http://www.philforhumanity.com/Infinity_Divided_by_Infinity.html

What does Infinity Divided by Infinity Equal?

At first, you may think that infinity divided by infinity equals one. After all, any number divided by itself is equal to one, however infinity is not a real or rational number. I am going to prove what infinity divided by infinity really equals, and you may not like the answer.

First, I am going to define this axiom (assumption) that infinity divided by infinity is equal to one:

$$\frac{\infty}{\infty} = 1$$

Since $\infty = \infty + \infty$, then we are going to substitute the first infinity in our axiom:

$$\frac{\infty + \infty}{\infty} = 1$$

The next step is to split this fraction into two fractions:

$$\frac{\infty}{\infty} + \frac{\infty}{\infty} = 1$$

Next, substitute the axiom twice into the equation, we get:

$$1 + 1 = 1$$

Finally, this can be rewritten as:

$$2 = 1$$

http://mathforum.org/library/drmath/view/53337.html

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Infinity over Infinity

Date: 4/10/96 at 18:50:50
From: "Jeremy Vigneault"
Subject: Infinity question

Dear Dr. Math,

I have a question maybe you can answer. My electronics teacher and I are having a disagreement. He says that Infinity divided by Infinity equals one. I most certainly disagree: I say that infinity over infinity is indeterminate, because infinity is a concept, not a finite number. Could you please help me in this discrepancy?

Date: 4/12/96 at 21:36:13
From: Doctor Syd
Subject: Re: Infinity question

Dear Jeremy,

Good work! You are correct. Many people are confused by infinity; you are right that is a concept and not a number the way that 28 is a number. There are sort of some different "sizes" of infinities, so this means that a quotient that looks like infinity over infinity can sometimes be a real number, and sometimes it is just infinity.

Maybe to prove your case to your teacher you could think about the following problem:

What is the limit as x approaches infinity of the expression x^2/(x-5)? If you "plug in" infinity for x in this expression you getinfinity over infinity, but if you apply L'Hopital's Rule, you see the the numerator dominates the denominator (you can see this without L'Hopital's Rule, too: x^2 gets big a lot faster than x does, right?), so the limit as x approaches infinity will be infinity, not one.

Good question! Hope this helped some.

-Doctor Syd, The Math Forum

[29.4] Observational Cosmology

$$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\, d\theta)^2 + (\varpi \sin\theta\, d\phi)^2\right]$$

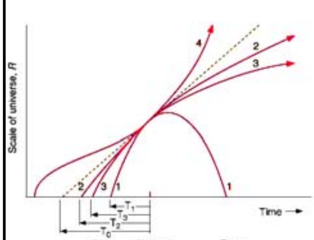
$$\left[\left(\frac{1}{R}\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho - \frac{1}{3}\Lambda c^2\right] R^2 = -kc^2$$

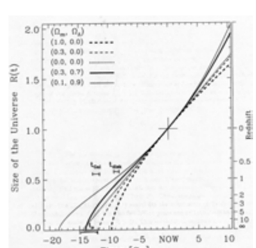
$$H_0^2(1-\Omega_0) = -kc^2$$

- Some theoretical parameter sets:
  - *R(t)* vs. *t*
  - $\Omega_{\Lambda,o}$ vs. $\Omega_{m,o}$
  - Curvature *k, dR/dt, d²R/dt²*

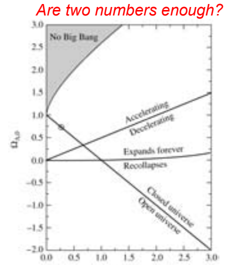
- But what can we actually *measure* that will tell us which universe we live in?

  As a function of *z*:
  - Apparent mag. of standard candles.
  - Angular sizes.
  - Space density of galaxies.

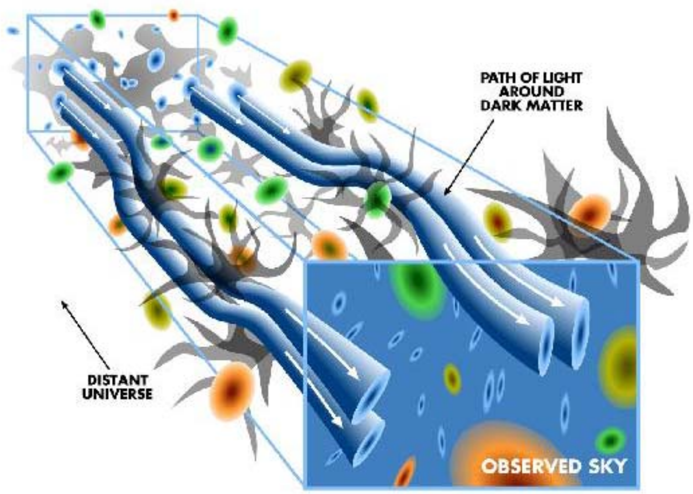




Are two numbers enough?



