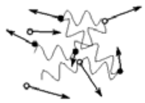
Black-Body Radiation



$$u_\lambda \, d\lambda = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda kT} - 1} \, d\lambda$$

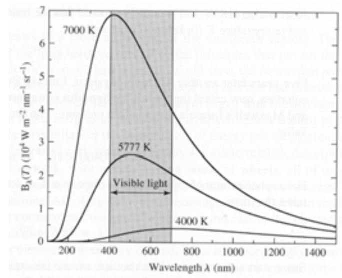


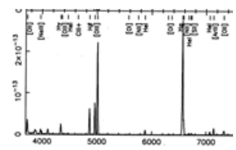
FIGURE 3.8 Blackbody spectrum [Planck function $B_\lambda(T)$].

[CO Sect. 3.4]

Black body

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

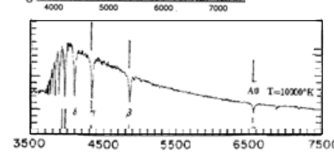
$L = A\sigma T^4$ $\qquad$ $\lambda_{\max} T = 0.002897755 \text{ m K}$



for *large* λ : (Rayleigh-Jeans tail)

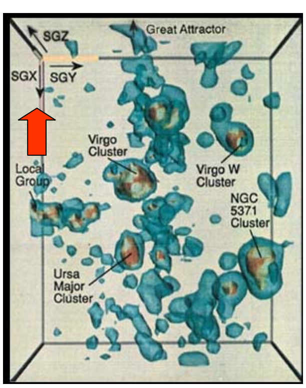
$$B_\lambda(T) \simeq \frac{2ckT}{\lambda^4}$$

also… dilute black body



Isotropy of the Cosmic Microwave Background

Local Group



Black body

$$B_\lambda(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

for *large* λ

$$B_\lambda(T) \simeq \frac{2ckT}{\lambda^4}$$

Dipole Anisotropy

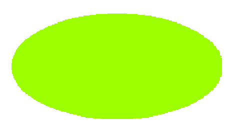
~ 1 part in 300.

$$T_{\text{moving}} = \frac{T_{\text{rest}}\sqrt{1 - v^2/c^2}}{1 - (v/c)\cos\theta}$$

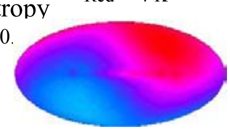
for $v << c$

$$T_{\text{moving}} \simeq T_{\text{rest}}\left(1 + \frac{v}{c}\cos\theta\right)$$

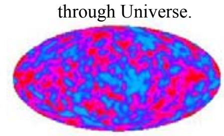
(29.62)



Blue = 0°K
Red = 4°K



Blue = 2.724°K
Red = 2.732°K
Dipole Anistropy
➔motion of Sun through Universe.



After removing dipole
Red – blue = 0.0002°K

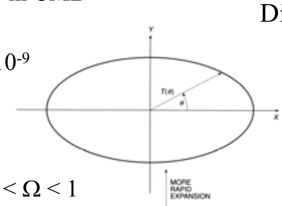
- Dipole Anistropy ~ 1 part in 300
  - 600 km/sec motion of Local Group in grav. field of larger scale mass concentrations.

Isotropy of the Cosmic Microwave Background

Is the universe really isotropic?

- No quadropole anisotropy in CMB

  ➔ $R_x/R_y = 1 \pm \varepsilon,\ \ \varepsilon < 10^{-9}$



- Vorticity:

  $(\omega/H_o) < 10^{-8}$ for $0.05 < \Omega < 1$

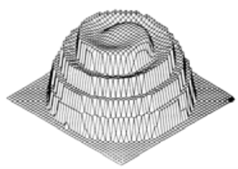
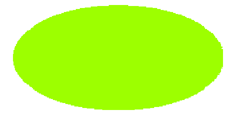
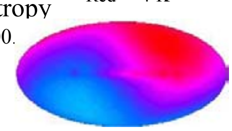


Fig. 8.6 The pattern of the CBR temperature $T(\theta, \phi)$ in a Bianchi type VII_0 model. Only one hemisphere, corresponding to $\pi/2 \leqslant \theta < \pi$, is shown. Note the 'spiral' pattern in $T(\theta, \phi)$ (from Barrow *et al.* (1985), with permission).

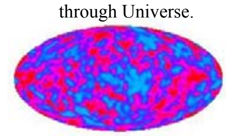


Blue = 0^oK
Red = 4^oK

Dipole Anisotropy
~ 1 part in 300.



Blue = 2.724^oK
Red = 2.732^oK
Dipole Anistropy
➔motion of Sun through Universe.



After removing dipole
Red – blue = 0.0002^oK

When did decoupling occur?

