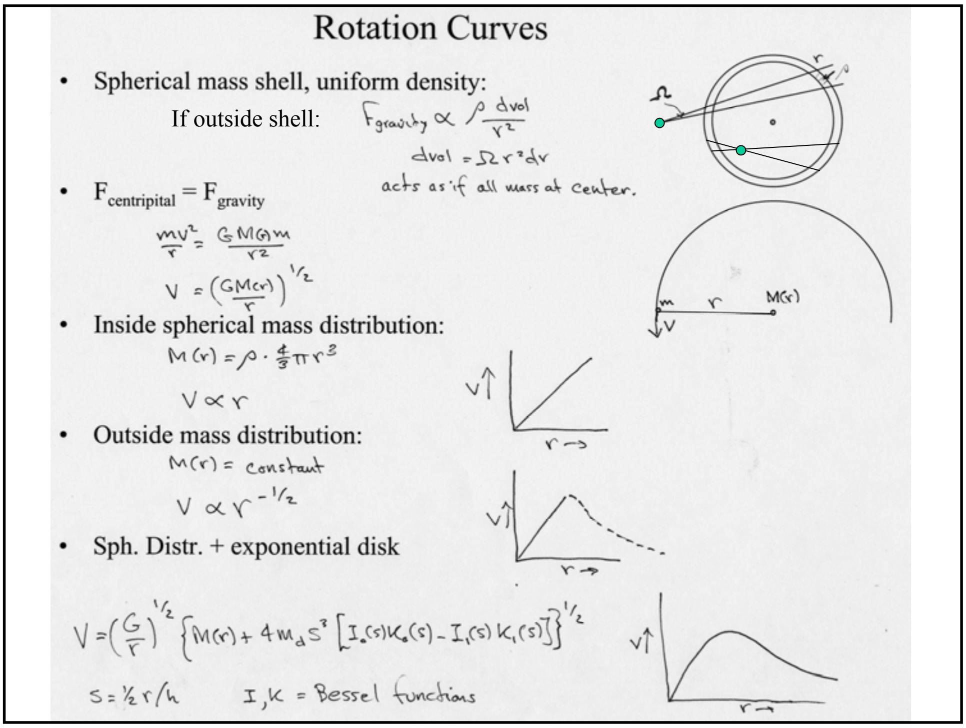
More about the game plan:

- NOW… Kinematics of spiral galaxies
  - Rotation curves ➔ mass distribution [CO 24.3]
  - *SKIP [CO 24.4 "The Galactic Center"]*
  - Spiral structure [CO 25.3]
- General properties of S and Irr galaxies [CO 25.2]
- E galaxies [CO 25.4]
- Midterm on Wednesday Oct. 3

Homework ~~5~~ 4
Assigned on Monday
Due Wednesday 26

Homework 5
= 2-3 spiral structure problems.
Maybe something about E galaxies.
Due Monday Oct 1.

NOTE: ALL OF THE SLIDES ARE ON THE COURSE WEB SITE
AS PDF FILES, 2 SLIDES PER PAGE.
www.pa.msu.edu/courses/ast308



Rotation Curves

- Spherical mass shell, uniform density:

  If outside shell: $F_{gravity} \propto \rho \frac{dvol}{r^2}$

  $dvol = \Omega r^2 dr$

  acts as if all mass at center.

- $F_{centripital} = F_{gravity}$

  $\frac{mv^2}{r} = \frac{GM(r)m}{r^2}$

  $V = \left(\frac{GM(r)}{r}\right)^{1/2}$

- Inside spherical mass distribution:

  $M(r) = \rho \cdot \frac{4}{3}\pi r^3$

  $V \propto r$

- Outside mass distribution:

  $M(r) = \text{constant}$

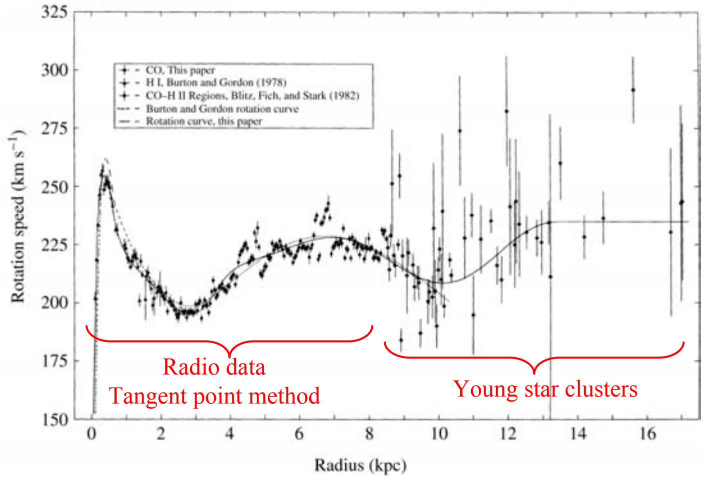
  $V \propto r^{-1/2}$

- Sph. Distr. + exponential disk

$$V = \left(\frac{G}{r}\right)^{1/2} \left\{M(r) + 4 m_d s^3 \left[I_0(s)K_0(s) - I_1(s)K_1(s)\right]\right\}^{1/2}$$

