More about the game plan:

• NOW… Kinematics of spiral galaxies
  • Rotation curves ➔ mass distribution [CO 24.3]
  • SKIP [CO 24.4 “The Galactic Center”]
• Spiral structure [CO 25.3]
• General properties of S and Irr galaxies [CO 25.2]
• E galaxies [CO 25.4]
• Midterm on Wednesday Oct. 3

NOTE: ALL OF THE SLIDES ARE ON THE COURSE WEB SITE AS PDF FILES, 2 SLIDES PER PAGE.
www.pa.msu.edu/courses/ast308

Rotation Curves

• Spherical mass shell, uniform density:
  If outside shell:
    \[ v^2 = \frac{\frac{GM}{\rho}}{r} \]
  \[ v = \left( \frac{GM}{\rho} \right)^{1/2} \]

• Inside spherical mass distribution:
  \[ M(r) = \rho \cdot \frac{4}{3} \pi r^3 \]
  \[ v \propto r \]

• Outside mass distribution:
  \[ M(r) = \text{constant} \]
  \[ v \propto r^{-1/2} \]

• Sph. Distr. + exponential disk
    \[ v = \left( \frac{C}{r} \right) \left[ M(0) + 4 \pi \rho_d r^2 \right]^{1/2} \]
    \[ \rho = \frac{\rho_d}{e} \quad I_K = \text{Bessel function} \]
The Milky Way’s Rotation Curve

Radio data
Tangent point method
Young star clusters

Rotation curves in other galaxies

- Vera Rubin & Kent Ford (late 1970’s)
- Image tube spectrograph
Density as shown by flat rotation curves

- \( \frac{dM}{dr} \sim \text{constant} 
- \) Unbounded mass distribution??
- NFW profile
  \[ \rho(r) = \frac{\rho_0}{(r/a)(1+r/a)^2} \]
  - Predicted for Cold dark matter (CDM)
  - Actual derived dark matter profiles often slightly different than this
  - What is CDM? Coming later in course.

\[ \text{Back to } f_{\text{gravitational}} = f_{\text{canoncial}} \]

\[ \frac{m v^2}{2} = \frac{GM}{r} \]
\[ M(r) = \frac{v^2 r}{G} \]
\[ \frac{dM(r)}{dr} = \frac{v^2}{G} \]
\[ \frac{d^2M(r)}{dr^2} = \frac{4\pi r^2 \rho}{G} \]
\[ \Rightarrow \rho(r) = \frac{v^2}{4\pi G r^2} \]
\[ v = \text{constant} \Rightarrow \rho(r) \propto \frac{1}{r^3} \]
\[ \text{Use } \rho(r) = \frac{C}{(r/a)^2} \text{ at } CO \]

Homework Assignment 5
Due Monday Oct. 1

- CO 2\text{nd} edition problems 25.13, 25.14, 15.16
  - Same as 1\text{st} edition problems 23.11, 23.12, 23.14

- There may be one addition derivation-type problem having to do with the stellar velocities found in E galaxies. It depends on whether I cover that in class with enough lead time.
What causes spiral structure?

- Winding up of arms
  - Due to differential rotation

- Stochastic, Self-Propagating Star Formation
  - Chain-reaction star formation
  - SN shells $\Rightarrow$ shock fronts $\Rightarrow$ density enhancements $\Rightarrow$ star formation $\Rightarrow$ more SN
  - Differential rotation then winds these regions up into spiral patterns

- Density Waves
  - Wave in gravitational potential
  - Orbital velocity of stars different than pattern speed
    - Stars, gas bunch up at position of spiral arms
    - Causes higher grav. potential

- Unclear if self-sustaining or forced.
Spiral Arms & the Interstate Highway

- Density wave
  - Spiral arms have higher density than space between arms
  - Excess gravitational attraction slows down gas, stars when they pass through spiral arm in course of their orbits.
  - ➔ spiral arms are a traffic jam
Retrograde Motion & Ptolemy’s Epicycles

- Trying to place Earth at center.
- Using only circular motions.
- Led to very complicated system.

Path of Mars, etc. as seen from Earth

U = potential energy
Φ = U/M

\[ M \ddot{R} = -\nabla U(R, z) = \frac{\partial U}{\partial R} \dot{R} - \frac{1}{R} \frac{\partial U}{\partial \phi} \dot{\phi} - \frac{\partial U}{\partial z} \dot{z} \]

Circular symmetry \( \Rightarrow \) independent of \( \phi \)

\[ \frac{d^2 \phi}{dt^2} = -\frac{\partial \Phi}{\partial R} \dot{R} - \frac{\partial \Phi}{\partial \theta} \dot{\theta} \]

\[ \frac{d^2 z}{dt^2} = \left( \ddot{R} - \frac{\dot{R}^2}{R} \right) \dot{R} + \frac{1}{R} \frac{\partial (R^2 \dot{\phi})}{\partial t} \dot{\theta} + \ddot{\theta} \]

Separate \( d^2 r/\text{d}t^2 \) into \( R, \phi, z \) components \( \Rightarrow \) 3 equations.

(25.17)

\[ \ddot{R} - \frac{\dot{R}^2}{R} = -\frac{\partial \Phi}{\partial R} \]

\[ \ddot{\phi} = 0 \]

\[ \ddot{z} = -\frac{\partial \Phi}{\partial z} \]

Define an effective potential:

\[ \Phi_{\text{eff}}(R, z) \equiv \Phi(R, z) + \frac{J_z^2}{2R^2} \]

Conservation of \( J_z \) \( \Rightarrow \) acceleration in \( \phi \) direction when \( r \) changes.