- Saha equation: <u>Collisional</u> ionization rate = Recombination rate

$$\frac{N_{H^+}}{N_{H^0}} = \frac{4}{N_e}\left(\frac{2\pi m_e k T}{h^2}\right)^{3/2} e^{-\chi_I/kT} \qquad (8.8)$$

- Solve for $\frac{N_{H^+}}{N_{H^0}} = 1$, using $T = \frac{T_o}{R}$, $N_e \sim \frac{\rho_o}{m_H}\frac{1}{R^3}$

  ➔ $R \sim 7.2\text{x}10^{-4}$
  $T \sim 3800$ K

- Taking composition and radiative transfer into account:

  $T_{dec} = 2970$K $\qquad z_{dec} = 1089$
  $R_{dec} = 9\text{x}10^{-4}$ $\qquad t_{dec} = 118{,}000$ yrs.

Decoupling also called "**re**combination"

The Radiation Era

$$T_0 = RT$$
$$T = \frac{T_0}{R}$$

Energy density (integrated over wavelength):

$$u = aT^4$$

$$\rho_{RAD} = \frac{u}{c^2} = \frac{aT^4}{c^2} = \frac{\rho_{RAD,0}}{R^4}$$

$$\rho_{matter} = \frac{\rho_{m,0}}{R^3}$$

$$\Rightarrow \frac{\rho_{RAD}}{\rho_{matter}} \propto \frac{1}{R}$$

$\rho_{matter} = \rho_{rad}$ at

$$R = \frac{aT_0^4}{\rho_0 c^2} = \frac{8\pi G a T_0^4}{3H_0^2 c^2 \Omega_0} \sim 2.5\times10^{-5}\,\Omega_0^{-1} h^{-2}$$

$$z = \frac{1}{R} - 1 \sim 4\times10^4\,\Omega_0 h^2$$

when $T = \frac{T_0}{R} \sim 1.1\times10^5\,\Omega_0 h^2$ K

$$Age = \frac{2}{3}\,\frac{1}{H_0(1+z)^{3/2}\,\Omega_0^{1/2}} \sim 3200 \text{ years}$$

for $\Omega_0 = 1$, $h = 0.71$

During the Radiation Era:

$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G \rho_{RAD}\right] R^2 = -kc^2$$

$k \sim 0$

$$\left(\frac{1}{R}\frac{dR}{dt}\right)^2 = \frac{8}{3}\pi G\,\frac{\rho_{RAD,0}}{R^4}$$

$$\left(\frac{dR}{dt}\right)^2 \propto \frac{1}{R^2}$$

$$R\,dR \propto dt$$

$$\boxed{R \propto t^{1/2}}$$

instead of $R \propto t^{2/3}$ in matter era.

also, $T_0 = RT \Rightarrow T \propto t^{-1/2}$

Terminology…

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho\right)R^2 = -kc^2$$

$$\left(\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\left(\rho_{matter} + \rho_{radiation}\right)\right)R^2 = -kc^2$$

$$\rho_{radiation} = \frac{u_{radiation}}{c^2}$$

$$\Omega_{radiation} = \frac{\rho_{radiation}}{\rho_{critical}}$$

PRIMORDIAL NUCLEOSYNTHESIS

Radiation era: $R(t) \propto t^{1/2}$; $RT = \text{constant} = 2.726\,°K \Rightarrow T(t) \propto t^{-1/2}$

$t = 10^{-4}\,s$, $T = 10^{12}\,K$ $\quad p^+ + e^- \rightleftarrows n + \nu_e$, etc. electrons from $\gamma \rightleftarrows e^- + e^+$

Statistical equilibrium $\quad \frac{n_n}{n_p} = e^{-\frac{\Delta E}{kT}} = e^{-\frac{1.5\times10^{10}K}{T}}$

$t \sim 1\,s$, $T \sim 1\times10^{10}\,K$

neutrons "freeze out" $\quad \frac{n_n}{n_p} = 0.223$

- redshifting of neutrinos
- lower γ energies.

subsequent β decays $\Rightarrow \quad \frac{n_n}{n_p} \sim 0.164$

$t \sim 3\,\text{min},\ T \sim 10^9\,K$

${}^4He$ production:

$$d + d \rightleftarrows {}^3He + {}^1H$$
$$ {}^3He + d \rightleftarrows {}^4He + n$$

$$d + d \rightleftarrows {}^3He + n$$
$$ {}^3He + d \rightleftarrows {}^4He + {}^1H$$

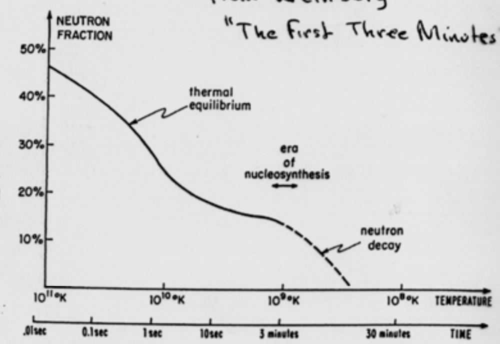
~all neutrons $\longrightarrow$ ${}^4He$

$\Rightarrow$ mass fraction of ${}^4He$

$$= \frac{2n_n}{n_n + n_p} \sim 0.281$$

$\boxed{\text{ROUGHLY INDEPENDENT OF EXACT DENSITY}}$

from Weinberg "The First Three Minutes"



Observed $\frac{{}^4He}{\text{Total}} = 0.228 \pm .005$ (Pagel et al. 1992)

or something close to that.