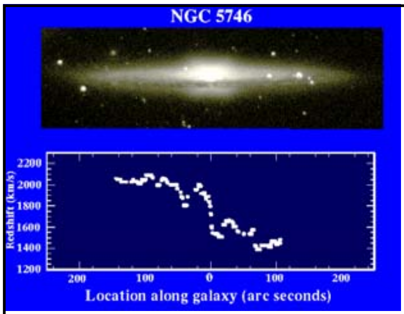
$s = \frac{1}{2} r/h$ I, K = Bessel functions

The Milky Way’s Rotation Curve

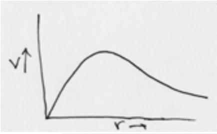


Radio data
Tangent point method

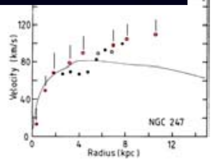
Young star clusters

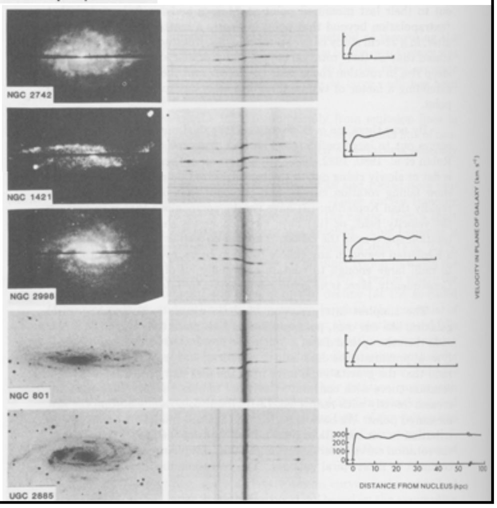


Rotation curves in other galaxies



- Vera Rubin & Kent Ford (late 1970’s)
- Image tube spectrograph





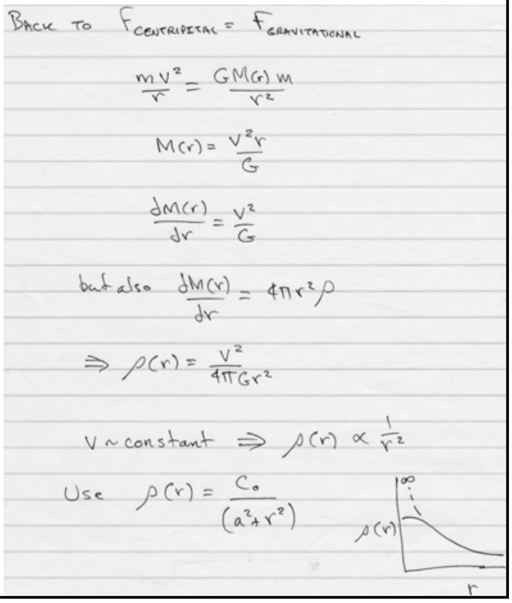
Density as shown by flat rotation curves

[CO pg. 917]

- dM(r)/dr ~ constant
    - Unbounded mass distribution??
- NFW profile

$$\rho(r) = \frac{\rho_0}{(r/a)(1 + r/a)^2}$$

- Predicted for Cold dark matter (CDM)
- Actual derived dark matter profiles often slightly different than this
- What is CDM? Coming later in course.



BACK TO $F_{\text{CENTRIPETAL}} = F_{\text{GRAVITATIONAL}}$

$$\frac{mv^2}{r} = \frac{GM(r)\,m}{r^2}$$

$$M(r) = \frac{v^2 r}{G}$$

$$\frac{dM(r)}{dr} = \frac{v^2}{G}$$

but also $\frac{dM(r)}{dr} = 4\pi r^2 \rho$

$$\Rightarrow \rho(r) = \frac{v^2}{4\pi G r^2}$$

$v \sim$ constant $\Rightarrow \rho(r) \propto \frac{1}{r^2}$

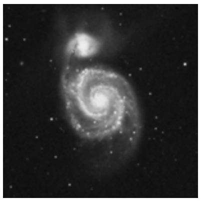
Use $\rho(r) = \frac{C_0}{(a^2 + r^2)}$

Homework Assignment 5
Due Monday Oct. 1

- CO 2nd edition problems 25.13, 25.14, 15.16
    - Same as 1st edition problems 23.11, 23.12, 23.14
- There may be one addition derivation-type problem having to do with the stellar velocities found in E galaxies. It depends on whether I cover that in class with enough lead time.

