Experiment 1
Introduction to 191 Lab

1. Introduction

In Physics 191 we will make extensive use of *Kaleidagraph* [*Kgraph*], a software package for graphing and data analysis, and *Excel*, for calculations. *Kgraph* is a substantial program but is nonetheless intuitive and extremely easy to use. Two major things that *Kgraph* does for us are:

1. Plotting high quality, easily customized graphs of your data.
2. Curve fitting, i.e., finding the parameters of a theoretical function that best describe your data. *Kgraph* provides estimates of the **statistical uncertainties of the fit parameters**, which are difficult to obtain with other commercial software (such as *Excel*).

2. Goals:

1. Familiarize yourself with basic calculations in *Excel*
2. Familiarize yourself with *Kaleidagraph's* plotting, calculation, and fitting options
3. Get a first impression of the style of labs and analysis for the class
4. Have a first look at histograms and standard deviation
5. Have a first look at fitting and residuals
6. Have a first try at searching for flaws ("systematic errors") in data.

3. Background

3.1 In this first lab, we will learn how to use *Excel* and *Kgraph* on a PC equipped with the Windows XP operating system. Your lab report will contain printouts of the graphs that you generate. **The Reference Guide (RG), and your notes made while performing this lab, will also serve as your guide for doing basic analysis operations.** You may write your report outside of class by hand or with Word. But you must **use your time carefully** in class to plot and analyze your data. Since you will be changing lab partners regularly, **take turns, and be sure each of you understands the software**. *KEEP this write-up for reference in future labs.*

3.2 For this laboratory exercise, you will analyze information from an actual mileage log kept for a Prius automobile. The log usually contains an entry for each tank of gas purchased (missing data were occasionally estimated). The Prius uses a hybrid engine system, with both a gasoline engine and electric motors. The gasoline engine ultimately provides all the power, but a large battery stores energy when the engine is not heavily loaded, or when the car is braking. The electric motor uses energy stored in the battery to provide extra power after a stop, when passing, when going in reverse, or even to allow the engine to be turned off temporarily. The battery and electric motor allows the gas engine to run nearer its optimum efficiency, part of why the Prius gets more miles per gallon than most cars.

The odometer measures the distance (in miles) a car has traveled by counting the number of turns of the tires. The amount of gas purchased is calculated by the gas pump. The Prius has a dashboard meter that displays the miles per gallon (mpg) being obtained at the moment, and since the last reset (in this mileage log, that usually means since the last tank of gas). These readouts use the same distance measurement as the odometer, but the gas consumption estimate is based on the time (measured in microseconds) that the computer has opened the fuel injectors. This assumes that the fuel pressure is constant and therefore the amount of fuel consumed is proportional to the time the injector was open.

4 Excel Data Calculations   Goal: print 1st page of modified spreadsheet

4.1 Use the desktop shortcut to the R drive to open R:\exp1_ref\Prius.xls, an *Excel* file containing the mileage log and some date information. **Now SAVE** to your computer's **U drive** space**:** File|Save As|*U:* (in Filename box)|Enter; pick your *section* folder and the *exp1* folder inside it. An alternative to typing *U:* is clicking the down arrow in the Save in box and selecting bps1263xxxx. The Days column calculates number of days since the last fill-up. The *month* column calculates the months, starting from the beginning of 2002, so **month 7.5 is the middle of July, 2002**, and month 20 is the beginning of August, 2003. The Month of year column gives the month, independent of year, so month 9.3 would be early September of 2002, or 2003, or 2004. Click on a cell and the formula used will be displayed.

4.2 Now you will calculate some other quantities from the original data using *Excel* formulas. See the Reference Guide chapter on *Excel* if you need help. Calculate Miles (the mileage since the last tank; but use 456 for the miles on the first tank of gas); MPG Calc = miles / gallons; miles/day; and calc/read = MPG Calc/mpg readout. For miles per day, you will get an error for a long day of freeway driving. Instead of just dividing by the days, we can patch it up by using a function which will keep us from dividing by 0. We suggest you use the Excel *max* function to divide by the larger of {the number of days, or 1.0}. See Help|max function. Print the 1st page (File|Print|**Print Page(s) From:1 To:1**). Then **save and close your file**: *Kgraph* won't open the file if *Excel* still has it open.

5. Kaleidagraph: Histograms and Statistics Goal: print 2 histograms and record statistics

5.1 **Navigating** *Kgraph*
5.1.1. See the Reference Guide for a terse introduction to basic *Kgraph*; use as needed.
5.1.2. You can modify an existing graph by double-clicking or right-clicking objects to bring up a menu for that object: for example, the title of a graph, or an axis label.
5.1.3 Kgraph has a good Help system; the >> spins you through successive related entries.
5.1.4. If you want to print out the "Statistics" screen, or anything else you can't figure out how to print from a Windows program, hold down ALT and hit the Print Screen key. A copy of the active window is now on the clipboard. Then open Word, and paste with Ctrl-V, and print the Word document. The statistics screen or other clipboard contents can also be pasted into a *Kgraph* Plot, or a *Kgraph* Layout window.

5.2 Begin your analysis of the mileage data by opening *Kgraph*, and using *Kgraph* to open the Prius.xls file you just created. *Kgraph* properly opens an *Excel* spreadsheet with column headings as long as the headings are in the first row.

5.3 Make a histogram plot of the MPG Calc column. The plot will consist of bars rising from the x axis. In a histogram, data points are considered as being grouped into "bins" (ranges of values, say x between 20 and 30, and so on). The extent of the bar along the x axis will represent the range of values of the variable in each "bin", and the height of the bar (along the y axis) will represent how many data points fell into each bin. For your first plot, make the histogram with a bin size of 5 miles per gallon. You can find more about histograms by looking up the term in the index of Taylor.

5.4 **For every plot in this entire class** you should: Give the plot a title which explains the reason for this plot, such as Temperature Dependence of Velocity (metric units); no two plots should have the same title. Include your name, section, experiment, and date in the upper right hand corner of the graph (Help|Find|Text Tool). You can later copy/paste this text to other graphs. Make sure that the graph labels, legends, etc. do not overwrite any significant part of the graph. Label the axes of every graph with names and units, e.g., for a histogram Number of Log Entries and Calculated MPG (mi/gal); or for a scatter plot, say Velocity (m/s) and Time (months). **Print your histogram and save it**.

5.5 Now make and print the histogram with a bin size of 2 miles per gallon. Does the new histogram look the same as the previous graph? Why or why not? Did you learn anything new by changing the bin size? **As you write in your lab notebook, or lab report, always include the number of the part of the write-up (e.g. 5.5).**

5.6 Now you will use one of the advanced functions of Kgraph to analyze the data. Click on the frame of the data sheet to highlight it; then click on the MPG Calc column, and then click Function|**Statistics** as described in the Reference Guide. Note the entries for Mean and Std Deviation. The Mean is just the average value; Std Deviation is an abbreviation for Standard Deviation, a measure of the scatter of the values around the average. Both are discussed in Taylor. Write these two values down in your notebook or paste the statistics block onto one of your histograms. On the histogram with 5 mpg bins, indicate the location of the mean, and draw a horizontal line stretching from (mean – standard deviation) to (mean + standard deviation).

6. Kaleidagraph: Scatter Plots and Fits Goal: print mpg plot, fuzzy parabola fits and residuals.

6.1 **Scatter Plot** Now you will seek a better understanding of the mpg data by plotting it as a function of time. Make a scatter plot of calculated mpg as a function of *month*, that is, mpg vs. time. **Whenever we say "y vs. x"** we mean **the first variable should be on the y (vertical) axis** and the second should be on the x (horizontal) axis. Do not use the Line Plot, as it is a "connect the dots" plot not well suited to our uses: by default it doesn't display much of the data, and the lines both over-lead the eye and get in the way of fits we will apply to the data. Make the data 18-point hollow diamonds (e.g. Plot|Variable Settings|Marker size|18). Label the axes of the graph with names and units, e.g., Velocity (m/s) and Temperature (K). Does this graph tell you anything about the reason for the structure you saw in the mpg histogram? How would you explain it? *Hint*: look at the definition of *month* in section 4.1.

6.2 Next you will "**fit**" a mathematical model. The "fitting" consists of writing down a mathematical expression ("curve") with some unknown parameters and letting Kgraph automatically try values of the parameters to find those that best "fit" the data. The mathematical method Kgraph uses to find the parameters is called "Least Squares". Least

squares fitting minimizes (finds the *Least*) the sum of the *Square* of the difference ("residual") between the measured data and the values predicted by the curve. This will be discussed in detail in Chapter 8 of Taylor.

To get a feel for fitting, you will now work through an example where the model is perfectly known, and see how well *Kgraph* reconstructs the true model. Open and save to your U: drive the file r:\exp1_ref\fuzzy_parabola.xls. Then close it and re-open the *copy* in *Kgraph*. Make a scatter plot of y = f(x) vs. x. Now fit the model $a + bx + cx^2$ to the data using the directions in the Reference Guide. Write down the values you obtain for the parameters a, b and c. The curve should pass perfectly through the data and Kgraph should report essentially zero uncertainty ("error") in the parameters. Check that you got the correct values by opening the fuzzy_parabola.xls file with *Excel*. Look at the equation used for the f(x) column and write down the correct values of the parameters in your notebook.

6.3 Now make a new plot of *fuzzy f* vs. x. The "fuzzy" value was created by adding a simulated "measurement error" to the value of *f*. (It is impossible to make a measurement which returns the exact "true" value). I used a random number between -1 and +1; the last column gives the difference between the fuzzy and real value.   Fit the *fuzzy f* data to the parabola. Print the plot with the fit, the values of the parameters and their uncertainties. Are they close to the correct values?

6.4 Now look at how the fit differs from the data by examining the "residuals", defined as the difference: residual = data- fit, at each data point. *Kgraph* uses fit-data, but that isn't a big problem. Make and print a histogram of the residuals (see Reference Guide). You should see values roughly between -1 and 1: the differences between the fit and the data are about the same as the "fuzz" we put in. If we'd looked at the residuals of the fit to the un-fuzzed f(x), the values would have been tiny. *Extra Credit:* plot the residuals for the un-fuzzed fit.

7.1 **Data Analysis.** Goal: print 2 fits to mpg vs. time; % difference between calc and readout

Now return to the mpg vs. month data. There isn't a precise theory of how mileage should vary with time, so the models will be approximate. You will make two fits to the data, one based on a cosine curve and the other on a 6th order polynomial. **Before doing so, predict** (write down before you do the fits!) which you think will do the better job, and why.

7.2 Make a fit with the **cosine** curve as described in the Reference Guide. You will need reasonable starting values for this fit to work properly. Print the plot with the fit curve and the fit parameters as labels for the figure. Also, histogram the residuals from this fit. Use the statistics function to calculate the standard deviation of the residuals: this, as you just saw, is a measure of how well the fit matches the data.

