Relationship between ϖ and redshift z

In principle

- R-W metric:

$$d_p(t) = R(t)\int_0^{\varpi}\frac{d\varpi'}{\sqrt{1-k\varpi'^2}}$$

Proper Distance

$$\int_{t_e}^{t_o}\frac{c\,dt}{R(t)} = \int_0^{\tilde{\omega}_e}\frac{d\tilde{\omega}}{\sqrt{1-k\tilde{\omega}^2}}$$

$$\int_{R(t_e)}^{R(t_o)}\frac{c\,dR}{R\frac{dR}{dt}} = \int_0^{\tilde{\omega}_e}\frac{d\tilde{\omega}}{\sqrt{1-k\tilde{\omega}^2}}$$

- Friedmann Eqn:

$$\rho(t) = \rho_m(t) + \rho_{rel}(t) + \rho_\Lambda(t)$$

$$\left(\frac{dR}{dt}\right)^2 - \frac{8}{3}\pi G\rho(t)R^2(t) = -kc^2$$

$$\frac{dR}{dt} = \sqrt{\frac{8}{3}\pi G\rho(t)R^2(t) - kc^2}$$

also: $R(t_o) = 1$ $\quad R(t_e) = \frac{1}{1+z}$

$$\int_{\frac{1}{1+z}}^{1}\frac{c\,dR}{R\sqrt{\cdots}} = \int_0^{\tilde{\omega}_e}\frac{d\tilde{\omega}}{\sqrt{1-k\tilde{\omega}^2}}$$

In practice

(because of that @#$% cosmological constant)

$$I(z) = H_0\int_{\frac{1}{1+z}}^{1}\frac{dR}{R(dR/dt)} = H_0\int_0^z\frac{dz'}{H(z')}. \qquad (29.167)$$

Using Eq. (29.122), we obtain

$$I(z) \equiv \int_0^z\frac{dz'}{\sqrt{\Omega_{m,0}(1+z')^3 + \Omega_{rel,0}(1+z')^4 + \Omega_{\Lambda,0} + (1-\Omega_0)(1+z')^2}}. \qquad (29.168)$$

With this definition of the integral $I(z)$, the present proper distance is

$$d_{p,0}(z) = \frac{c}{H_0}I(z). \qquad (29.169)$$

$$\varpi(z) = \frac{c}{H_0}S(z)$$

$$\begin{aligned} S(z) &= I(z) && (\Omega_0 = 1)\\ &= \frac{1}{\sqrt{\Omega_0 - 1}}\sin\left[I(z)\sqrt{\Omega_0 - 1}\right] && (\Omega_0 > 1)\\ &= \frac{1}{\sqrt{1-\Omega_0}}\sinh\left[I(z)\sqrt{1-\Omega_0}\right] && (\Omega_0 < 1).\end{aligned}$$

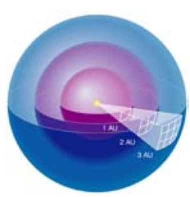
$$I(z) = \int_0^z\left\{1 - (1+q_0)z' + \left[\frac{1}{2} + 2q_0 + \frac{3}{2}q_0^2 + \frac{1}{2}(1-\Omega_0)\right]z'^2 + \cdots\right\}dz' \qquad (29.178)$$

$$I(z) = z - \frac{1}{2}(1+q_0)z^2 + \left[\frac{1}{6} + \frac{2}{3}q_0 + \frac{1}{2}q_0^2 + \frac{1}{6}(1-\Omega_0)\right]z^3 + \cdots. \qquad (29.179)$$

$$\boxed{d_{p,o} \sim \quad \varpi \simeq \frac{cz}{H_0}\left[1 - \frac{1}{2}(1+q_0)z\right] \qquad (\text{for } z \ll 1).}$$

Eqns. 29.180, 29.181, true for all models

Luminosity Distance



$$F = \frac{L}{4\pi d^2}$$

$$F = \frac{L}{4\pi\varpi^2(1+z)^2}$$

Redshift ➔ $(1+z)$
Time dilation ➔ $(1+z)$

$$d_L = \varpi(1+z)$$

About right…

$$d_L(z) \simeq \frac{cz}{H_0}\left[1 + \frac{1}{2}(1-q_0)z\right] \qquad (\text{for } z \ll 1)$$

$$m - M \simeq 5\log_{10}\left[\frac{c}{(100\ \text{km s}^{-1}\ \text{Mpc}^{-1})(10\ \text{pc})}\right] - 5\log_{10}(h)$$
$$+ 5\log_{10}(z) + 5\log_{10}\left[1 + \frac{1}{2}(1-q_0)z\right] \qquad (\text{for } z \ll 1).$$