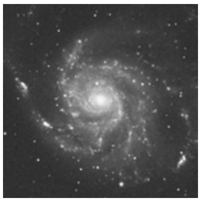
Spiral Structure

[CO 25.3]



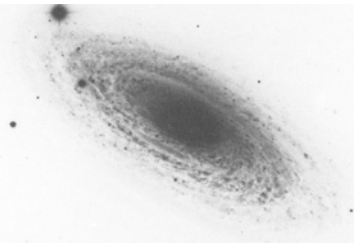
Grand design (10%)

M51



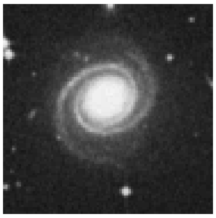
Multi-arm (60%)

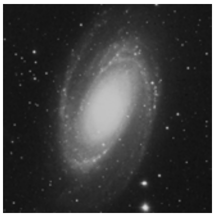
M101



Flocculent (30%)

NGC 2841

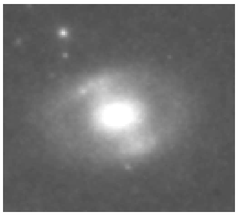




Inner rings

NGC 7096

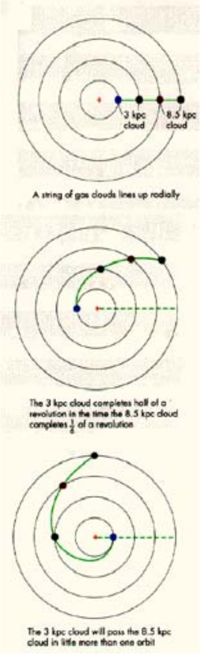
M81



Outer Ring

NGC 4340

What causes spiral structure?

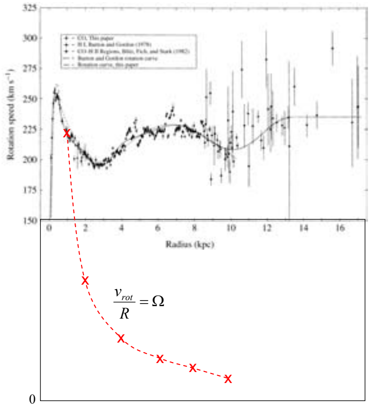


A string of gas clouds lines up radially

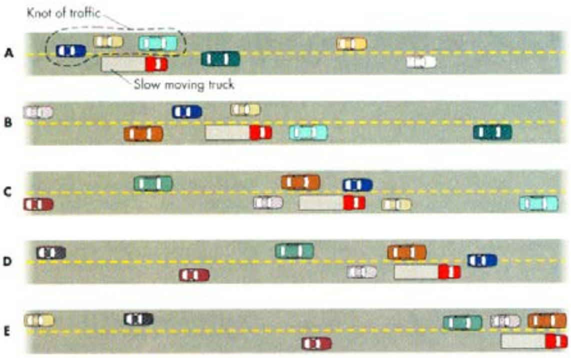
The 3 kpc cloud completes half of a revolution in the time the 8.5 kpc cloud completes $\frac{1}{6}$ of a revolution

The 3 kpc cloud will pass the 8.5 kpc cloud in little more than one orbit

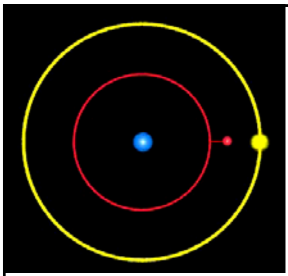
- Winding up of arms
  - Due to differential rotation
- Stochastic, Self-Propagating Star Formation
  - Chain-reaction star formation
  - SN shells ➔ shock fronts ➔ density enhancements ➔ star formation ➔more SN
  - Differential rotation then winds these regions up into spiral patterns
- Density Waves
  - Wave in gravitational potential
  - Orbital velocity of stars different than pattern speed
    - Stars, gas bunch up at position of spiral arms
    - Causes higher grav. potential
  - Unclear if self-sustaining or forced.



Spiral Arms & the Interstate Highway



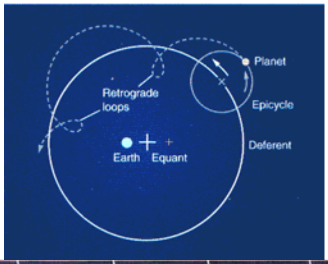
- Density wave
  - Spiral arms have higher density than space between arms
  - Excess gravitational attraction slows down gas, stars when they pass through spiral arm in course of their orbits.
  - ➔ spiral arms are a traffic jam

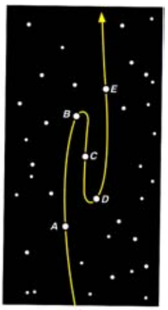


Retrograde Motion & Ptolemy's Epicycles

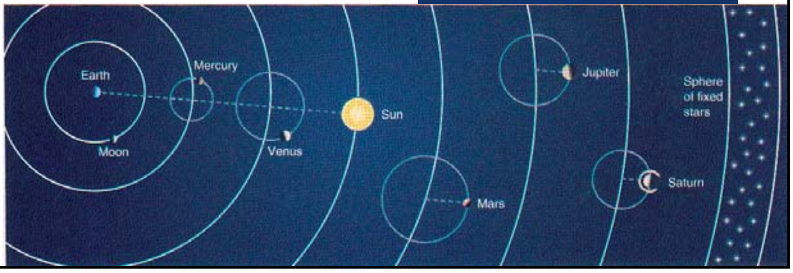
Ptolemy
Alexandria, 140AD

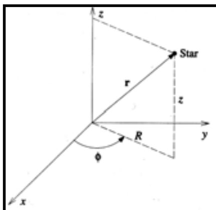
- Trying to place *Earth* at center.
- Using only circular motions.
- Led to very complicated system.