7.3 Perform another fit to the data: a 7-parameter fit, to a **6th order polynomial**. You may need to make two plots in order to show both fits. Print this fit as well. Find the standard deviation of the residuals of this fit. For your report, discuss which fit actually looks "better" to your eye as a representation of the data. Does that correspond to the plot with the smaller standard deviation of residuals? *Extra Credit:* You could also make a scatterplot of the residuals vs. time. Ideally, there would be no longer any obvious time dependence of the residuals if the fit had captured the main time dependence. Is that the case here?

7.4 Now **cross check** of the two estimates of fuel consumption to see if they're roughly equivalent, and how close they are to each other. You can approach this question with the techniques you've learned: plotting or examining statistics for the column mpg calc/readout, or making a scatter plot of the calculated vs. readout miles per gallon and performing a fit. Choose one of these techniques and find quantitatively, by how many % the two estimates differ typically, and how much (in %) individual values vary about the main trend.

7.5 *Extra Credit Questions.* How might you study the mileage for highway as compared to city driving? The data between Jan 1 and Aug 1 2003 correspond to a time in New Mexico rather than Michigan; are there noticeable differences in the data due to the location? What might cause these differences? What do you think is the capacity of the gas tank?

7.6 <mark>**Finishing up:**</mark> As you **exit Kaleidagraph, select "None"** in the dialogue box when you are asked whether you want to save changes. **It is advisable to back up files from the U Drive to a USB flash drive** if you will need your data for next week (**the U drive is not backed up!**):
      1) Insert USB flash drive in front of the computer, or on USB extender
      2) Open the U drive by double-clicking on the shortcut
      3) Select in the Address box of this window your flash drive (F: or G:)
      4) Again open the U drive by double-clicking on the shortcut (gives a second window)
      5) Drag the files or folders from the U: window to the flash drive window
      6) **Left**-click Safely Remove Hardware icon in system tray (bottom right)
      7) Click Remove F; after it says it's OK to do so, actually remove your flash drive.
      8) Next time: Pick your program | Open |Type F: in "filename" box and hit Enter
To save to **AFS** space, open Internet Explorer to ftp://afs.msu.edu then log in and follow directions to get an Windows Explorer window pointing to your area. Double click on the U drive, and copy and paste files or folders between these two windows. To create folders or rename files in this AFS window, you'll to use need a computer outside the lab.

Finally, **Log Out** from Windows (this will erase all changes you made except for the files you have saved on the U drive or your USB flash drive, and clean up so the next student sees the same starting point as you did).

8. Summary Tables In each lab report you should present your critical results in summary tables which organize your results for you, and for your readers. In many experiments you will also need to organize your data recording in tables. At the beginning, we will give you explicit tables to use; as time goes on, you should be able to think through for yourself what tables you will need. For this experiment, your summary tables should look like the following:

Quantity	Mean	Standard Deviation
MPG Calculated		
MPG Cosine Fit Residuals		
MPG Polynomial Fit Residuals		
MPG Calculated / MPG Readout (%)		

f(x) fits	a	b	c
true parabola parameters			

f(x) fit parameters			
fuzzy f(x) fit parameters			
uncertainty of fuzzy fit parameters			

9. Questions to be discussed in your report:

9.1 Why should one look at histograms with different bin widths?

9.2 How well do the two numbers Mean and Standard Deviation summarize your MGP histogram data? What features do they describe, and what do they miss?

9.3 What do you think causes the structure you see in the finer-binned mpg histogram?

9.4 Does the Prius go farther on a gallon of gas than the vehicle you are most familiar with? If possible, give a quantitative comparison, and state how reliable your conclusion is.

9.5 Are the parameter "errors" (the uncertainties given *Kgraph*) in the parabola fits of about the size of the difference between the "fuzzed" parameters and the original ones? Is this what you would expect, roughly?

9.6 Which fit to the mileage data was better, the cosine fit or the polynomial fit? On what (numerical) basis did you decide? Was that consistent with your prediction?

9.7 Which fit, the cosine fit or the polynomial fit, would be more likely to correctly describe the data for the next year? *Hint*: look at the behavior at the edge(s) of the plot.

10. **Searching for Flaws**: Where could the measurements be misleading? Any experimental procedure is subject to errors of execution, mistaken assumptions, or biases. Part of our job as experimentalists is to think about them. At the beginning of the term, we will guide your thinking about these, but you should strive to be able to do this more independently as the term progresses. Here we will suggest some sources of uncertainty in the data and procedures. For items in **bold italics**, hints are given below: but try by yourself first!

10.1 What would happen to the miles per gallon if the tires were under-inflated? By about how many percent might under-inflation affect the estimated distance traveled? Would under-inflation cause a measurement to be too low, or too high?

10.2 Would tire under-inflation affect the ratio of mpg calculated divided by mpg readout?

10.3 The calculated mpg depends on the precision of the measurement of gasoline volume by the gas station pump. What do you estimate for the fractional % of uncertainty in the number gallons?

10.4 The calculated mpg in effect assumes that the gas tank is filled back up to the same level after each filling. This is usually determined by the automatic shutoff. How accurately (in gallons) do you think the shutoff valve at different gas stations measures "tank full"? What % uncertainty would this translate to for a typical tank of gas for this car?

10.5 How accurately (in %) would you guess that a manufacturer would want to have calibrated the fuel consumption? How accurate would it have to be, to be small compared to the uncertainties due to tire inflation?

10.6 From these considerations, and the data, to what % accuracy do you think the mpg is known? For a measurement of 45mpg, how many mpg would this uncertainty be?

10.7 Is the scatter in the data about the fits larger or smaller than your estimate from 10.6? If larger, what other effects might be causing such variations from one tank to the next?

10.8 Is the mpg according to the readout systematically different from the calculated value? By how many % If so, which measurement would you tend to believe more? Why?

11. Improvements: What was the muddiest point of the lab? Where could the write-up be improved? Be specific—state exactly where and how should it be improved.

12. Reading and Homework: Turn these exercises in with your lab report. It's due next week even though class isn't meeting to do a lab. Your instructor will discuss how to turn it in.

Read Taylor chapter 1, 2 (you may skip sections 2.6 and 2.9), 4.2, and 5.1 . Read section 1.3 and chapter 2 especially carefully—they give the ***purpose of uncertainty calculations***, to be able to make ***quantitative comparisons*** between measured values and between measurements and predictions. See also the handout on the important points on uncertainties.

Homework 1: Work through the ***Excel* Tutorial** in the Reference Guide and print out both the resulting spreadsheet results, and the formulas. Do **Taylor problems** 2.4, 2.6, 4.1, and 5.4. In problem 4.1, use your calculator, *Excel,* or *Kaleidagraph* to check your values.

Hints for section 10:

10.1 Speedometers are often off by 2 - 5 mph at 65 mph, so a 3 - 8% error in the speedometer is not out of the question. However, that's the speedometer, which is perhaps more complicated than the odometer. The odometer works by counting the revolutions of the tires and converting turns into distance traveled. This assumes the tires are of constant circumference. The accuracy implied by our uncertainty was $\approx$ 1mi/ 456mi = 0.2% which seems awfully strict when you think of the change of shape in the tires when they are fully inflated vs. under-inflated. You might estimate the tire height (diameter) might change 1/2 inch out of about 2 feet, or about 2 %, due to changes in tire pressure, from being warm or cold, or over or under-inflated. (This alone might give a 1 mpg apparent decrease for highway driving, which keeps the tires warm and records fewer apparent miles (axle revolutions) than actual miles, giving a lowered mpg).

10.3 For a first estimate use the recording error of .1 gal; for 11.3 gal that's 0.9% (though the pump records more decimal places, that's all that was recorded in the log). Assume that all pumps are calibrated by the state to measure the same within well less than the 0.9 % measurement error on a tank (at least we hope so). What about the definition of a full tank? Is that repeatable from pump to pump? What is its uncertainty? Hard to guess again but it might be another 0.1 gallon, or perhaps even more.   This latter might not matter so much in the long run: if the pump cut off early for one tank, you'd miss some of the gas you actually used for those miles. But the next tank, you'd probably record some gas for miles which really were

recorded for the previous tank. Over time it would average out, but this uncertainty would contribute to the uncertainty of any single tank.

10.5 See 10.1 above

10.7 If the scatter of measurements is large compared to your estimated error (as seen by either the calculated vs. readout data, or the size of the fit residuals), then the measurement was dominated by sources of uncertainty other than those you have thought about. These are sometimes called uncontrolled variables. They might not have anything to do with the measurement process itself, but rather changes in what is actually being measured. A driver might be more interested in speed than good mileage on a particular trip. Weather, traffic, road conditions, or the load carried might vary.

Reference Guide for PHY191

Bring this Reference Guide to class each week

Contents

1. Kaleidagraph Essentials
2. Excel Introduction
3. Significant Figures
4. Uncertainty Calculations
5. Log Plots: Guessing Functional Relations
6. Using a Vernier Caliper
7. Writing a Good Lab Report
8. A Sample Lab

1. Kaleidagraph Essentials

Histograms: use menus Gallery|Stat|Histogram|*pick data*|New.   To **Change Bin Size**:
Plot|Plot Options|SpecifyingTheBinSize|OK , then Plot|Axis Options|*enter new bin size*|OK

Statistics Highlight data column, then Function|Statistics and either write down results and |OK,
or |Clipboard|OK and then paste where you want them (Plot, Layout, or Word).

y vs. x scatterplot (*x is horizontal!*): Gallery|Linear|Scatter|*select x and y variables*|NewPlot

Curve Fitting CurveFit|General|EditGeneral|Add|NewFit|*give new fitname in box*|Edit|
Entering equation: say want to fit a parabola y = parabola a + b x + c x^2. Kgraph's convention
for the first fit parameter is m1 (and so on). So you could enter in the Edit box:
        m1 + m2*x + m3*x^2; m1=1; m2=1; m3=1
where m1 stands for your parameter a, m2 for b, m3 for c. The **extra equations** give ***nonzero***
guesses for the parameters which the fit routine **must have** to start from. Now, having entered
the expression you are fitting, OK|OK|CurveFit|General|*fitname*|*check the box*|OK
If you'd wanted an equation of the form C + a cos (k x + b), you'd use m1+m2*cos(m3*x + m4).
To **display the fit equation**, Plot|DisplayEquation. This shows you the fit parameters, and their
uncertainties (labeled "errors"). The value of m1 is the best fit value of a, etc.
Moving it: *Click ParamsBlock*|Cut(Ctrl-X)|Ctrl-L|Layout|SelectPlot|*plot*|Paste(Ctrl-V)|Drag
ParamsBlock;   Ctrl-L is equivalent to Windows|ShowLayout|*KgraphLayout*

Plot Resizing: Right-click within plot | SetPlotSize | *click both Axis and Frame size | Enlarge
Frame size Y*, but to less than paper size | drag Fit Parameters block to below the plot.

Residuals: To make a new column in the data window containing fit residuals, ***immediately*** after
you've performed the fit (or redone the fit): CurveFit|General|*fitname*|View|CopyResiduals.
Find the right data window by clicking on the small grid in the upper right of the plot window;
you can **edit** the data window **column heading** by double-clicking it.

