right)^{\frac{2}{3}}$$

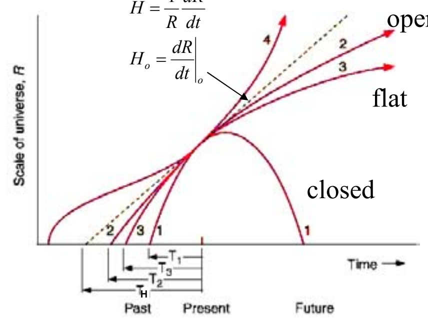
For Ω≠1, see parametric solutions in CO [29.32-29.39]

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2\right]R^2 = -kc^2$$

Includes cosmological constant

$$H = \frac{1}{R}\frac{dR}{dt}$$

$$H_o = \left.\frac{dR}{dt}\right|_o$$



For flat universe:

$$\frac{t_1}{t_2} = \left(\frac{R_1}{R_2}\right)^{3/2}$$

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Homework:
[CO 29.7] = [27.9 in 1st ed]
Due Oct. 17