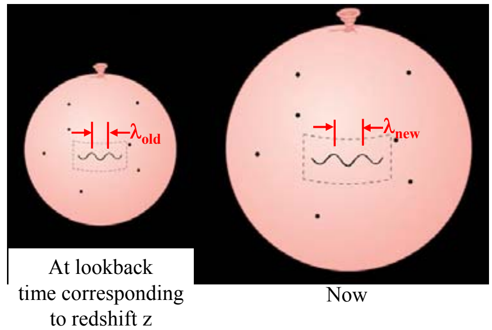
Redshift z vs. $R(t)$

$$z = \frac{\lambda_{new} - \lambda_{old}}{\lambda_{old}} = \frac{\lambda_{new}}{\lambda_{old}} - 1$$

$$R(t) = \frac{\lambda_{old}}{\lambda_{new}} = \frac{1}{1+z}$$

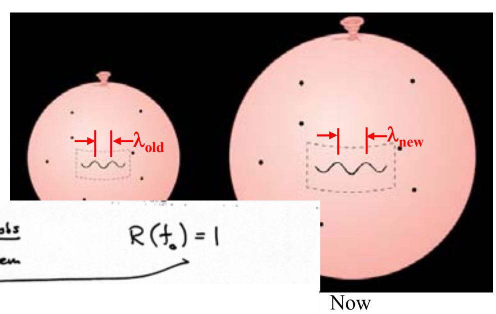


Redshift ➔ scale factor $R(t)$ at time light was emitted.

Redshift z vs. $R(t)$

$$z = \frac{\lambda_{new} - \lambda_{old}}{\lambda_{old}} = \frac{\lambda_{new}}{\lambda_{old}} - 1$$

$$R(t) = \frac{\lambda_{old}}{\lambda_{new}} = \frac{1}{1+z}$$



$R = \left(\frac{3}{2}\right)^{2/3}\left(\frac{t}{t_H}\right)^{2/3}$ $\quad 1+z = \frac{R_{obs}}{R_{em}}$ $\quad R(t_0) = 1$

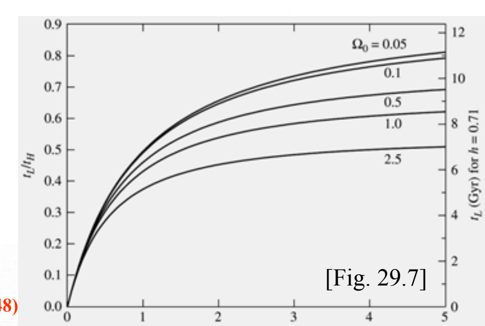
Age vs. Redshift

$$\frac{t(z)}{t(H)} = \frac{2}{3}\frac{1}{(1+z)^{3/2}} \quad \text{(for } k=0\text{)} \qquad \textbf{(29.40)}$$

Look back time →

$$t_{lookback} = t_0 - t(z) \qquad \textbf{(29.47)}$$

$$\frac{t_L}{t_H} = \frac{2}{3}\left[1 - \frac{1}{(1+z)^{3/2}}\right] \quad \text{for } k=0 \qquad \textbf{(29.48)}$$



[Fig. 29.7]

All Universes ~ "flat" ($\rho \sim \rho_c$) at early times.

- Homework problem 29.9 will show: $\Omega(t) = \dfrac{\rho(t)}{\rho_c(t)} = 1 + \dfrac{kc^2}{(dR/dt)^2}$ (29.194)

  $dR/dt \rightarrow \infty$ as $t \rightarrow 0$

- In terms of redshift, for large z : $1 + z \gg \left| \dfrac{1}{\Omega_0} - 1 \right|$

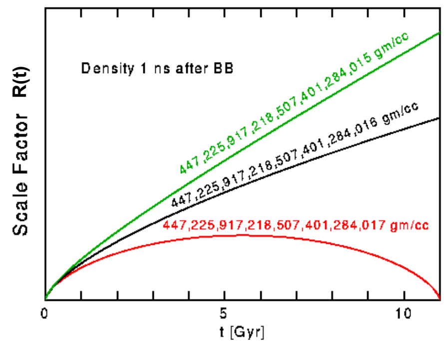
  $\dfrac{t(z)}{t_H} = \dfrac{2}{3} \dfrac{1}{(1+z)^{3/2}\,\Omega_0^{1/2}}$ (29.43) for all values of k.

- Tiny departures from ($\rho = \rho_c$) at small t (large z) grow into much larger departures, of size presently considered possible.