If the function fit doesn't look much like the data, you probably need to give **better initial
values** than 1. Do this by CurveFit|General|*fitname*|Define| then edit the initial values in the box:
    m1 + m2*cos(m3*x + m4); m1=45; m2=10; m3=30; m4=180
Why might these be better values? The C constant (m1) should be near the average *y* value; the
m2 should be the amount above or below the average; m3 should be such that the range Δx of
one cycle of x has Δx*m3=360 degrees (**if Degrees is checked**) or 2π if Radians is checked; and
180 (Degrees) would be appropriate for a maximum the middle of the range.

Remove a previous fit: CurveFit|General|*fitname*|Deselect

Multiple Plots per page: Windows|ShowLayout|*click on layout*| then a Layout menu appears.
Layout|SelectPlot|*click on plot*| for the plots you want on the page (2 or 3 will be big enough to
read). Then Layout|ArrangeLayout| and select say 2 or 3 rows and one column to arrange your
plots. You can drag, resize, or delete the plots; add text; or paste a clipboard.

It's really best to just jump in, but the Kgraph tutorial (examples 2&4) offer more detail.

2. Introduction to EXCEL (and Tutorial)

This introduction will show you the basics you need for this lab. Open *Excel* by double-clicking the icon with the mouse. Then open a new spreadsheet by File | New (and possibly Blank Workbook).   Files that end in ".xls," are Excel spreadsheets. **You can copy and paste data from the *Excel* spreadsheet to the *Kgraph* spreadsheet.** *Kgraph* can open a spreadsheet too, and if it's just data and single-cell column headers in row 1, it will label the columns.

The *Excel* spreadsheet is made up of rectangles called "cells." To enter text into a cell, click on the cell with the mouse, type the desired text, and press Return or Enter. To execute a **formula** in a cell, you must **always first type "="** (an **equals** sign), followed by the desired formula. For example, to compute 5+6, type "**=5+6**" into an empty cell and press "Enter." The answer, "11," appears in the cell where you entered the formula. The formula you entered in the cell appears at the top of the screen under the menu bar. This works for any cell: to see a formula, just click on the cell and the formula for that cell appears at the top of the spreadsheet.

Now we will fill a **column A** labeled X with numbers ranging from 0 to 1.4 in steps of 0.2. Do not take the time to do this all by hand! You can have *Excel* do it for you. In cell **A1** enter X and type Enter. In the next cell of the column, **A2**, enter 0, our first value. Then in **A3**, we can give *Excel* a formula we want it to follow in order to fill in the rest of the column. In each cell, we want it to add 0.2 to the value in the cell just above. That will fill in the column in steps of 0.2. Therefore in cell **A3** enter the formula "=A2+0.2" and press return. Highlight 7 cells of the column, <u>starting with the cell that has the formula in it</u>, **A3**. Then go to Edit | Fill | Down and then let go of the mouse button. Your column should be filled in increments of 0.2.

In the next **column B** (Label it Sin(x) in **B1**) you are going to compute the sine of X using a formula. In **B2** type "=Sin(A2)" and hit Return. By typing **A2** in the parentheses we are telling *Excel* to take the sine of the value (in radians) which it finds in **A2**. Then you can use the same process as above to "fill-down" the entire column. If you do this just as described above, *Excel* knows to start with **A2**, take the sine of each successive value in the A-column, and place the new value in each successive cell of the B-column. Now repeat this same procedure, only this time for cosine X in the **C-column**.

<center>More hints:</center>

● If in doubt, use parentheses to make sure things get calculated in the right order. For example, 3 + 5/2 results in 5.5, but (3 + 5)/2 results in 4. It would have been better to use 3 + (5/2) in Excel for the first case.

● To **check your spreadsheet formulas**, type Ctrl-` (that is, hold down the Ctrl key and type the ` key (which has a ~ above it). If you now print your spreadsheet, you can see all the formulas. Save it with a new name, adjust the column width, and use page setup to print landscape, say to fit to 2 pages wide x 1 tall. You can use Print Preview before printing to check.

● Typing Ctrl-` again toggles back to showing the calculated numbers.

● Instead of typing a cell number (say **O27**) you can click on the cell while entering the formula.

● You can change formatting with the % and .00 → .0 and .0 → .00 buttons.

● The formula =A17 copies the contents of cell A17 to the cell containing the formula.

Copying formulas: The menu command Edit | Fill | Down is equivalent to:

click the cell with the formula	cell will be highlighted, with dark borders
hover mouse on lower right corner	mouse pointer turns from a hollow to a solid cross
drag the mouse down over more cells	the formula is "copied"

When the formula is copied, the cell references change automatically! So if your formula in A3 was A2+.2, when it is filled or dragged down to cell A4, it becomes A3 + .2 ; if you are doing the same set of calculations on a whole column of numbers, you can set up the formulas in the first row, then highlight and drag down the formulas in several columns at once.

A similar process allows you to copy formulas sideways (across columns), or copy and paste into another column. Put 100 in D2. Next, enter =D2 in D3 to copy D2's contents. Now highlight A3:A8, hit copy (Ctrl-c) and then move the mouse to D4 and paste (Ctrl-v). This fills the **D column** with numbers spaced by .2, but starting from 100.

Sometimes this automatic renumbering isn't what you want to do, so you need to be able to stop Excel from adjusting cell numbers. One way to always refer to single cell is to give it a name. Type 10 into cell E1. Highlight E1, then click your mouse in the name box (it will say E1) left of where the formula appears. Type *fred* there and when you use the name *fred* in your formulas and it will always refer to that specific cell. Enter in E3 the formula =*fred*^A3 and copy it down through E4:E9. You should see numbers ranging from 1.58 through 25.12 in the **E column**.

When your spreadsheet is complete, make sure you save and print!

Common operations in Excel
you can substitute cells for numbers with these operations

What?	Mathematics	Excel Equation	Value
Addition	$11 + 12$	$= 11 + 12$	23
Subtraction	$29 - 21$	$= 29 - 21$	8
Multiplication	30×15	$= 30 * 15$	450
Division	$44 / 12$	$= 44/12$	3.66666667
Example	$3 + \dfrac{4}{5 \times 2} - 3 \times 7$	$= 3 + (4/(5*2) - (3*7))$	17.6
Power	$6^{.3}$	$= 6^\wedge(.3)$	1.7117
Exponential	$e^{6.1}$	$= \exp(6.1)$	445.86
Square Root	$\sqrt{3^2 + 4^2}$	$= \text{sqrt} (3^\wedge 2 + 4^\wedge 2)$	5
Sine or Cosine	$\text{Sin}(x)$ or $\text{Cos}(x)$	$= \sin(x)$ or $= \cos(x)$	x = .5 (radians): sin = .479
Pi	$\text{Cos} (2 \pi x)$	$= \cos(2*pi()*x)$	.809 for x = .1

Operations on groups of cells: use as an example cells   A3…A9 containing   .2, .4, …. 1.4 The group can be denoted by A3:A9, or you can highlight a group of cells while entering the formula

Sum	$\sum a_i$	$= \text{sum}(A3:A9)$	5.6
Mean		$= \text{average}(A3:A9)$	0.8
Standard Deviation		$= \text{stdev}(A3:A9)$	. 316

3. Significant Figures

In calculations, it is always important to distinguish significant figures (s.f.) from insignificant figures in the final presented numbers. The rules in Taylor section 2.2 describe things rather well. We add the following comments.

1. The smallest digit in the **final** reported value should be of the same order as the smallest significant digit of the uncertainty. Since the error of the measurement is only an <u>estimate</u> of the uncertainty of the measurement, we keep only the two most significant digits (when available); we keep 2 s.f. so we can calculate t values to 2 s.f.   Examples:

Incorrect	Correct
517.436 m ± 12.34 cm (avoid mixed units)	517.44 ± 0.12 m
24.3441364 ± 0.0023 m/s	24.3442 ± 0.0023 m/s
12385 s ± 241 s	12390 ± 240 s or better $(1.239 ± .024) \times 10^4$ s

2. **Perform intermediate calculations to a few more figures** than you know to be significant (this avoids inadvertent loss of significance) and apply rule 1 for final results.

Rules 3-5 describe which digits are the **minimum** that are known to be significant. These rules also help when you have no explicit uncertainty calculation at the end, and are just calculating from inputs with a fixed number of significant digits.

3. When adding or subtracting two numbers, the result should have the same number of the significant digits <u>after the decimal point</u> as the least precise summand. Example:

4. When multiplying or dividing, the result should have the same <u>total</u> number of the significant digits as the least precise multiplier. Example:
$$1234.3 \times 23.45 \approx 28940$$
5. For other operations (raising to power, square root, exponent, sine) a rough and ready rule is to keep at least as many significant digits in the final result as you had in the input.
Example: Sin(3.567 radians) = - .4127
Complex functions can increase **or** decrease the number of actual significant digits by 1, depending on the function and the argument. For example
   Sin(3.567 radians) = - .4127;   Sin(3.568) = -.4136   lost nearly 1 digit: .4127 ± .0009
   But   √6.567 = 2.5626;   √6.568 = 2.5628   gained 1 digit: 2.5626 ± .0002
Raising to powers > 1 may lose significant digits, and powers < 1 may gain them.

6. More subtle points: when you average many measurements, the standard deviation of the mean becomes small compared to the least significant digit of individual measurements, indicating that the mean has more significant digits than the individual measurements. This will normally be covered by following rules 1 and 2. Similarly, if you wish **to calculate a *t* value to 2 significant digits** (in order to compare it with 2.0, as suggested in the Uncertainty Calculations section below), you will need to **estimate your uncertainty to 2 significant digits**.

4. Uncertainty Calculations: Critical Things to Know

Justify your uncertainty: Give a specific reason you chose δx as the uncertainty for the measurement of x. See examples in Taylor §1.5; §3.1-3.2; and §1.6, §4.1-4.6 for standard deviation and standard deviation of a mean for repeatable measurements.

Compatibility (§1.3, 2.4-2.5)**: The whole point of quantitative measurement with uncertainties is to test hypotheses, and compare results.** Say you measure q, and you compare it to p (the expected value). Define the discrepancy as the difference of your result from the result expected by some hypothesis:

$\boxed{\textbf{D = q – p = measured - expected}}$.

The best way to describe the degree of discrepancy of p and q is in terms of the number of standard deviations (the "t value") of their difference from expectations:

$\boxed{t = \textbf{D} / \boldsymbol{\delta}\textbf{D}}$ δD is the uncertainty of D (its standard deviation, for Gaussian uncertainties).

The "**two standard deviations**" rule says p and q are compatible as long as $|t| \le 2$.
Typically $\delta D = \sqrt{(\delta q^2 + \delta p^2)}$; or just δq if p is well known (so δp is tiny). Best practice is to calculate *t*, then say something like "the difference is 1.6 times its uncertainty, so the measurements are compatible by the 2 standard deviation rule." If $|t| > 2$, we would call p and q statistically incompatible, or call their difference statistically significant. To obtain a *t* value of 2 significant digits, keep 2 s.f. in uncertainties.