The actual path of light through space.
(sketched in 3D space)



[29.4] Redshift and Cosmological Time Dilation

Use neg. sq. root:

$$\frac{-c\,dt}{R(t)} = \frac{d\varpi}{\sqrt{1-k\varpi^2}}$$

so that ϖ decreases with t.

$$(ds)^2 = 0 = (c\,dt)^2 - R^2(t)\left[\frac{d\tilde{\varpi}}{\sqrt{1-k\tilde{\varpi}^2}} + \cdots\right]$$

For two radially travelling wavecrests, emitted at t_e, $t_e+\Delta t_e$

$$\int_{t_e}^{t_0} \frac{dt}{R(t)} = \frac{1}{c}\int_0^{\tilde{\varpi}_e} \frac{d\tilde{\varpi}}{\sqrt{1-k\tilde{\varpi}^2}}$$

$$= \int_{t_e+\Delta t_e}^{t_0+\Delta t_0} \frac{dt}{R(t)}$$

$\int \frac{dt}{R(t)}$ ↓ t_e, t_0, $t_e+\Delta t_e$, $t_0+\Delta t_0$

(See pg. 1200)

$$\Rightarrow \int_{t_0}^{t_0+\Delta t_0} \frac{dt}{R(t)} = \int_{t_e}^{t_e+\Delta t_e} \frac{dt}{R(t)}$$

$$\frac{\Delta t_e}{R(t_e)} = \frac{\Delta t_0}{R(t_0)}$$ time dilation

$$\frac{\Delta t_0}{\Delta t_e} = 1+z$$

$$\Rightarrow \frac{R(t_0)}{R(t_e)} = \frac{\lambda_0}{\lambda_e} = 1+z$$ [CO 29.142]

cosmological redshift

Proper distance

= the *current* distance to a distant object.

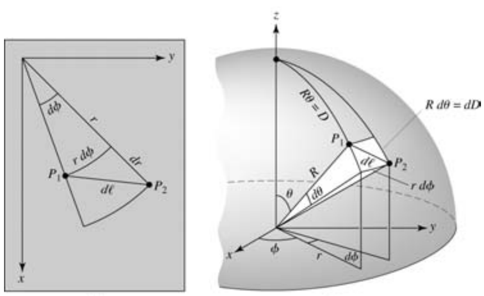
$$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\,d\theta)^2 + (\varpi\sin\theta\,d\phi)^2\right]$$

$dt = 0$, proper distance $d_p(t) = \text{sqrt}(-ds^2)$

Random sidetrack in [CO]:

$$d_p(t) = R(t)\int_{t_e}^{t_0}\frac{c\,dt'}{R(t')}$$

$$d_p(t) = R(t)\int_0^{\varpi}\frac{d\varpi'}{\sqrt{1-k\varpi'^2}}$$



Flat: $d_{p,0} = \varpi$

Closed: $d_{p,0} = \frac{1}{\sqrt{k}}\sin^{-1}(\varpi\sqrt{k})$

Open: $d_{p,0} = \frac{1}{\sqrt{|k|}}\sinh^{-1}(\varpi\sqrt{|k|})$

The particle horizon

Horizon distance = distance a photon has traveled since $t = 0$.

$$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\,d\theta)^2 + (\varpi\sin\theta\,d\phi)^2\right]$$

$$d_h(t) = R(t)\int_0^t \frac{c\,dt'}{R(t')}$$

Radiation dominated flat universe: $R \propto t^{1/2}$ ➔ $d_h(t) = 2ct$

Matter dominated flat universe: $R \propto t^{2/3}$ ➔ $d_h(t) = 3ct$

Matter dominated flat universe in terms of *redshift* ➔ $d_h(z) = \frac{2c}{H_0\sqrt{\Omega_{m,0}}}\frac{1}{(1+z)^{3/2}}$

Including Ω_Λ ➔ $d_h(t) = \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{1/3}\sinh^{2/3}\left(\frac{3}{2}H_0 t\sqrt{\Omega_{\Lambda,0}}\right)\int_0^t \frac{c\,dt'}{\left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)^{1/3}\sinh^{2/3}\left(\frac{3}{2}H_0 t'\sqrt{\Omega_{\Lambda,0}}\right)}$

= 14.6 Gpc (WMAP)

[29.158]

The paths of photons in terms of proper distance.

$$(ds)^2 = (c\,dt)^2 - R^2(t)\left[\left(\frac{d\varpi}{\sqrt{1-k\varpi^2}}\right)^2 + (\varpi\,d\theta)^2 + (\varpi\sin\theta\,d\phi)^2\right]$$

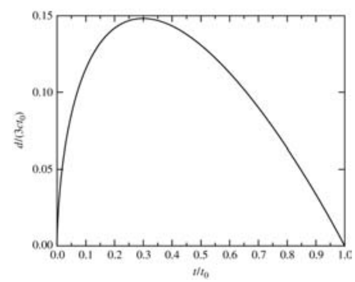
$$\int_0^t \frac{c\,dt'}{R(t')} = \int_{\varpi}^{\varpi_e} d\varpi'$$

Matter dominated flat universe:

$$R(t) = \left(\frac{t}{t_0}\right)^{2/3}$$

$$\varpi = \varpi_e - 3ct_0\left(\frac{t}{t_0}\right)^{1/3}$$

At $t = t_o$, $\varpi = 0$ ➔ $\varpi_e = 3ct_0$



$$R(t)\varpi = d_p(t) = 3ct_0\left[\left(\frac{t}{t_0}\right)^{2/3} - \left(\frac{t}{t_0}\right)\right]$$

[29.165]