Homework:
[CO 29.9] = [27.11 in 1st ed]
Due Oct. 24

~ 0 in comparison to ρ when R is small

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2\right] R^2 = -kc^2$$



The deceleration parameter q_o

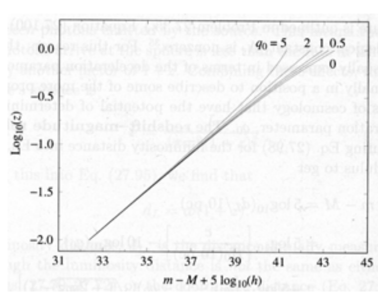
(29.54)

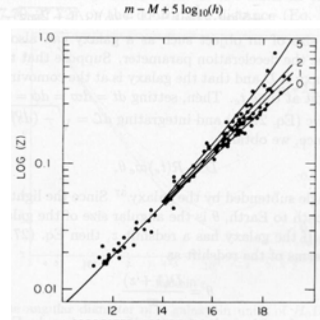
$$q(t) = -\frac{R(t)\,[d^2R(t)/dt^2]}{[dR(t)/dt]^2}$$

$$= \frac{1}{2}\Omega(t)$$

$$q_0 = \frac{1}{2}\Omega_0$$

q_o	= 0	empty
	< 0.5	open
	= 0.5	flat
	> 0.5	closed





Including Pressure

[pp. 1160-1161 2nd ed. only]

- For a fluid undergoing *adiabatic* expansion (no transfer of heat):

Work done is $dU = -PdV$

$$\frac{dU}{dt} = -\frac{4}{3}\pi P \frac{d(r^3)}{dt}$$

$$u = \frac{U}{\frac{4}{3}\pi r^3} \rightarrow \frac{d(r^3 u)}{dt} = -P\frac{d(r^3)}{dt}$$

$$\rho = \frac{u}{c^2} \rightarrow \frac{d(r^3 \rho)}{dt} = -\frac{P}{c^2}\frac{d(r^3)}{dt}$$

$$\frac{d(R^3 \rho)}{dt} = -\frac{P}{c^2}\frac{d(R^3)}{dt}$$

Friedman Equation (Energy)

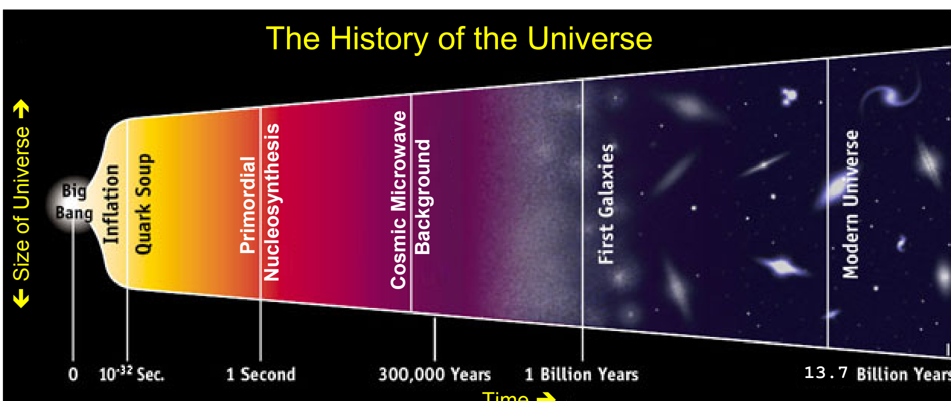
$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right]R^2 = -kc^2$$

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right]R^3 = -kc^2 R$$

Time derivative + algebra

Acceleration Equation (Force): $$\frac{d^2R}{dt^2} = -\frac{4}{3}\pi G\left(\rho + \frac{3P}{c^2}\right)R$$

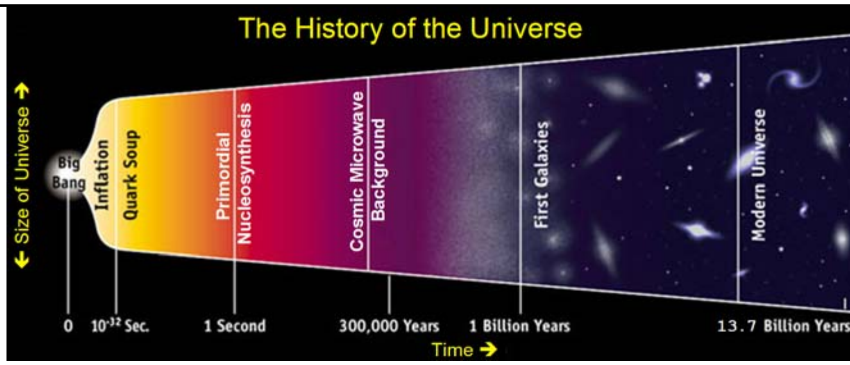
Homework:
[CO 29.12]
= derive
acceleration eqn.
Due Oct. 24



Planck time | Inflation | Formation of H, He, Li | Decoupling of CMB | Galaxy Formation | Now