   If your uncertainties are Gaussian, and correctly estimated, and the assumptions (hypothesis) leading to the expected value are also correct, a $|t| > 2$ deviation would occur by chance only about 5% of the time. So large |t| values suggest real disagreement from what you expected, while small |t| values are compatible—not proven to disagree. If you measure poorly (δD is large), your result is compatible with almost anything: not a very useful measurement.

   Occasionally we use a simpler criterion compares |D| with $\delta q + \delta p$ (the worst case for δD, but allowing only 1 standard deviation difference): this is just "do the error bars touch".

   We are often also interested in the **fractional deviation** the measured value from what we expected, which is just D/p = (q-p)/p . The D(%) (**% deviation** or **% difference)** is the same thing expressed in percent. D/p or D(%) is all we can report if we don't know δD. But just because the percent difference is small, does not necessarily make it insignificantly different statistically. That's what the *t* criterion is for.

Know the Uncertainty Calculation Formulae (§3.3-3.7; 3.11) **on inside covers of Taylor, and how/when to use them**.   Some hints:
For q = x ± y,   x, y, q, dx, dy, and dq must have the **same units** (so I can add q + dq: error bars)
The fraction uncertainties $\delta q/q$, $\delta x/x$, $\delta y/y$ all have NO UNITS (can write as a fraction, or as %, but watch the factor of 100!)
        But to get δq, **don't forget to multiply q × ($\delta q/q$)**
How to check your calculations to see if they make sense:
q = x + y   always must have: $\delta q > \max(\delta x, \delta y)$
q = x*y or x/y always must have $\delta q/q > \max(\delta x/x, \delta y/y)$

Independent measurement: no relationship in the *imperfections* between the measurements; e.g. 2 students measure the same distance each with a different, but good, ruler. A measurement dominated by a systematic error (same shrunken ruler used by both students) would produce results that aren't independent. See Chapter 4; needed to apply Chapter 3 formulas.

Random: you expect to get slightly different values each time you measure it: due to reading uncertainties, varying judgments, uncontrollable factors, or inherent properties of the measurement.

Standard Deviation and Standard Deviation of the Mean

The standard deviation (σ) is a measure of the uncertainty of any single measurement. The standard deviation of the mean, $\sigma_m = \sigma/\sqrt{N}$, is a measure of the uncertainty of an average of N such measurements. Clearly, the average is better known than a single measurement.

Examples

q = x/y Often easiest to do in terms of %

       x = 10 $\delta x = .10$ y = 2.7 $\delta y = .20$ so q = 3.7

      $\delta q/q = \sqrt{(1\% + 8\%)} \approx 8\%$ so $\delta q \approx .08 \times q \approx .3$ (notice 8% $\rightarrow$ .08, the factor of 100)

Whip out your calculator now: Let's try r=10   and $\delta r = .1$, so what's the fractional error for r?

      $\delta r/r = 1\%$ Now say q= r^2 then what's $\delta q/q = ?$

From Eq 3.23, 3.26:

      $\delta q / q = (|dq/dr| \, \delta r) / r^2 = 2 \, \delta r / r = 2\%$

For comparison, you can calculate changes in q directly (the most general way, rather than the Chapter 3 formulas, which rely on first derivative approximations):

      $(q + \delta q)/q = (r+\delta r)^2 / r^2 = 102.01/100 = 1.0201 = (q + \delta q) / q$, so $\delta q/q = 2.01\%$ (same as $\delta q \rightarrow 0$)

A More Complicated Example Calculation (See Step by Step: Taylor Chapter 3.8)

$q = x^2 \, y + z^{1/3}$ $x = 10 \pm .1$ $y = 20 \pm .2$ $z = 10000 \pm 1800$

              $\delta x/x = 1\%$ $\delta y/y = 1\%$ $\delta z/z = 18\%$

let w = $z^{1/3} = 15.8$ $x^2 y = 2000$ and q = 2015.8

Let's start with the product term: $x^2 y$

$\delta (x^2 y) / (x^2 y) = \sqrt{\{ (\delta x^2/x^2)^2 + (\delta y/y)^2 \}} = \sqrt{\{ (2 \times 1\%)^2 + (1\%)^2 \}} = 2.2 \% \approx 2\%$

      notice we have used $\delta x^2 / x^2 = 2 \, \delta x/x$: **the 2 goes inside the parentheses!**

so $\delta x^2 y = x^2 y \times (\delta x^2 y / x^2 y) = 2000 \times (2\%) = 40$

Now $\delta w/w = 1/3 \, (\delta z/z) = 1/3 \times 18\% = 6\%$, so $\delta w = 6\% \times w \approx .9$

      notice that 6% is NOT rounded up to 10%, nor is .948 rounded up to 1

      in each instance we keep the first significant digit, though a real calculation, it might make sense to keep two digits or even an extra digit. Here w has *more* significant digits than z: $15.8 \pm .9$ compared to $(10.0 \pm 1.8) \times 10^3$!

Finally, since $q = x^2 y + w$, $\delta q = \sqrt{\{ (40)^2 + (.9)^2 \}} \approx 40$

So q = 2015.8 ± 40, or $2020 \pm 40 = (2.02 \pm .04) \times 10^3$ after significant figures.

5. Log Plots and Guessing Functional Relationships

In Kgraph, from Plot | Axis Options you can choose Linear (default) or Log (logarithmic) for either the x or y axis. Let's see how this might help you: For this discussion, we will assume logarithmic means natural logarithms, log to the base e. See also Taylor §8.6 . In Kgraph you can select Log from Plot | Axis Options, or make a new data column with, e.g. ln(c5) or log(c5). When using log scales, you should also use Plot | Axis Options to choose minima and maxima so that your displayed data fills much of the plot area, not just a small corner.

1) Semi-log plots

Suppose the true relationship is something like $y = y(x) = e^{-ax}$ (the constant a might be positive or negative). If you plot y on a log scale and x on a linear scale, this is a semi-log plot (logarithmic on only one axis). Then you would be looking at Ln(y) vs. x, and substituting the true relationship y(x), your plot would display -ax vs. x.

That is, **exponential relationships produce straight lines on a semi-log plot** and the slope is equal to the coefficient of x in the exponent. To find the slope, you could perform either the nonlinear fit y = Exp(- a x), or a linear fit to the semi-log plot, fitting Ln(y) = -a x. These fits will give somewhat different answers as equal weighting in y assumes different uncertainties than equal weighting in Ln(y).

2) Log-Log plots

Suppose now the relationship between y and x is of the form $y^p = a\, x^q$. A linear relationship (y = a x, with no additive constant) is a special case with p = q = 1. Assuming y and x are positive, you can take the p-th root and recast of this relationship as $y = y(x) = b\, x^u$, where $b = a^{1/p}$ and u = q/p.

If you plot both x and y on a logarithmic scale, it's a log-log plot. Then you would be looking at

        Ln(y) vs. Ln(x) , which, on substituting the true dependence y(x) would yield

        Ln(b) + u Ln(x)   vs. Ln(x)

That is, **power relationships** of the form $y^p = a\, x^q$ **produce straight lines on a log-log plot**. The slope is the power u, to which x is raised to give y. For $y = x^2$, on a linear plot you get a parabola—not straight. But a log y vs. log x plot (for x > 0 !) gives a straight line of slope u=2.

You can find this slope by rise over run by measuring distances directly on the graph:

        $u = [\, Ln(y_1) - Ln(y_o)\,] / [Ln(x_1) - Ln(x_0)\,]$

This works directly from the plot, as long as the y and x scales are the same. It's the same as

        $u = Ln(y_1/y_o) / Ln(x_1/x_o)$

which makes it obvious that the units cancel, and all that matters is that a given factor (e, or 10, say) takes up the same amount of space on the y and x axes. You could also use either formula with coordinates of points on the line.

Or you could perform either the nonlinear fit $y = b\, x^u$, or a linear fit to the log-log plot, fitting Ln(y) =b + u Ln(x). These fits will give somewhat different answers as equal weighting in x or y assumes different uncertainties than equal weighting in Ln(x) or Ln(y).

To really test these relationships well, you should ideally:

        Compare the fit quality with an alternative, such as a straight line

        Have more than 3 data points

        Have data covering at least a factor of e^2 (= 7.4) to 10 in range of x and y.

6. Using a Vernier Caliper

A vernier caliper consists of a high quality metal ruler with a special vernier scale attached which allows the ruler to be read with greater precision than would otherwise be possible. The vernier scale provides a means of making measurements of distance (or length) to an accuracy of a tenth of a millimeter or better. Although this section will be devoted to the use of the vernier caliper, vernier scales can also be used to make accurate measurements of other quantities, such as angular displacements.
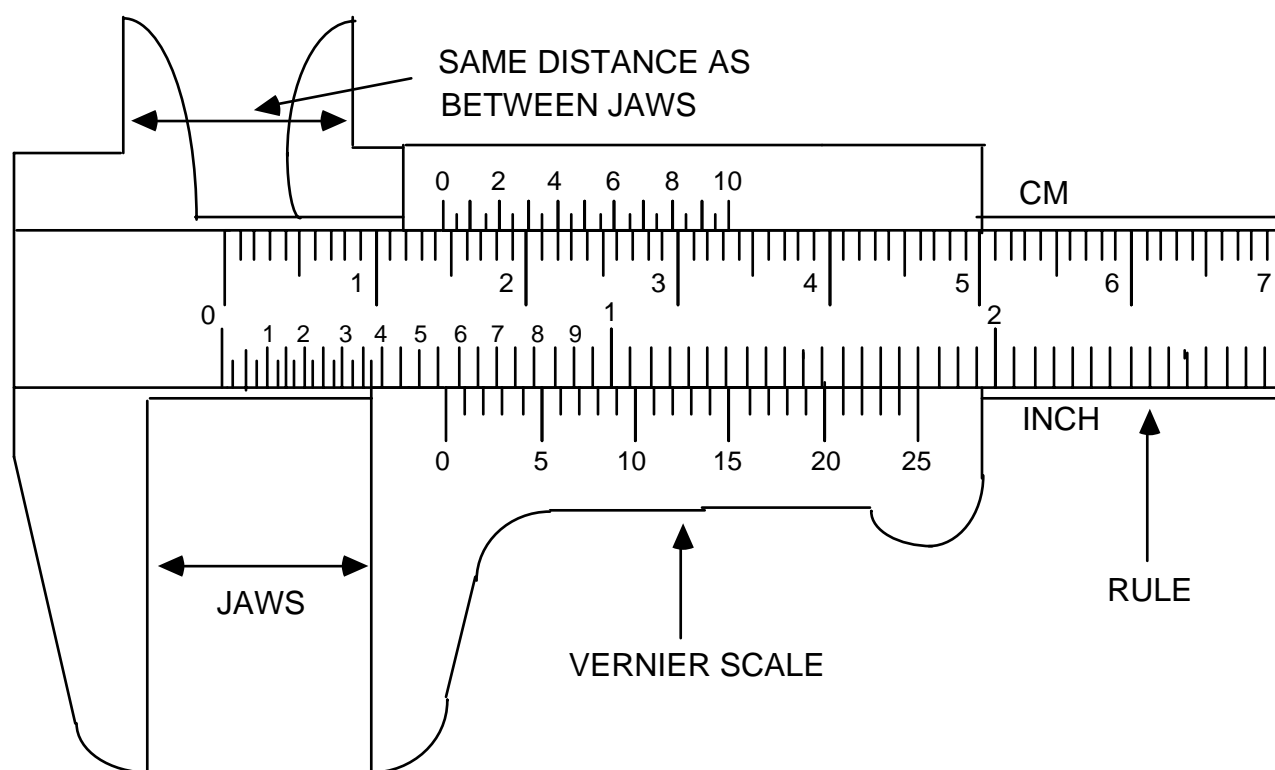
Figure 1: Vernier Caliper

Looking at the vernier caliper in Fig. 1, notice that while the units on the rule portion are similar to those on an ordinary metric ruler, the gradations on the vernier scale are slightly different. The number of vernier gradations is always one more than the number on rule for the same distance. The line on the vernier which is aligned with one on the rule tells us the fraction of the units on the rule.