Path of Mars, etc. as seen from Earth





Epicycles… the short form.

For lurid details, see [CO 25.3 (23.3 1st Ed.)]

Ptolemy
Alexandria, 140AD

U = potential energy
$\Phi = U/M$

$$M\frac{d^2\mathbf{r}}{dt^2} = -\nabla U(R,z) = -\frac{\partial U}{\partial R}\hat{\mathbf{e}}_R - \frac{1}{R}\frac{\partial U}{\partial \phi}\hat{\mathbf{e}}_\phi - \frac{\partial U}{\partial z}\hat{\mathbf{e}}_z$$

Circular symmetry ➔ independent of ϕ

$$\frac{d^2\mathbf{r}}{dt^2} = -\frac{\partial \Phi}{\partial R}\hat{\mathbf{e}}_R - \frac{\partial \Phi}{\partial z}\hat{\mathbf{e}}_z \qquad (25.17)$$

$$\frac{d^2\mathbf{r}}{dt^2} = \left(\ddot{R} - R\dot{\phi}^2\right)\hat{\mathbf{e}}_R + \frac{1}{R}\frac{\partial\left(R^2\dot{\phi}\right)}{\partial t}\hat{\mathbf{e}}_\phi + \ddot{z}\hat{\mathbf{e}}_z$$

Separate d^{2}r/dt^2 into R, ϕ, z components ➔ 3 equations. (25.19-25.21)

$$\ddot{R} - R\dot{\phi}^2 = -\frac{\partial \Phi}{\partial R},$$

$$\frac{1}{R}\frac{\partial\left(R^2\dot{\phi}\right)}{\partial t} = 0,$$

$$\ddot{z} = -\frac{\partial \Phi}{\partial z}.$$

Conservation of specific angular momentum
$J_Z = R^2\, d\phi/dt$

Define an effective potential:

$$\Phi_{\text{eff}}(R,z) \equiv \Phi(R,z) + \frac{J_z^2}{2R^2}$$

$$\ddot{R} = -\frac{\partial \Phi_{\text{eff}}}{\partial R}$$

$$\ddot{z} = -\frac{\partial \Phi_{\text{eff}}}{\partial z}$$

Conservation of J_Z ➔ acceleration in ϕ direction when r changes.

Taylor series expansion around position of minimum Φ_{eff} (circular orbit):

$$\Phi_{\text{eff}}(R,z) = \Phi_{\text{eff},m} + \cancel{\frac{\partial \Phi_{\text{eff}}}{\partial R}}\bigg|_m \rho + \cancel{\frac{\partial \Phi_{\text{eff}}}{\partial z}}\bigg|_m z$$

$$+\frac{1}{2}\cancel{\frac{\partial^2 \Phi_{\text{eff}}}{\partial R\partial z}}\bigg|_m \rho z + \frac{1}{2}\frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2}\bigg|_m \rho^2 + \frac{1}{2}\frac{\partial^2 \Phi_{\text{eff}}}{\partial z^2}\bigg|_m z^2 + \cdots$$

$$\Phi_{\text{eff}}(R,z) \simeq \Phi_{\text{eff},m} + \frac{1}{2}\kappa^2\rho^2 + \frac{1}{2}\nu^2 z^2. \qquad (25.31)$$

$$\ddot{\rho} \simeq -\kappa^2\rho \qquad \kappa^2 \equiv \frac{\partial^2 \Phi_{\text{eff}}}{\partial R^2}\bigg|_m$$

$$\ddot{z} \simeq -\nu^2 z \qquad \nu^2 \equiv \frac{\partial^2 \Phi_{\text{eff}}}{\partial z^2}\bigg|_m$$

(25.32-25.33)

Harmonic oscillation in R, ϕ, z about circular orbit (Epicycles)

$$\ddot{\rho} \simeq -\kappa^2 \rho$$

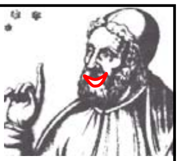
$$\ddot{z} \simeq -\nu^2 z$$

$$\rho(t) = R(t) - R_m = A_R \sin \kappa t$$

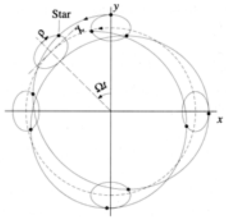
$$z(t) = A_z \sin(\nu t + \zeta)$$

$$\phi(t) = \phi_0 + \frac{J_z}{R_m^2} t + \frac{2J_z}{\kappa R_m^3} A_R \cos \kappa t = \phi_0 + \Omega t + \frac{2\Omega}{\kappa R_m} A_R \cos \kappa t$$

R_m = R at min. Φ_{eff}
Ω = circular ang. vel.