Epicycles… the short form.

For lurid details, see [CO 25.3 (23.3 1st Ed.)]

Conservation of specific angular momentum \( J_z = R^2 \dot{\phi}/\text{d}t \)

Taylor series expansion around position of minimum \( \Phi_{\text{eff}} \) (circular orbit):

\[ \Phi_{\text{eff}}(R, z) \approx \Phi_{\text{eff},m} + \left. \frac{\partial \Phi_{\text{eff}}}{\partial R} \right|_{m} \rho + \left. \frac{\partial^2 \Phi_{\text{eff}}}{\partial z^2} \right|_{m} \dot{z}^2 + \cdots \]

(25.31)

\[ \rho \approx -\kappa^2 \rho \]

(25.32-25.33)
Harmonic oscillation in $R, \phi, z$ about circular orbit (Epicycles)

\[ \dot{\rho} \simeq -\kappa^2 \rho \]
\[ \ddot{z} \simeq -\nu^2 z \]

\[ \rho(t) = R(t) - R_m = A_R \sin \kappa t \]
\[ z(t) = A_z \sin (\nu t + \zeta) \]
\[ \phi(t) = \phi_0 + \frac{J_R}{R_m^2} t + \frac{2J_z}{\kappa R_m} A_R \cos \kappa t = \phi_0 + \Omega t + \frac{2\Omega}{\kappa R_m} A_R \cos \kappa t \]

In inertial frame:

Orbits closed if:

\[ m (\Omega - \Omega_{lp}) = n \kappa \]
\[ \Omega_{lp}(R) = \Omega(R) - \frac{n}{m} \kappa(R) \]

Viewed from frame rotating with $\Omega_{lp}$:

Two ways to line up closed elliptical orbits (as seen from frame rotating with $\Omega_{lp}$)

Basic nature of a density wave


- At each $R_m$, stars’ positions in epicycles are forced into a specific pattern by gravitational potential of spiral arm.
- Sum of positions of stars at this $R_m$ forms an ellipse rotating at pattern speed.
- Spiral density pattern is sum of many ellipses, all rotating at same pattern speed.
Some Solar System Resonance Phenomena

- Asteroid orbits
- Gaps in Saturn's rings

Lin & Shu's theory

- Resultant gravitational field due to stars and gas
- Total material needed to maintain the resultant gravitational field

Density response of the stellar disk

Density response of the gaseous disk

Total response in the distribution of the matter in the disk

This equation serves to specify the wave properties completely

Figure 12.26. The calculational scheme used to calculate the normal modes of oscillation of a disk galaxy. (After C. C. Lin)

Perturbed form of collisionless Boltzmann equation. “quite complicated”

Hydrodynamics

Angular velocity $\Omega_0$

Milky Way

Figure 12.27. Contours of equal density contour above the average value around a circle in a typical spiral mode. The dotted line gives the radius where the material rotates at the same speed as the wave pattern
Density wave theory interprets most spirals as 2-armed
• 4-armed pattern is $n / m = 1 / 4$
• exists over a narrow range of radius.
  ➔ less likely to be seen.

• Actual 4-armed spirals are superposition of two 2-armed patterns

Spiral Structure

Grand design (10%)  
M51
Multi-arm (60%)  
M101
Flocculent (30%)  
NGC 2841

Inner rings  
NGC 7096
Outer Ring  
NGC 4340
M81 spiral structure at different wavelengths

UV: hot stars

Visible: stars + obscuration

Near IR: late-type stars

21 cm: HI

Old red population shows small but real spiral density enhancement.

Passage of gas through spiral arms

Calculated streamlines for gas
Response of gas to density waves

- Simple pendulum model
  - Each pendulum = 1 gas cloud
  - For large amplitude forcing, pendulums collide.
  - → shock fronts in spiral arms

- HI map (right) shows velocity jumps at spiral arms.

Orbits in Barred Spirals

- Gas avoids “co-rotation” radius in barred potential.
- Causes “Fig-8” shape in rotation curve.
Bars appear to be easily excited instability in disks

Trailing vs. leading spirals
Which is the near side of the galaxy?
Molecular clouds on inner edges of arms

CO contours over red image

CO contours over 21 cm map

Swing Amplification

Position of (leading) spiral density enhancement

Epicyclic orbit of star

While it is swing around, the spiral arm moves at about same angular speed as star.

Automatically converts any leading spirals into much stronger trailing spirals.

Time steps = \( \frac{1}{2} \) of crotation period at CR.
Spiral Structure of the Milky Way

Recent model
- mix of
  - 2-armed mode
  - 4-armed mode
- Sun at ~ co-rotation radius.

From HI (21 cm observations) assuming circular rotation.

Map of nearby young objects

N-body simulation

Hard to measure, because we are inside it.

Summary: Density Waves?
- Evidence showing density waves do occur.
  - Old, red stars show spiral density perturbation.
  - Molecular clouds form on inner edges of spiral arms.
  - HI gas flow shows discontinuity due to shocks at inner edges of spiral arms.
  - Bright young stars also in narrow arms.
    - Observed width Δθ ~ t_s(Ω - Ω_0), as predicted.
- Are these waves self-sustaining over 10^10 years? Problems:
  - Lin-Shu theory is linear; does not predict whether waves will grow or decay.
  - How are density waves initially formed?
- The usual interpretation
  - Density waves need a driving force
    - Satellite galaxy at co-rotation radius (M51)
    - Bars
  - Otherwise, act to prolong life of transitory phenomena.
  - Other mechanisms probably also important.
    - Swing-amplification efficiently builds up temporary trailing spirals.