To use the vernier caliper:

 (1)   Roll the thumb wheel until the jaws are completely closed (touching each other). Now check whether the caliper is reading exactly zero. If not, record the caliper reading, and subtract this number from each measurement you make with the caliper.

 (2)   Use either the inside edges of the jaws, or the outside edges of the two prongs at the top of the caliper to make your measurement.   Do <u>not</u> use the tips of the prongs. Roll the thumb wheel until these surfaces line up with the end points of the distance you are measuring.

 (3)   To read the caliper:

(a) record the numbers which correspond to the last line on the rule which falls before the index line (marked as 0) on the vernier scale. In Fig 1, this would be 1.4 cm since the index line falls after the 1.4 cm line and before the 1.5 cm line.

(b) count to the right on the vernier scale until you reach a vernier line which lines up with a line on the rule and record the number of this vernier line as your last digit. In Fig 1, it is the vernier line marked 4 (rather than the 4.5 line, say) which is aligned with one on the rule. The full vernier scale corresponds to one small division on the main scale, that is, .1 mm, so the vernier tells us to add .4 * .1 = .04cm, so the whole distance is 1.440 cm.

Alas, the inch scale vernier of Fig 1 is not marked correctly; the correct corresponding reading would be .567 inches, so the inch vernier should have lined up at about 17 instead of 25: .55 inches + .017inches).

The following examples show vernier scales similar to that on the vernier caliper, to allow you to test your ability to read a vernier caliper.
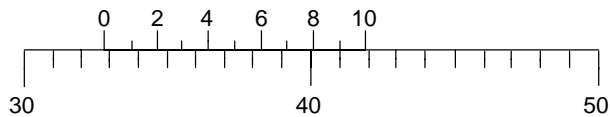
Ans: 32.9
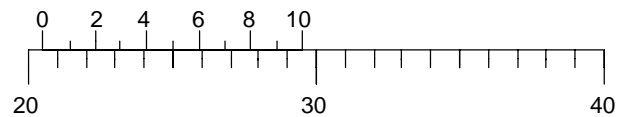
Ans:
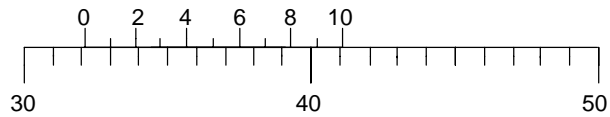
Ans: 32.1
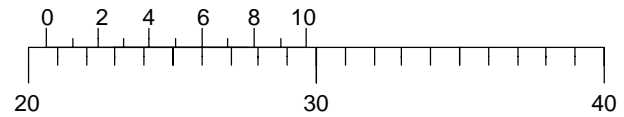
Ans:
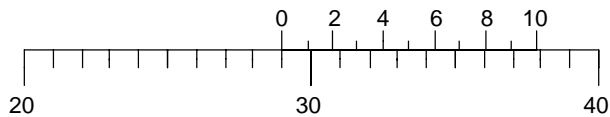
Ans:
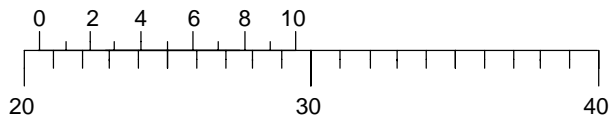
Ans:
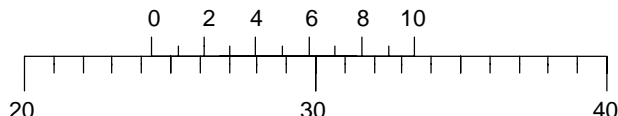
Ans:

Ans:

7. Writing a good lab report

The Notebook: Write what section number (say 4.1.1) of the lab write-up you are working on! Record your original measurements (with units) in your lab book. Include your estimates of uncertainty and their justification for the measured quantities (it might be more uncertain than the finest you can read the instrument!).

Answer all questions posed by the lab write-up, either as you proceed, in the notebook, or in your report (again, with a write-up section number for reference. All your work should be in the lab notebook - including any mistakes or duplicate measurements. Your lab notebook records what exactly you did, including any false steps. Errors should be neatly crossed out and a note should be recorded in the lab book indicating the nature of the mistake. This is the method used by practicing scientists for the recording of their experimental measurements. Write in pen—no pencils or erasers in lab. Write legibly; it can't be graded if it can't be read.

Analysis of results with Kaleidagraph and Excel

Calculate derived quantities from the original data and uncertainties
Plot data with correct labeling and uncertainties (error bars)
Class info in upper right corner, a descriptive title, labeled axes with units, etc.
Titles should distinguish graphs from each other.
Check that your results make sense: smooth graphs; consistency in tables
Find best fit lines, curves, and fit results as needed

### Testing for statistical consistency: learn to use formulae in Taylor

Use uncertainties to compare two results (calculate the t value)
Use uncertainties to compare a result with an expected value (calculate the t value)

Lab reports

The goal is to clearly communicate your analysis and results. You do not have to do everything in Word: by hand is fine, provided it can be read. Or you can do text in Word, but write in equations and sample calculations to save time. Your report will consist of: the report proper (written or typed); pages from your lab notebook (the duplicate sheets or the photocopies) and printouts of spreadsheets or graphs.

In the upper right hand corner of the first page put:
    Your name
    Your partner's name
    PHY191 section xxx (your section's number)
    Experiment #

DO NOT make your grader hunt for things! Follow the section order of the lab write-up. Staple things in an order so the report can be read. Notebook pages and figures should be near where they are referred to in the text. Use the order: **report text for a section, then notebook pages, then figures, then spreadsheets, then back to text for the next section**; it won't be perfect, but the idea is to make it as easy to follow as possible. Section numbers from the lab write-up for answers to questions are critical; it is good to add them to printouts as well. If there's more than one figure in a section, to refer to them in the text you could either assign figure numbers (Fig 6 and 7) or call them something like Fig 5.4.3 A and 5.4.3 B.

For each part of experiment, **in the order given by the lab manual,** your report should have
1. Formulae and a sample calculation for each calculation type
    a. In words, why did you choose this formula (e.g. "independent random errors")
    b. The sample calculation is required
2. Use **summary tables** to organize results (often, a spreadsheet printout) . These really help you to focus on the final results.
    a. Give a summary table for each part of lab
    b. Use an overall summary at the end if comparing results across parts

For example:

	δx(cm)	L(cm)	W(cm)	H(cm)	Mass(g)	V(cm^3)	Density g/cm^3	Material	Expected Density
Ruler	.1	1.5	2.0	1.3	100.0±.1	3.9±.4	2.5±.3	Aluminum	2.7
Caliper	.01	1.47	2.03	1.31	100.0±.1	3.92±.04	2.61±.07		

3. Note whether results differed from your predictions; why?

At the end of your report, include the final discussion of the experiment:

Answer the questions asked in the lab manual
Make quantitative compatibility comparisons where relevant
            (for multiple measurements or experiment vs. theory)
What do you conclude based on the above comparisons?
What did you learn about physics from this lab? About procedure?
Give the **muddiest point(s)** of the lab:Give specifics!
        not "this was bogus" but
        "the readings didn't cover standard deviation but it was needed in part 4b"
        or. suggest an improvement to the techniques, the lab manual, or Taylor

8. A Sample Report

Below is a good example of a lab report for 191 on an experiment we aren't doing. It is not perfect, but should give you a feeling for what a good lab report might look like. The "Objectives" section isn't usually needed. Here it replaces a lab write-up, to help you read the report. Notice:

> Equations and sample calculations
> Lab book pages, tables, and graphs inserted as needed in the text
> Good labeling and referencing of tables and figures

I would have liked clearer comparisons of the measured values and expectations, along the lines of:

Page 29 or 31 We expect g = 9.804, and measure 9.780 ± .019 cm/s^2 . Our measurement of g is consistent with expectations since the t value of the discrepancy is t = -.024/.019 = -1.3 standard deviations, less than 2 standard deviations from the expected result.

Page 35 *as the report text says, the rate of change is compatible with 0; better would have been to say:* Then the coefficient is -.021 ± .017 J/kg s, which, though it is nonzero, is also statistically consistent with zero (t = -1.2)

Page 36 *Ideally the student would have had time to perform the fits with a custom function, in which case the uncertainty in the parameter would have been available. Then they might have written:*
Here is a table with the various parameters of the quadratic fits, and their sums. The sum of each is expected to be zero. The uncertainties on the sum of the terms come from:

$$\delta(a + b) = \sqrt{ (\delta a)^2 + (\delta b)^2} \text{ and } t = (a + b - 0) / \delta(a + b)$$

Term	PE/m	KE/m	sum	Error(sum)	t
constant	0	0.015	0.015	0.02	0.7
Linear	-2.06	2.18	0.12	0.07	1.7
quadratic	-47.86	47.52	-0.34	0.13	-2.6

The constant and linear terms are consistent with zero, but the quadratic term is not consistent with being zero, because |t| > 2.

Roshan Foadi
(Jim Linnemann)
PHY 191 Sec. 8
Experiment 2

Energy of a Free-Falling Body

Objectives:

1) To observe the changes of the potential energy (PE), the kinetic energy (KE), and the total energy (TE), of a free-falling body.
2) To verify, graphically, that the TE remains constant in time.
3) To compare the theoretical predictions for the time dependence of the potential energy and the kinetic energy, with the experimental results.

PART 1

In this part we want to test the conservation of the TE of a free-falling object. In the Behr apparatus, an object is released from a given height. Falling down, it marks periodically a tape, with period $\tau=1/60$ s. The latter quantity is given with negligible error. We ignore the first and the last marks on the tape, because they are affected by the systematic errors connected to the motion beginning and the motion end. Therefore, the second mark on the tape corresponds to the 0th point in Tab.1. We assign the instant $t=0$ s to the 0th point, so that the nth point corresponds to $t=n/60$ s. We measure the distance of the nth point from the zero-tick of a meter stick, and denote it by y_n. The error on y_n is $\delta y=0.0008$ m, as explained in the lab book.