In inertial frame:

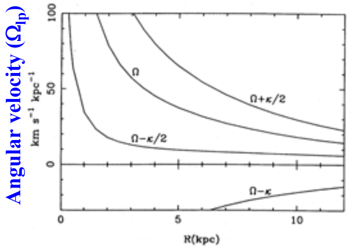


“local pattern speed”

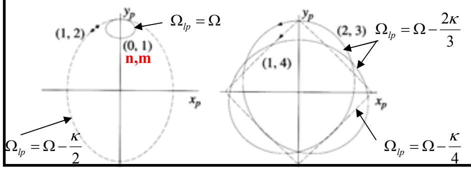
Orbits closed if:
$m(\Omega - \Omega_{lp}) = n\kappa$

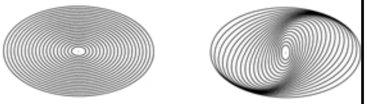
$$\Omega_{\ell p}(R) = \Omega(R) - \frac{n}{m}\kappa(R)$$

Ω = the rotation curve for Milky Way:



Viewed from frame rotating with Ω_{lp}:



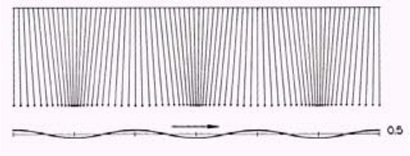


Two ways to line up closed elliptical orbits
(as seen from frame rotating with Ω_{lp})

Basic nature of a density wave

From: Toomre, Annual Review of Astronomy & Astrophysics, 1977 Vol. 15, 437.

Pendulum example



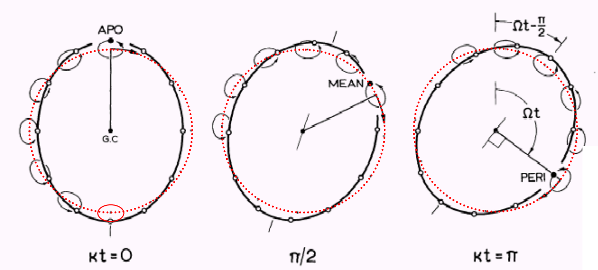
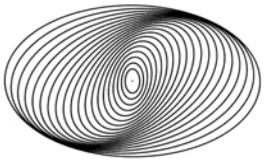


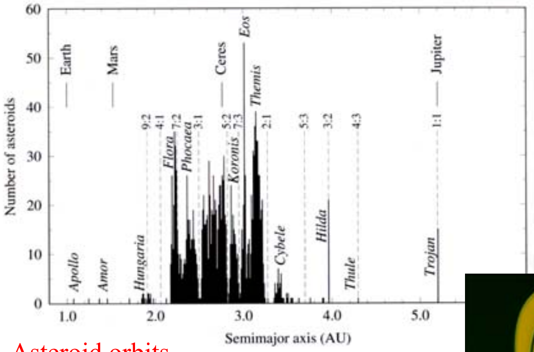
Figure 2 Slow $m = 2$ kinematic wave on a ring of test particles, all revolving clockwis (like the 12 shown) with mean angular speed Ω in strictly similar and nearly circular orbits The small elliptical “epicycles,” traversed counterclockwise in the above sequence of snapshots separated in time by exactly one-quarter of the period $2\pi/\kappa$ of radial travel along each orbit, depict the apparent motions of these particles relative to their mean orbital positions or “guiding centers.” Drawn for the case $\kappa = \sqrt{2}\Omega$—or one where the rotation speed $V(r) = r\Omega(r) = \text{const}$ at neighboring radii—the diagram emphasizes that the oval *locus* of such independent orbiters advances in longitude considerably more slowly than the particles themselves. That precession rate equals $\Omega - \kappa/2$, as one can verify at once by comparing the last frame with the first.

- At each R_m, stars’ positions in epicycles are forced into a specific pattern by gravitational potential of spiral arm.
- Sum of positions of stars at this R_m forms an ellipse rotating at pattern speed.

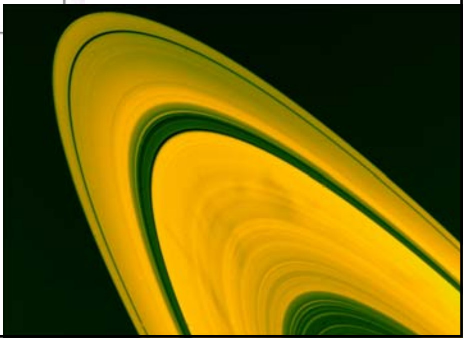


- Spiral density pattern is sum of many ellipses, all rotating at same pattern speed.

Some Solar System Resonance Phenomena

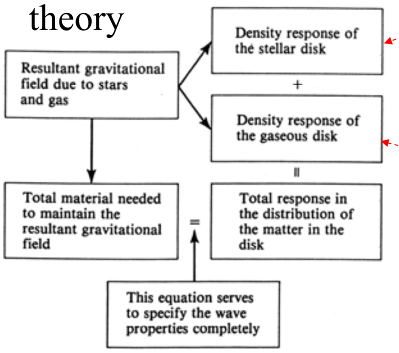


Asteroid orbits



Gaps in Saturn's rings

Lin & Shu's theory



Resultant gravitational field due to stars and gas

Density response of the stellar disk

+

Density response of the gaseous disk

=

Total response in the distribution of the matter in the disk

Total material needed to maintain the resultant gravitational field

=

This equation serves to specify the wave properties completely

Perturbed form of collisionless Boltzmann equation. "quite complicated"

Hydrodynamics

Figure 12.26. The calculational scheme used to calculate the normal modes of oscillation of a disk galaxy. (After C. C. Lin.)