Hot (10^{32} K) → (10^9 K) → (3000 K) → Cool (3 K)

High density → Low density



Event	Age of U.	Redshift	°K
Planck time; Gravity separated out	10^{-43} sec	10^{74}	10^{32}
Strong nuclear force separated out	10^{-38} sec	10^{71}	10^{29}
Inflation	10^{-36} sec	---	---
Electromagentic, Weak nuclear forces	10^{-12} sec	10^{14}	10^{15}
Nucleosynthesis of H, He, Li	1 sec - 3 min	10^{8}	10^{9}
Decoupling of CMB	300,000 yrs	1100	3000
Galaxy Formation	950 million yrs	6	19
Now	13.7 billion yrs	0	2.73

hot → cool

high density → low density

Cosmology in 1946

λ_o means observed at present time!

- Big-Bang Nucleosynthesis
  - Cooling of Universe (Alpher & Herman, 1948)
    - Radiation energy density $u_{rad} \propto \dfrac{1}{R(t)^4}$

because $$E_{phot}(t) = \frac{hc}{\lambda(t)} = \frac{hc}{\lambda_o R(t)}$$

  - Big-Bang Nucleosynthesis
    - Alpher, Bethe & Gamow (αβγ) paper --- all elements built in Big Bang?
    - Later found: can't get much past ^{4}He
- Steady State Model
  - Bondi, Gold & Hoyle
  - "Perfect" Cosmological Principle – universe same at all points and at all times
    - U has always been here.
  - Nucleosynthesis in stars
    - B^2FH

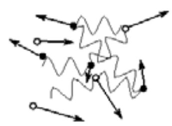
A Prediction

λ_o means observed at present time!

$$u_{rad} \propto \frac{1}{R(t)^4}$$

$$E_{phot}(t) = \frac{hc}{\lambda(t)} = \frac{hc}{\lambda_o R(t)}$$

- Hot universe ➔ filled with free electrons
- Electron opacity ➔ black body radiation field



$$u_\lambda \, d\lambda = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda kT} - 1} \, d\lambda$$

- Cooling universe: at some point, $e^- + H^+$ ➔ H^0
- Universe becomes transparent.
- ➔ relic of black body radiation field should be observable today.

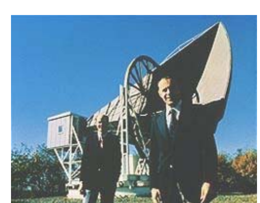
Redshifted radiation ➔ black body radiation field for a lower temperature

$$u_\lambda \, d\lambda = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda kT} - 1} \, d\lambda$$

u_o , λ_o, T = present (observed) values
$u(R)$, λ, $T(R)$ = values when $R=R(t)$

$\lambda = R\lambda_o$
$d\lambda = Rd\lambda o$

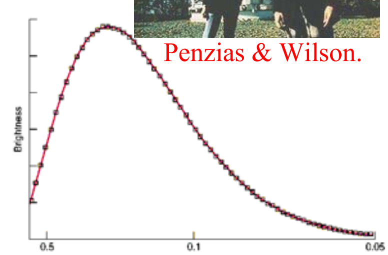
$$\begin{aligned} u_0 \, d\lambda_0 &= R^4 u(R) \, d\lambda(R) \\ &= R^4 \frac{8\pi hc/R^5\lambda_0^5}{e^{hc/R\lambda_0 kT(R)} - 1} R d\lambda_0 \\ &= \frac{8\pi hc/\lambda_0^5}{e^{hc/\lambda_0 k[RT(R)]} - 1} \, d\lambda_0. \end{aligned}$$



Penzias & Wilson.

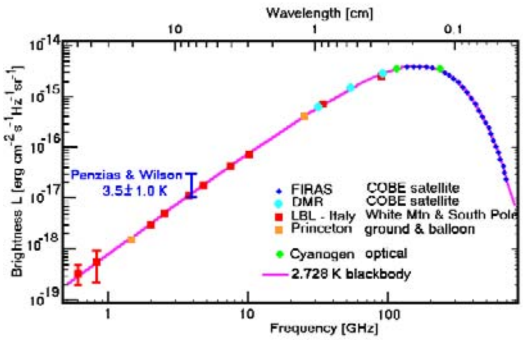
- Both shape *and* energy density are predicted.