Equations Used

The instantaneous velocity at the nth mark is

$$V_m = \frac{y_{m+1} - y_m}{2\tau} \qquad \left(ex: \ V_1 = \frac{1.29 \cdot 10^{-2} - 10^{-4}}{2 \cdot \frac{1}{60}} \ \frac{m}{s} = 3.84 \cdot 10^{-1} \frac{m}{s} \right)$$

The corresponding error is

$$\delta V_m = \frac{2\delta y}{2\tau} = \frac{\delta y}{\tau} = \frac{8.0 \cdot 10^{-4} \, m}{\frac{1}{60} \, s} = 4.80 \cdot 10^{-2} \frac{m}{s}$$

Name: ROSHAN FOADI   Date: 9/20/2004

Experiment: CONSERVATION OF ENERGY   001

- After we run the Behr free fall apparatus, following the directions given in the "Experimental Procedure" section, we measure the position of each point, taking the second point to be $y=0$, and ignoring the last point. We ignore the first and last points because they are affected by systematic errors. We find that the points do not fall on a straight line, probably due to the tape oscillation. In order to solve this inconvenient, we measure the distance of the second point from the tape border, and look for a far away point which is at the same distance from the border; this happens for the 27th point.

same distance from the tape border
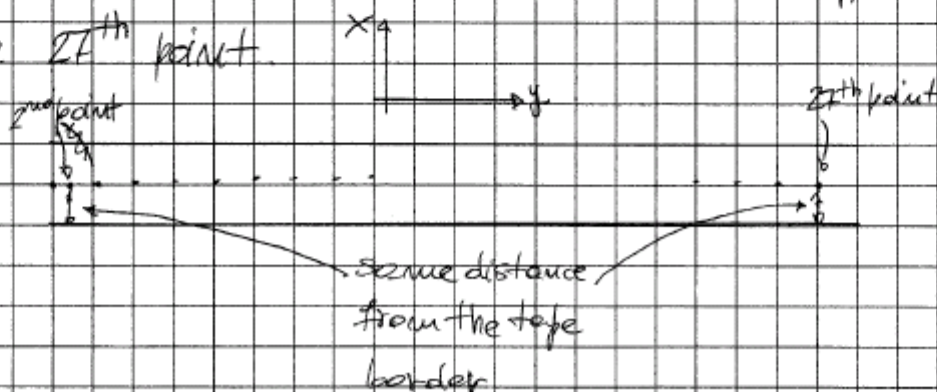
Then we place a meter stick (which we suppose to be reaso

Name: ROSHAN FOADI   Date: 9/20/2004

Experiment: CONSERVATION OF ENERGY   002

mably straight) on the tape, in such a way that it is
tangent to the 2nd and the 27th point. We assume
that the oscillations are purely transversal, that is,
along the x-direction, in such a way that they do
not affect our measurements in a sensible way. Measuring
the thickness of the ties with a caliper, we find that
they are $\simeq 0.54 \div 0.60$ mm thick. Therefore we assign
an error, in the y measurement, of ~~////~~ 1.6 mm, that is,
$\Delta y = 0.8$ mm (in $y \pm \Delta y$). The values are reported
below.

#	y (mm)	#	y (mm)	#	y (mm) $\underset{\text{y (27)}}{}$
2	0.1	15	275.8	28	72.4
3	5.3	16	315.9	29	148.2
4	~~////~~ 12.9	17	358.5	30	226.3
5	22.6	18	404.5	31	307.0
6	36.3	19	452.9	32	391.6
7	51.4	20	504.0	33	477.0
8	70.1	21	557.8		
9	91.5	22	614.0		
10	115.4	23	673.4		
11	141.9	24	735.4		
12	171.1	25	798.8		
13	203.4	26	867.3		
14	237.9	27	957.9		

Name: KOSHANTOADI   Date: 9/20/2004

Experiment: CONSERVATION OF ENERGY   003

The points after the 27th ~~have been measured~~ are measured by repositioning the meter stick, with the zero at the 27th point

. We create a table with KaleidaGraph. The table is shown in the lab report, where we also include the details of how the various quantities have been calculated.

Monday, September 20 10:52 PM 2004

Page #1 - Data 1

R. Foadi

(J. Linnemann)

PHY 191 Sec 8

Table 1

	t(s)	y(m)	v(m/s)	PE/m(J/kg)	KE/m(J/kg)	TE/m(J/kg)	TE/m Res.(J/kg)	TE/m Chauvenet
0	0.0000	0.00010000		0.050884				
1	0.016667	0.0053000	0.38400	0.0000	0.073728	0.073728	-0.0027846	-0.092662
2	0.033333	0.012900	0.51900	-0.074369	0.13468	0.060311	0.010778	0.35867
3	0.050000	0.022600	0.70200	-0.16929	0.24640	0.077115	-0.0058784	-0.19561
4	0.066667	0.036300	0.86400	-0.30335	0.37325	0.069901	0.0014820	0.049315
5	0.083333	0.051400	1.0140	-0.45111	0.51410	0.062991	0.0085378	0.28411
6	0.10000	0.070100	1.2030	-0.63409	0.72360	0.089511	-0.017835	-0.59350
7	0.11667	0.091500	1.3590	-0.84350	0.92344	0.079939	-0.0081173	-0.27012
8	0.13333	0.11540	1.5120	-1.0774	1.1431	0.065699	0.0062687	0.20860
9	0.15000	0.14190	1.6710	-1.3367	1.3961	0.059435	0.012679	0.42193
10	0.16667	0.17110	1.8450	-1.6224	1.7020	0.079593	-0.0073324	-0.24400
11	0.18333	0.20340	2.0040	-1.9385	2.0080	0.069520	0.0028869	0.096068
12	0.20000	0.23790	2.1720	-2.2761	2.3588	0.082708	-0.010154	-0.33790
13	0.21667	0.27580	2.3400	-2.6470	2.7378	0.090849	-0.018149	-0.60396
14	0.23333	0.31590	2.4810	-3.0393	3.0777	0.038335	0.034511	1.1484
15	0.25000	0.35850	2.6580	-3.4562	3.5325	0.076279	-0.0032860	-0.10935
16	0.26667	0.40450	2.8320	-3.9063	4.0101	0.10378	-0.030641	-1.0196
17	0.28333	0.45290	2.9850	-4.3799	4.4551	0.075167	-0.0018818	-0.062619
18	0.30000	0.50400	3.1470	-4.8800	4.9518	0.071826	0.0016063	0.053452
19	0.31667	0.55780	3.3000	-5.4064	5.4450	0.038566	0.035012	1.1651
20	0.33333	0.61400	3.4680	-5.9564	6.0135	0.057138	0.016586	0.55193
21	0.35000	0.67340	3.6420	-6.5376	6.6321	0.094456	-0.020585	-0.68500
22	0.36667	0.73540	3.7920	-7.1443	7.1896	0.045311	0.028706	0.95524
23	0.38333	0.79980	3.9570	-7.7745	7.8289	0.054425	0.019739	0.65685
24	0.40000	0.86730	4.1430	-8.4350	8.5822	0.14721	-0.072900	-2.4259
25	0.41667	0.93790	4.2900	-9.1259	9.2021	0.076186	-0.0017298	-0.057561
26	0.43333	1.0103	4.4460	-9.8343	9.8835	0.049130	0.025472	0.84764
27	0.45000	1.0861	4.6170	-10.576	10.658	0.082285	-0.0075358	-0.25077
28	0.46667	1.1642	4.7640	-11.340	11.348	0.0075483	0.067347	2.2411
29	0.48333	1.2449	4.9590	-12.130	12.296	0.16586	-0.090817	-3.0221
30	0.50000	1.3295	5.1000	-12.958	13.005	0.047173	0.028016	0.93227
31	0.51667	1.4149		-13.794				
32								
33							-0.090817	-3.0221
34							0.067347	2.2411

Monday, September 20 10:52 PM 2004

Page #2 - "Data 1"

Table 1 (cont.)

	t(s)	y(m)	v(m/s)	PE/m(J/kg)	KE/m(J/kg)	TE/m(J/kg)	TE/m Res.(J/kg)	TE/m Chauvenet
35							-1.4901e-08	-4.9586e-07
36							30.000	998.30
37							-4.9671e-10	-1.6529e-08
38							-0.00012391	-0.0041235
39							0.029546	0.98319
40							0.030051	1.00000
41							0.00090306	0.030051
42							0.0054865	0.18257
43							-0.93075	-30.972
44							2.3822	79.271

The relationship between position and time, and between velocity and time, in a free falling object, are

$$y(t) = y_0 + v_0 t + \frac{1}{2} g t^2$$

$$V(t) = V_0 + g t$$

where, v_0 is the velocity at the instant $t=0$, and g is the gravitational acceleration. In Fig.1 we plot velocity vs. time. A linear fit gives the value of g. The error is given by the formulas reported in Taylor, Chapt.8:

$$\delta V_0 = \sqrt{\frac{N-1}{N-2} \sigma^2 \left(\sum_{m=1}^{N} t_m^2\right) / \Delta} \quad , \quad \delta g = \sqrt{\frac{N(N-1)}{N-2} \frac{\sigma^2}{\Delta}}$$

σ = standard deviation of the residuals = $8.295 \cdot 10^{-3} \frac{m}{s}$

N = # of measurement = 23

$$\Delta = N \left(\sum_{m=1}^{N} t_m^2\right) - \left(\sum_{m=1}^{N} t_m\right)^2 = 4.644$$

We obtain:

$$g = (9.780 \pm 0.019) \ m/s^2$$

We calculate the potential energy per unit mass, PE_n/m, at each point n. The formula is

$$PE_n/m = -g\left(y_n - y_0\right) \qquad \left(ex: \ PE_{20}/m = -9.78 \frac{m}{s^2} \left(0.614 \cdot 10^{-4}\right)m \right.$$

$$\left. = -5.956 \frac{J}{kg} \right)$$

The error associated to, PE_n/m is

$$\delta \frac{PE_n}{m} = \delta g \left(y_n - y_0\right) + g \cdot 2 \delta y \qquad \left(ex: \ \delta \frac{PE_{20}}{m} = \cdots \right)$$