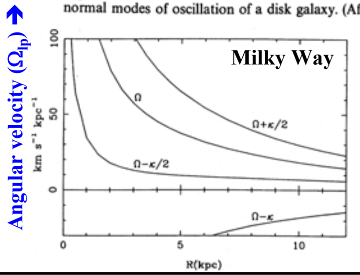
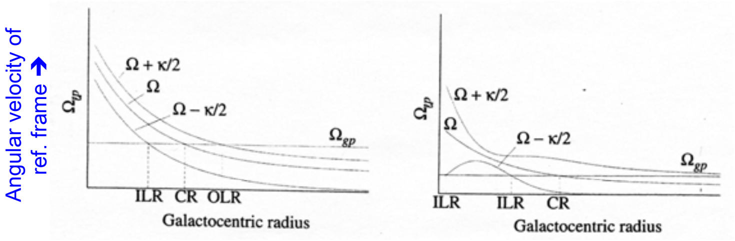


Figure 12.27. Contours of equal density excesses above the average value around a circle in a typical spiral mode. The dashed circle gives the radius where the material rotates at the same speed as the wave pattern.

Inner Lindblad Resonance (ILR)
Co-rotation Radius
Outer Lindblad Resonance (OLR)

} Important in all disk galaxies

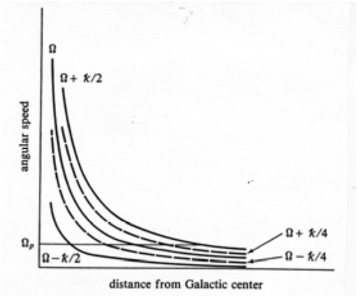


Density waves cannot propagate across ILR or OLR

Density wave theory interprets most spirals as 2-armed

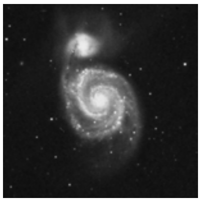
- 4-armed pattern is n / m = 1 / 4
- exists over a narrow range of radius.
  ➔ less likely to be seen.

•Actual 4-armed spirals are superposition of two 2-armed patterns



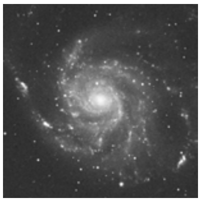
Spiral Structure

[CO 25.3]



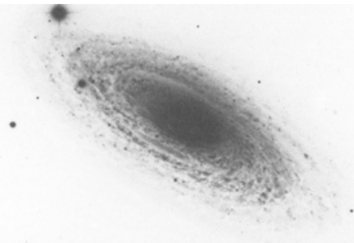
Grand design (10%)

M51



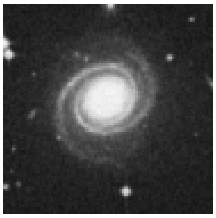
Multi-arm (60%)

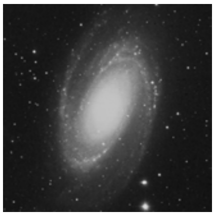
M101



Flocculent (30%)

NGC 2841

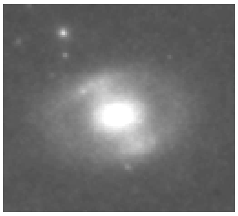




Inner rings

NGC 7096

M81

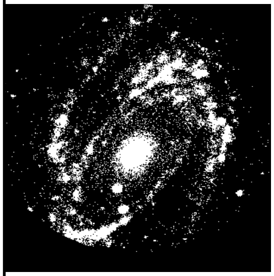


Outer Ring

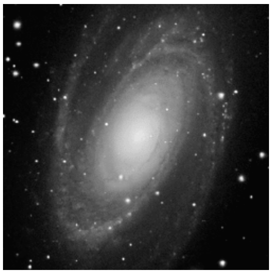
NGC 4340

M81 spiral structure at different wavelengths

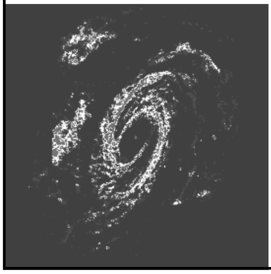
UV: hot stars



Visible: stars + obscuration



Near IR: late-type stars



21 cm: HI

Old red population shows small but real spiral density enhancement.