In practice

(because of that @#$% cosmological constant)

$$d_L(z) = \frac{c}{H_0}(1+z)S(z)$$

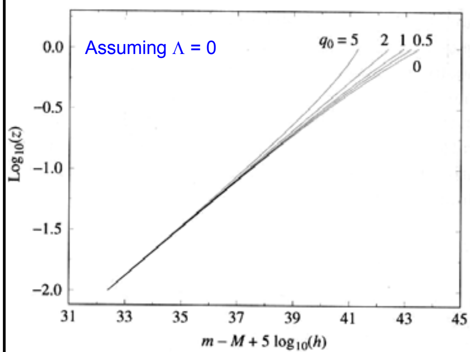
$$\begin{aligned} S(z) &= I(z) && (\Omega_0 = 1)\\ &= \frac{1}{\sqrt{\Omega_0 - 1}}\sin\left[I(z)\sqrt{\Omega_0 - 1}\right] && (\Omega_0 > 1)\\ &= \frac{1}{\sqrt{1-\Omega_0}}\sinh\left[I(z)\sqrt{1-\Omega_0}\right] && (\Omega_0 < 1).\end{aligned}$$

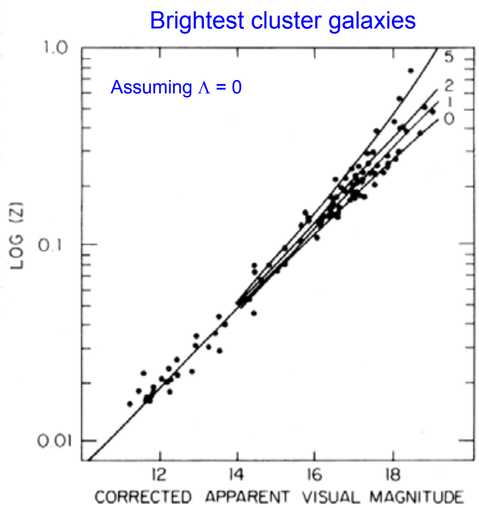
$$\boxed{m - M = 5\log_{10}(d_L/10\ \text{pc})}$$

$$m - M = 5\log_{10}\left[\frac{c}{(100\ \text{km s}^{-1}\ \text{Mpc}^{-1})(10\ \text{pc})}\right] - 5\log_{10}(h)$$
$$+ 5\log_{10}(1+z) + 5\log_{10}[S(z)]$$
$$= 42.38 - 5\log_{10}(h) + 5\log_{10}(1+z) + 5\log_{10}[S(z)].$$

$$\boxed{m - M \simeq 42.38 - 5\log_{10}(h) + 5\log_{10}(z) + 1.086(1-q_0)z \qquad (\text{for } z \ll 1). \qquad (29.188)}$$

Old Redshift-Magnitude Results

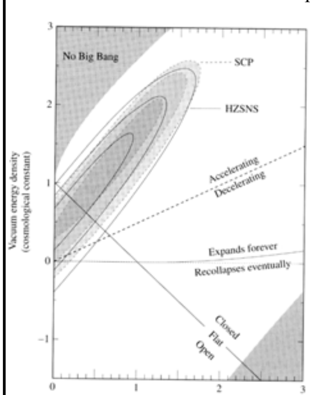


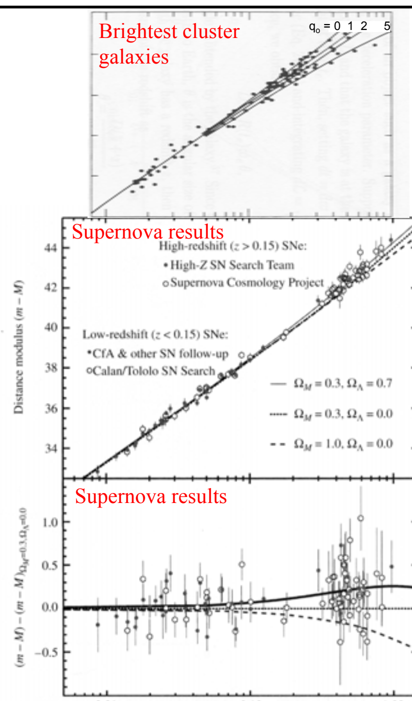


- Curves diverge at larger z
  - ➔ use very luminous objects which are observable at large z.
- Best bet = brightest cluster galaxies
  - But big correction for evolution of stellar population.

q_o – the accelerating universe

- Type Ia supernovae are best standard candles.
  - Least scatter in luminosity
- 2 independent groups get same answer
  - Supernova Cosmology Project
    - Perlmutter et al. 1999 ApJ 517, 565
  - High-*z* Supernova Search
    - Garnavich et al. ApJ 509, 74
- Found *acceleration*
  - Not deceleration as expected.





[CO fig. 29.27]