Figure 1

R.Foadi

T. Linnemann

PY 191 Sec. 8

Velocity vs. Time

— y = 0.21139 + 9.7802x   R= 0.99997

v(m/s)

t(s)

	v Res.(m/s)
Minimum	-0.013586044
Maximum	0.018400311
Sum	-1.4901161e-07
Points	23
Mean	-6.4787657e-09
Median	-0.0015451908
RMS	0.0081123583
Std Deviation	0.0082946812
Variance	6.8801737e-05
Std Error	0.0017295606
Skewness	0.34539494
Kurtosis	-0.45711055
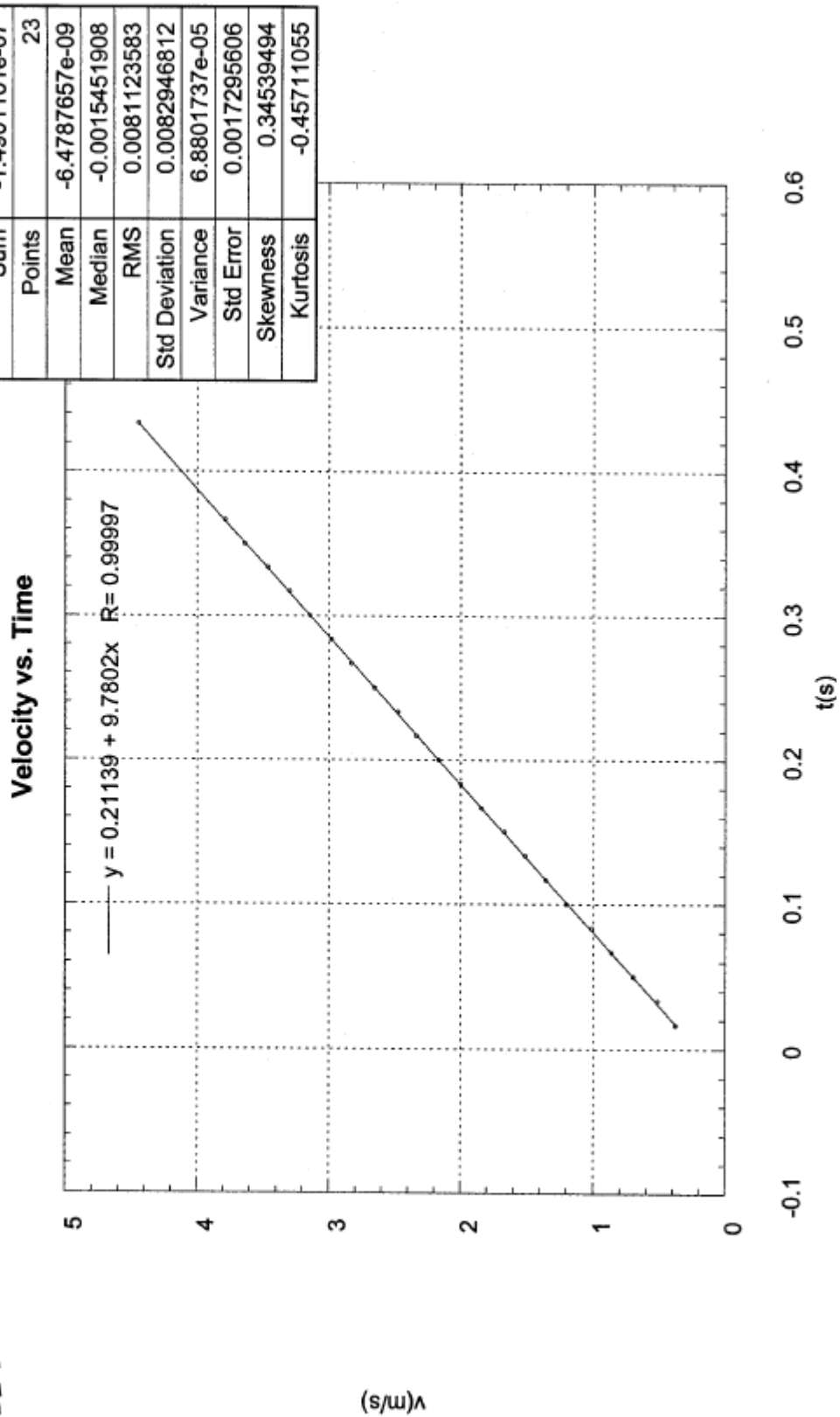
Then we calculate the kinetic energy per unit mass, KE_n/m at each point n:

$$\frac{KE_m}{m} = \frac{1}{2} V_m^2 \quad \left(ex. \quad \frac{KE_{10}}{m} = \frac{1}{2} \cdot (1.711 \cdot 10^{-12})^2 \frac{m^2}{s^2} = -1.622 \, J \right)$$

The error is

$$\delta \frac{KE_m}{m} = \frac{1}{2} \cdot 2 V_m \, \delta V_m = V_m \, \delta V_m = \frac{V_m}{\tau} \delta y \quad \left(ex: \frac{\delta KE_{10}}{m} = \ldots \right)$$

Finally, we calculate the total energy per unit mass, TE_n /m at each point n, which is given by

$$TE_m/m = PE_m/m + KE_m/m \quad \left(ex. : \frac{TE_{10}}{m} = -12.130 \, J + 12.136 \, J \right)$$
$$= 1.659 \cdot 10^{-1} J$$

with the experimental error

$$\delta \frac{TE_m}{m} = \delta \frac{PE_m}{m} + \delta \frac{KE_m}{m} \quad \left(ex. : \delta \frac{TE_{10}}{m} = \ldots \right)$$

Results

All of these quantities are reported in Tab.1. In Fig. 1 we show the plot of TE/m vs. time. Based on the Chauvenet criterion, we reject the 24th, the 28th and the 29th points, which are sensibly different from the other ones. We also discard the 0th point, since its velocity is unknown: it cannot be zero, because we ignored the first mark on the tape. As a consequence, also the total energy is not zero, as it is clear from Fig.1. The new data, after the wrong numbers have been rejected, are shown in Tab.2. Note that we can assign a velocity to an even lower number of points. The value we get for g is smaller than what we expect (g=9.804 m/s^2). A possible explanation for this discrepancy lies in the apparatus oscillations during the free fall. All the quantities in Tab.2 have been calculated based on the obtained value of g. In Fig.2 we show the new TE/m vs. time plot, with the statistic of the residuals. The constant term, in the linear fit, gives the initial TE/m. Its error is

Page #1 - 'Data 2'   Monday, September 20 10:52 PM 2004

#	t(s)	y(m)	v(m/s)	v Res.(m/s)	PE/m(J/kg)	PE/m Res.(J/kg)	PE/m Err.(J/kg)	KE/m(J/kg)	KE/m Res.(J/kg)	KE/m Err.(J/kg)
0	0.0000	0.00010000			0.0000	-0.00093944	0.0078260			
1	0.016667	0.0053000	0.38400	-0.0096031	-0.050857	0.0020530	0.0079243	0.073728	-0.0083408	0.018432
2	0.033333	0.012900	0.51900	0.018400	-0.12519	0.0020272	0.0080679	0.13468	0.0066895	0.024912
3	0.050000	0.022600	0.70200	-0.0015963	-0.22005	-0.0039509	0.0082511	0.24640	-0.0026504	0.033696
4	0.066667	0.036300	0.86400	-0.00059289	-0.35404	0.0027009	0.0085100	0.37325	-0.00071609	0.041472
5	0.083333	0.051400	1.0140	0.012410	-0.50172	-0.0034456	0.0087953	0.51410	0.013613	0.048672
6	0.10000	0.070100	1.2030	-0.013586	-0.68461	-0.0012947	0.0091486	0.72360	-0.014316	0.057744
7	0.11667	0.091500	1.3590	-0.0065826	-0.89391	0.00068784	0.0095529	0.92344	-0.0061752	0.065232
8	0.13333	0.11540	1.5120	0.0034208	-1.1277	0.00063002	0.010004	1.1431	0.0085688	0.072576
9	0.15000	0.14190	1.6710	0.0074241	-1.3868	-0.00048995	0.010505	1.3961	0.016294	0.080208
10	0.16667	0.17110	1.8450	-0.0035725	-1.6724	-0.0016938	0.011057	1.7020	-0.0024251	0.088560
11	0.18333	0.20340	2.0040	0.00043106	-1.9883	0.00059342	0.011667	2.0080	0.0051510	0.096192
12	0.20000	0.23790	2.1720	-0.0045657	-2.3257	-0.0022616	0.012319	2.3588	-0.0056627	0.10426
13	0.21667	0.27580	2.3400	-0.0095623	-2.6964	0.0016451	0.013035	2.7378	-0.018302	0.11232
14	0.23333	0.31590	2.4810	0.012441	-3.0886	0.00057745	0.013792	3.0777	0.034586	0.11909
15	0.25000	0.35850	2.6580	-0.0015554	-3.5052	-0.0025301	0.014597	3.5325	-0.0010495	0.12758
16	0.26667	0.40450	2.8320	-0.012552	-3.9551	0.00087118	0.015466	4.0101	-0.033114	0.13594
17	0.28333	0.45290	2.9850	-0.0025487	-4.4285	0.0010028	0.016381	4.4551	-0.0061502	0.14328
18	0.30000	0.50400	3.1470	-0.0015452	-4.9282	0.0010495	0.017346	4.9518	-0.0044799	0.15106
19	0.31667	0.55780	3.3000	0.0084581	-5.4544	0.0010133	0.018363	5.4450	0.027086	0.15840
20	0.33333	0.61400	3.4680	0.0034616	-6.0041	-0.0020428	0.019424	6.0135	0.0097346	0.16646
21	0.35000	0.67340	3.6420	-0.0075352	-6.5850	-0.00045967	0.020547	6.6321	-0.031277	0.17482
22	0.36667	0.73540	3.7920	0.0054684	-7.1914	-0.00027657	0.021718	7.1896	0.015131	0.18202
23	0.38333	0.79980			-7.8212	-0.0031114	0.022935			
24										
25	0.41667	0.93790			-9.1719	0.0027027	0.025544			
26	0.43333	1.0103	4.4460	0.0034819	-9.8800	-0.0014467	0.026912	9.8835	0.0011234	0.21341
27	0.45000	1.0861			-10.621	0.00074577	0.028344			
28										
29										
30	0.50000	1.3295			-13.002	0.0048599	0.032942			
31	0.51667	1.4149			-13.837	-0.00050192	0.034556			

P.Foadi   PHY 191 - Sec. 8

(T.Linnemann)

Table 2

Monday, September 20 10:52 PM 2004

Table 2
(cont.)