Passage of gas through spiral arms

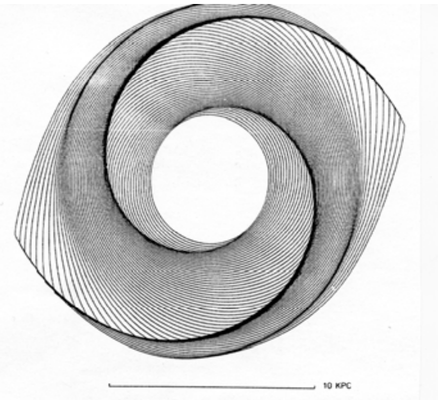
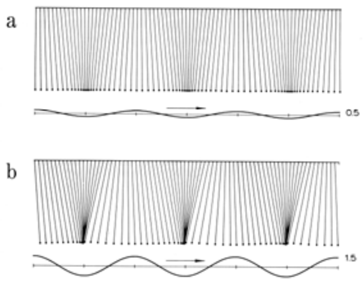
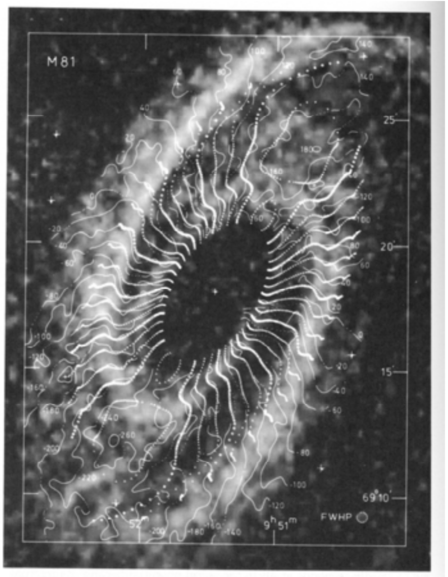


Figure 12.33. The streamlines of gas from a theoretical model of the spiral galaxy M81 (NGC3031, type Sb). (From H. Visser, *Astr. Ap.*, **88**, 1980, 159.)

Calculated streamlines for gas

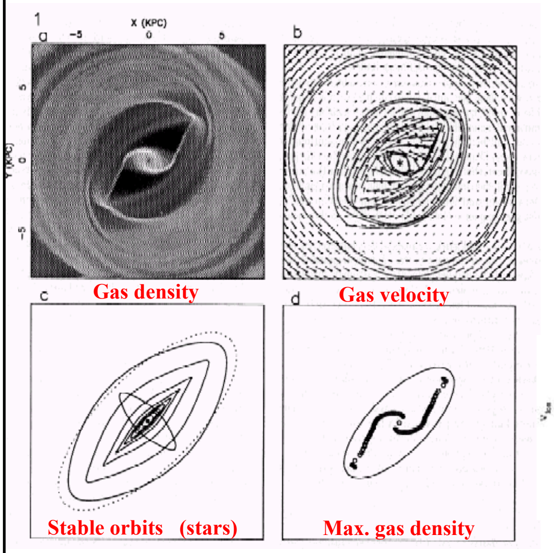
Response of gas to density waves





- Simple pendulum model
  - Each pendulum = 1 gas cloud
  - For large amplitude forcing, pendulums collide.
  - ➔ shock fronts in spiral arms
- HI map (right) shows velocity jumps at spiral arms.

Orbits in Barred Spirals



Gas density

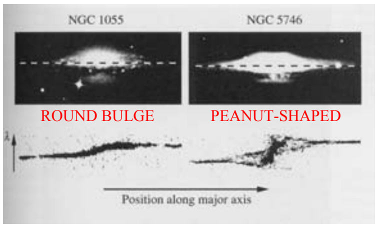
Gas velocity

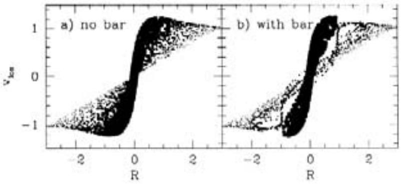
Stable orbits (stars)

Max. gas density

Kinematics of gas, in [NII]

[BM] Fig 4.60

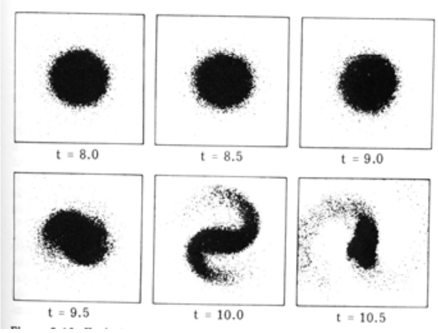




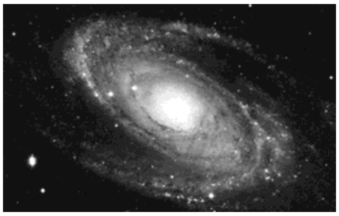
movie
Movie

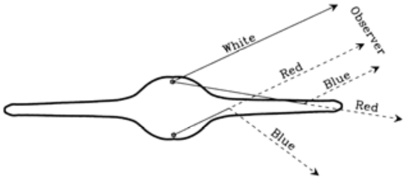
- Gas avoids “co-rotation” radius in barred potential.
- Causes “Fig-8” shape in rotation curve.

Bars appear to be easily excited instability in disks

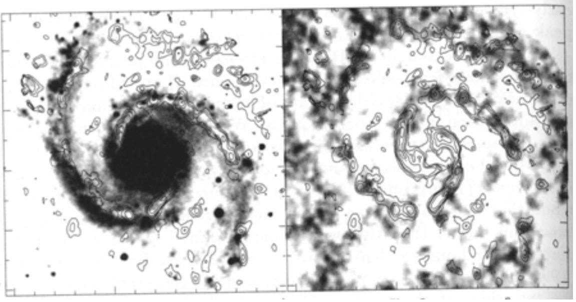


Trailing vs. leading spirals
Which is the near side of the galaxy?