$$T_0 = RT(R)$$

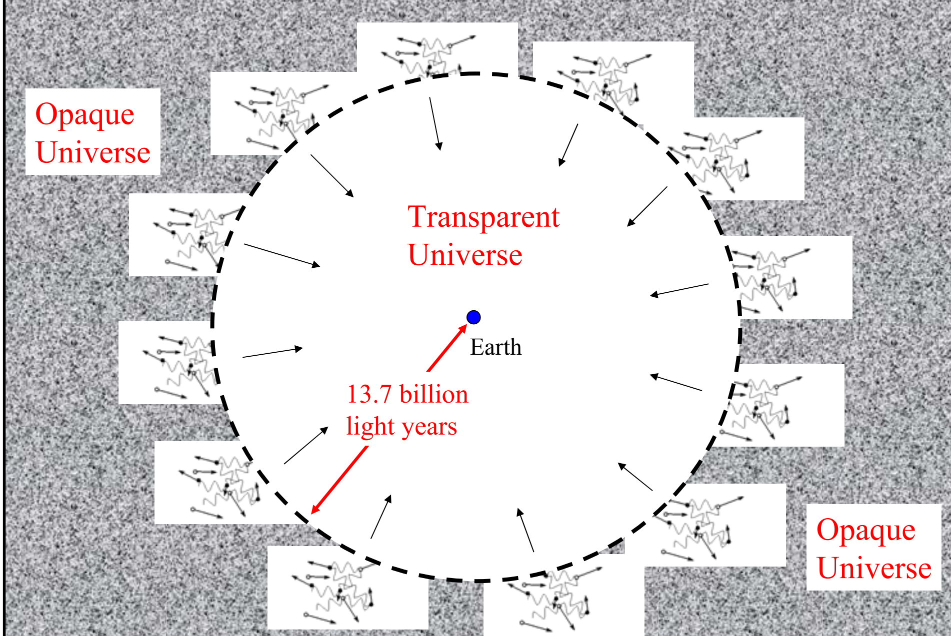


How COBE Changed Things

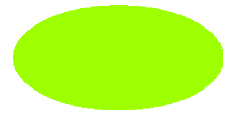
- COBE satellite (1991).



A Snapshot in ***Lookback*** Time

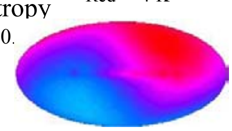


Isotropy of the Cosmic Microwave Background

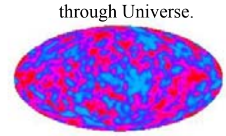


Blue = 0°K
Red = 4°K

Dipole Anisotropy
~ 1 part in 300.



Blue = 2.724°K
Red = 2.732°K
Dipole Anistropy
→ motion of Sun through Universe.

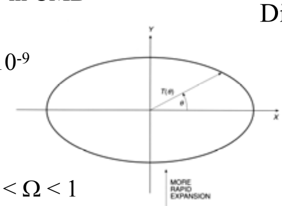


After removing dipole
Red – blue = 0.0002°K

Is the universe really isotropic?

- No quadropole anisotropy in CMB

  → $R_x/R_y = 1 \pm \varepsilon,\ \ \varepsilon < 10^{-9}$



- Vorticity:

  $(\omega/H_o) < 10^{-8}$ for $0.05 < \Omega < 1$

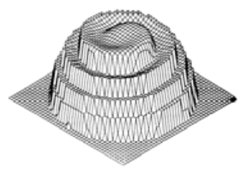
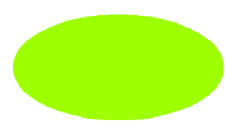


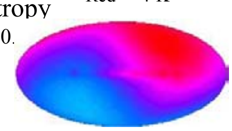
Fig. 8.6 The pattern of the CBR temperature $T(\theta, \phi)$ in a Bianchi type VII_0 model. Only one hemisphere, corresponding to $\pi/2 \leqslant \theta < \pi$, is shown. Note the 'spiral' pattern in $T(\theta, \phi)$ (from Barrow *et al.* (1985), with permission).

Isotropy of the Cosmic Microwave Background

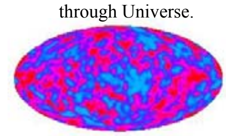


Blue = 0°K
Red = 4°K

Dipole Anisotropy
~ 1 part in 300.

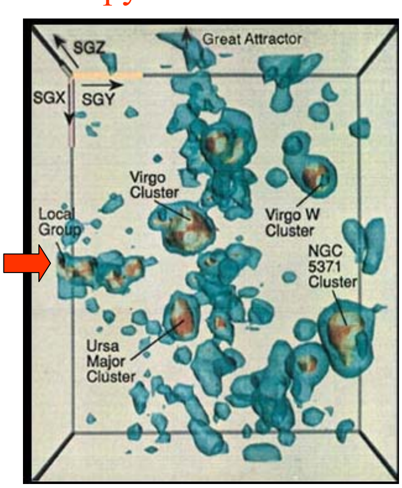


Blue = 2.724°K
Red = 2.732°K
Dipole Anistropy
→ motion of Sun through Universe.



After removing dipole
Red – blue = 0.0002°K

Local Group →



- Dipole Anistropy ~ 1 part in 300
  - 600 km/sec motion of Local Group in grav. field of larger scale mass concentrations.