Page #2 - "Data 2"

	TE/m(J/kg)	TE/m Res. (J/kg)
0		
1	0.022871	0.0017434
2	0.0094939	0.014775
3	0.026348	-0.0024242
4	0.019205	0.0043729
5	0.012374	0.010858
6	0.038990	-0.016104
7	0.029530	-0.0069891
8	0.015415	0.0067806
9	0.0092883	0.012562
10	0.029598	-0.0080939
11	0.019693	0.0014654
12	0.033060	-0.012247
13	0.041399	-0.020931
14	-0.010907	0.031029
15	0.027258	-0.0074816
16	0.054999	-0.035568
17	0.026638	-0.0075521
18	0.023562	-0.0048220
19	-0.0094180	0.027812
20	0.0094466	0.0086022
21	0.047073	-0.029370
22	-0.0017490	0.019107
23		
24		
25		
26	0.0035000	0.012475
27		
28		
29		
30		
31		

Figure 2

	TE/m Res.(J/kg)
Minimum	-0.035568327
Maximum	0.031028835
Sum	0
Points	23
Mean	0
Median	0.0014654323
RMS	0.016321382
Std Deviation	0.0166882
Variance	0.00027849603
Std Error	0.0034797303
Skewness	-0.19529332
Kurtosis	-0.29367177

TE per Unit Mass vs. Time

y = 0.02496 - 0.020733x   R= 0.14401

R.Foodi

. Limmermann)

191 Sec. 8
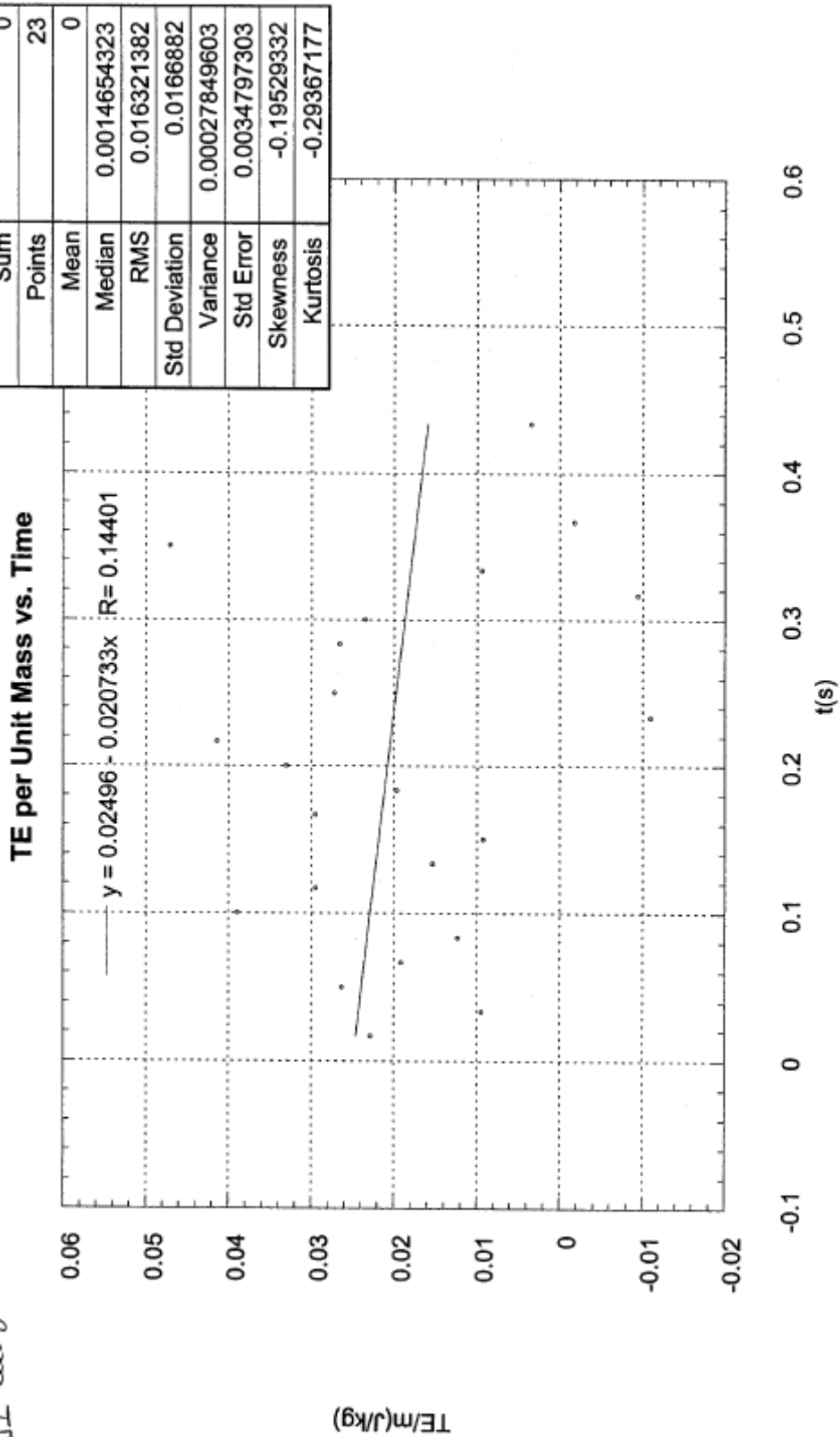
The linear coefficient gives the TE average rate of change. Its error is

$$\delta \text{rate} = \sqrt{\frac{N(N-1)}{N-2}\frac{\sigma^2}{\Delta}} \quad , \quad \sigma = 1.669 \cdot 10^{-2} \frac{J}{kg \, s}$$

We summarize the results for this part in the table below. We observe that the rate of change is compatible with zero, but a negative value is more likely. This energy loss is probably due to the friction acting on the falling body. We also observe a clear oscillatory pattern. These oscillations are mainly along the tape x-axis, as we assumed in the lab book. They affect the speed measurement, because one has to take into account the total distance between the two points, not only their vertical displacement. Therefore, a more accurate measure would require one column for the x-position, in addition to the column for the y-position of Tab.2.

g(m/s^2)	Err.(m/s^2)	TE/m(t=0)(10^-2 J/kg)	Err.(10^-2 J/kg)	Rate (10^-2 J/kg*s)	Err.(10^-2 J/kg*s)
9.78	0.019	2.5	1.2	-2.1	4

PART II

In this part we compare the theoretical prediction for the PE and the KE as functions of time, with the experimental curves.

Equations Used

Inserting the equations for $y(t)$ and $v(t)$ in the formulas for the PE and the KE, we obtain

$$\frac{PE}{m} = -g(y(t) - y_0) = -g\left(v_0 t + \frac{1}{2}gt^2\right) = -gv_0 t - \frac{1}{2}g^2 t^2$$

$$\frac{KE}{m} = \frac{1}{2}v^2 = \frac{1}{2}(v_0 + gt)^2 = \frac{1}{2}v_0^2 + gv_0 t + \frac{1}{2}g^2 t^2$$

NOTE : THE ABS. VALUE OF THE LINEAR AND QUADRATIC COEFFICIENTS ARE IDENTICAL

Results

The *PE/m* vs. time, and the *KE/m* vs. time plots are in Fig.3 and Fig.4, respectively, with the statistic of the residuals. The absolute values of the linear and quadratic terms, from the PE and the KE fits, do not coincide. The main reason for this, once again, is the oscillations of the apparatus, which affect the speed measurements, mainly because we ignored the x-displacements. They also affect the y-position measurements, since an oscillation is never purely transverse. However, the single fits for *PE/m* and *KE/m* are excellent, as the correlation coefficients show.

Questions for Preliminary Discussion

1) Discuss the time *PE=TE* and the time *KE=TE* for an object in free fall. What happens at these points? That is, what are the position, speed, and acceleration of the object at these points?

The condition *PE=TE* corresponds to *KE=0*: this happens at the beginning of the motion, right after the object is released. At that instant the speed is zero, and the acceleration is *g*. The condition *KE=TE* corresponds to *PE=0*: this happens at the end of the motion, right before the object touches the ground. At that instant the speed has its maximum value, $v_{max}=2TE/m$, and the acceleration is of course still *g*.

2) A student says that the TE of a body is *2.3 J*. A different student says that the TE of the same object is *532.2 J*. What do you think is the reason of this discrepancy?

The TE of an object is defined only up to a constant, which is arbitrary. For example, if we decide that the PE at the top of the free-fall motion is zero, then *TE=0*. In the same way, if we decide that the PE at the bottom of the free-fall motion is zero, then $TE=(1/2)m\ v_{max}^{2}$.

3)..QUESTION...
....
....?

....ANSWER...
....
....
....

.
.
.

Figure 3

Foadi

Limne mi Quill

191 Sec. 8

PE per Unit Mass vs. Time

	PE/m Res.(J/kg)
Minimum	-0.0050191879
Maximum	0.0048599243
Sum	-0.0058024026
Points	29
Mean	-0.00020008285
Median	0.0005774498
RMS	0.0022077222
Std Deviation	0.0022375538
Variance	5.0066471e-06
Std Error	0.00041550331
Skewness	-0.13384489
Kurtosis	-0.2577745

$Y = M0 + M1*x + \dots M8*x^8 + M9*x^9$

M0	-0.00093944
M1	-2.062
M2	-47.859
R	1

PE/m(J/kg)

t(s)

Figure 4

Foadi
Limmena(2mm)
191 Sec. 8

KE per Unit Mass vs. Time

	KE/m Res.(J/kg)
Minimum	-0.033114195
Maximum	0.034585714
Sum	0.0033183321
Points	23
Mean	0.00014427531
Median	-0.0010495186
RMS	0.015710973
Std Deviation	0.016063395
Variance	0.00025803265
Std Error	0.0033494494
Skewness	-0.069965118
Kurtosis	0.18976413

$Y = M0 + M1*x + ... M8*x^8 + M9*x^9$

M0	0.015838
M1	2.1809
M2	47.523
R	0.99998

Axes: KE/m(J/kg) vs. t(s)

.
.
.
.
.
.

Questions to be Discussed

1) Does *TE/mass* depend linearly on *t*? If not, what kind of time dependence does it
   show?

The plot *TE/mass* vs. *t* is shown in Fig.2. We observe that the dots form, approximately, a
sinusoid whose middle point falls down roughly linearly. The reason for this behavior is
in the systematic error due to the apparatus oscillation, which we (mistakenly) neglected.

2) Does the fit curves for *PE/mass* and *KE/mass* fall within the experimental
   uncertainties?

The curves, for *KE/mass,* and, especially, for *PE/mass* are very accurate: there is no
point, in each plot, that does not contain the curve. We were careful not to overestimate
the error, and tried to be as accurate and precise as possible in determining the mark
positions on the tape. In this way we reduced the only source of casual error to the
minimum, since the time intervals are extremely accurate.

3)..................................*QUESTION*..
....
....?

.............................*ANSWER*...
....
....
....

.
.
.
.

Conclusions

In the first part of this experiment we tested the conservation of the TE for a free-falling
body. First, we found the gravitational acceleration, by linearly fitting the velocity vs.
time plot. The obtained value of *g*, $g=9.78\ m/s^2$, was slightly below its known value,

$g=9.80$ m/s^2. (The reasons for this discrepancy have been discussed in the Exp.2 Lab Report.) Then, we calculated the TE, and plotted it vs. time. Our results show that some energy is lost during the motion, most likely because of frictional forces acting in the Behr apparatus. Also, we found that the motion was affected by oscillations, which propagated up to the measured values of the PE, the KE, and the TE. Based on these observations, we believe that the experiment can be improved by: (1) Trying to keep the apparatus stable. (2) Measuring also the x-component of the motion, and including it in the formulas for the KE and the TE. (3) Reducing the apparatus friction, by using a specific lubricant.

In the second part of the experiment, we compared the theoretical predictions for the PE and the KE of the falling body (per unit mass), with the data fits. The absolute values of the coefficients, in the quadratic fits, should be the same, for both *PE/m* and *KE/m*. Our results show some discrepancy (we did not calculate the error), which are probably due to the systematic sources of error discussed above: oscillation of the apparatus, and frictional forces. Our suggestions, summarized in the points (1)-(3) of the last paragraph, should improve also this part of the experiment. However, the quadratic fits are excellent, for both *PE/m* and *KE/m*, with coefficients of correlation very close to unity: this means that casual errors have been reduced to a very high level of accuracy. Thus the systematic errors are mainly responsible for the large uncertainties in the TE and its rate of change, (as the oscillatory pattern of Fig.2 clearly shows) , and the discrepancies between the coefficients of the *PE/m* and *KE/m* fits.