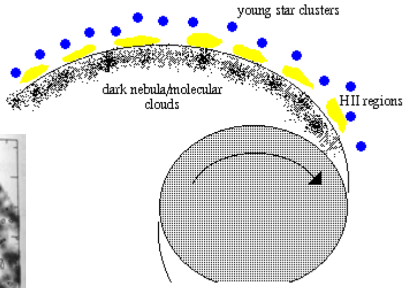


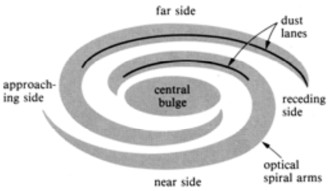
Molecular clouds on inner edges of arms



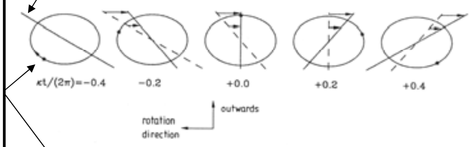
CO contours over red image

CO contours over 21 cm map





Swing Amplification

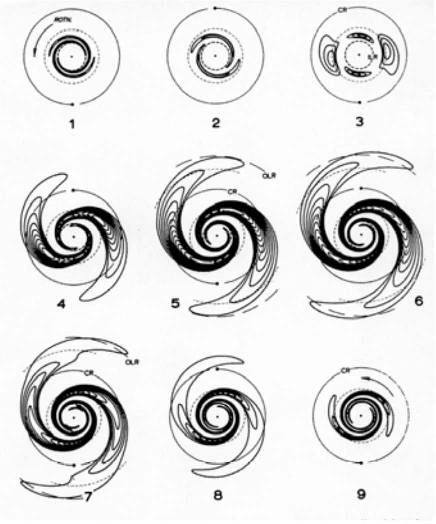


Position of (leading) spiral density enhancement

Epicyclic orbit of star

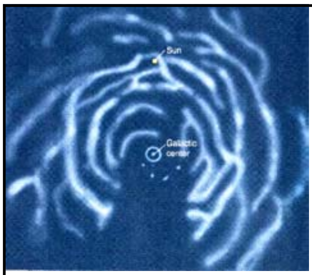
While it is swing around, the spiral arm moves at about same angular speed as star.

Automatically converts any leading spirals into much stronger trailing spirals.



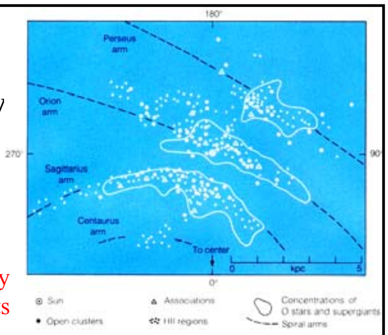
Time steps = ½ of crotation period at CR.

Spiral Structure of the Milky Way



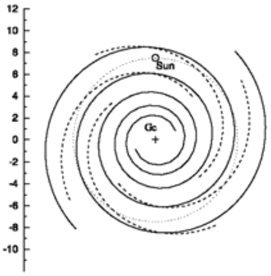
From HI (21 cm observations) assuming circular rotation.

Hard to measure, because we are inside it.



Map of nearby young objects

- Recent model
  - Lepine et al (2001) ApJ 546, 234.
- ➔ mix of
  - 2-armed mode
  - 4-armed mode
- Sun at ~ co-rotation radius.



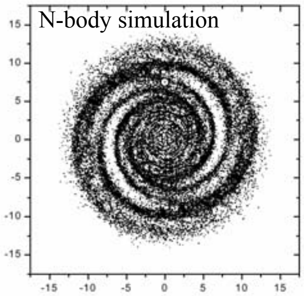


FIG. 3.—Visible structure of the Galaxy derived for the best model (superposition of 2 + 4 self-sustained wave harmonics) by means of cloud-particle simulation. The scale is indicated in kpc. Note that the model is not valid for radii smaller than about 2.5 kpc.

Summary: Density Waves?

- Evidence showing density waves *do* occur.
  - Old, red stars show spiral density perturbation.
  - Molecular clouds form on inner edges of spiral arms.
  - HI gas flow shows discontinuity due to shocks at inner edges of spiral arms.
  - Bright young stars also in narrow arms.
    - Observed width $\Delta\theta \sim t_*(\Omega - \Omega_p)$, as predicted.
- Are these waves self-sustaining over 10^{10} years? Problems:
  - Lin-Shu theory is linear; does not predict whether waves will grow or decay.
  - How are density waves initially formed?
- The usual interpretation
  - Density waves need a driving force
    - Satellite galaxy at co-rotation radius (M51)
    - Bars
  - Otherwise, act to prolong life of transitory phenomena.
  - Other mechanisms probably also important.
    - Swing-amplification efficiently builds up temporary trailing spirals.