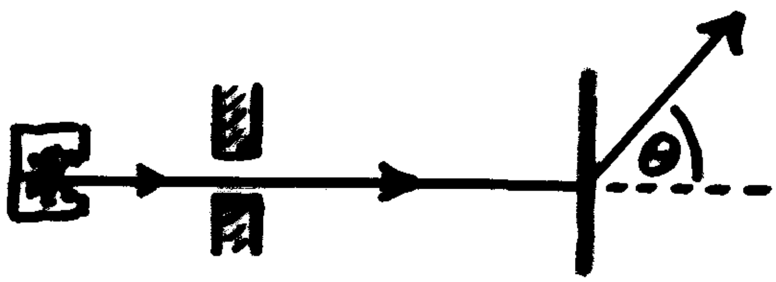
Atomic Structure

Thornton and Rex, Ch. 4

RUTHERFORD SCATTERING

I HAVE ALREADY DESCRIBED THE EXPERIMENT, PERFORMED BY GEIGER AND MARSDEN UNDER THE SUPERVISION OF ERNEST RUTHERFORD, WHICH SHOWED THAT RADIOACTIVE α-RAYS WERE SCATTERED FROM THIN GOLD TARGETS THROUGH VERY LARGE ANGLES.

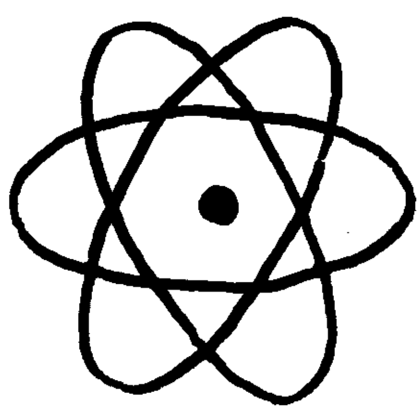


RUTHERFORD REPORTED IN 1911 THAT THE RESULTS OF THE EXPERIMENT WERE NOT CONSISTENT WITH THE α-RAY SCATTERING FROM A HOMOGENEOUS ATOMIC STRUCTURE (LIKE J.J. THOMSON'S PLUM-PUDDING MODEL). RUTHERFORD WROTE *"IT SEEMS SIMPLEST TO SUPPOSE THAT THE ATOM CONTAINS A CENTRAL CHARGE DISTRIBUTED THROUGH A VERY SMALL VOLUME, AND THAT THE LARGE SINGLE DEFLECTIONS ARE DUE TO THE CENTRAL CHARGE AS A WHOLE, AND NOT TO ITS CONSTITUENTS."*

RUTHERFORD WORKED OUT THE SCATTERING EXPECTED FOR THE α-RAYS AS A FUNCTION OF ANGLE, THICKNESS OF MATERIAL, VELOCITY AND CHARGE.

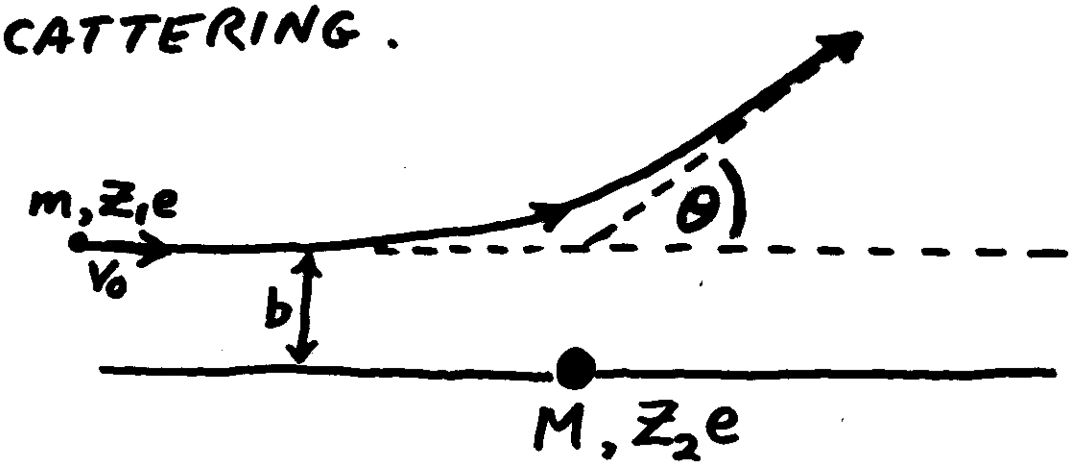
GEIGER AND MARSDEN IMMEDIATELY BEGAN AN EXPERIMENTAL INVESTIGATION OF RUTHERFORD'S IDEAS AND REPORTED IN 1913 *"WE HAVE COMPLETELY VERIFIED THE THEORY GIVEN BY PROF. RUTHERFORD."* IN THAT SAME YEAR, RUTHERFORD COINED THE USE OF THE WORD NUCLEUS FOR THE CENTRAL CHARGED CORE AND DEFINITELY DECIDED THAT THE CORE (CONTAINING MOST OF THE MASS) WAS POSITIVELY CHARGED, SURROUNDED BY THE NEGATIVE ELECTRONS.

THE POPULAR PICTURE OF THE ATOM TODAY IS DUE TO RUTHERFORD:-



RUTHERFORD'S SCATTERING CALCULATIONS

RUTHERFORD ASSUMED THAT THE SCATTERING OCCURRED BECAUSE THE POSITIVELY-CHARGED α-RAY PASSED CLOSE TO A (RELATIVELY MASSIVE) POSITIVE CHARGE. THE CLOSER THE DISTANCE, THE LARGER IS THE SCATTERING.



b IS CALLED THE <u>IMPACT PARAMETER</u>. IT IS THE CLOSEST DISTANCE OF APPROACH BETWEEN THE BEAM PARTICLE AND THE TARGET IF THE PROJECTILE HAD CONTINUED IN A STRAIGHT LINE.

ON PAGES 132-133 AN EQUATION RELATING THE IMPACT PARAMETER AND THE SCATTERING ANGLE IS DERIVED :-

$$b = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0} \frac{1}{2K} \cot\frac{\theta}{2}$$

WHERE $Z_1 e$ AND $Z_2 e$ ARE THE SIZES OF THE COLLIDING CHARGES, AND K IS THE KINETIC ENERGY OF THE INCIDENT PARTICLE.

$\theta = 0$	$\cot\frac{\theta}{2} = \infty$	$b = \infty$
$\theta = 90^\circ$	" $= 1$	
$\theta = 180^\circ$	" $= 0$	$b = 0$

i.e. SHOWING THAT SMALLER IMPACT PARAMETERS CORRESPOND TO LARGER SCATTERING ANGLES.

ANY PARTICLE HITTING AN AREA πb^2 CLOSE TO THE NUCLEUS WILL BE SCATTERED THROUGH AN ANGLE OF θ OR GREATER. WE DEFINE THIS AREA AS A <u>CROSS SECTION</u>, σ

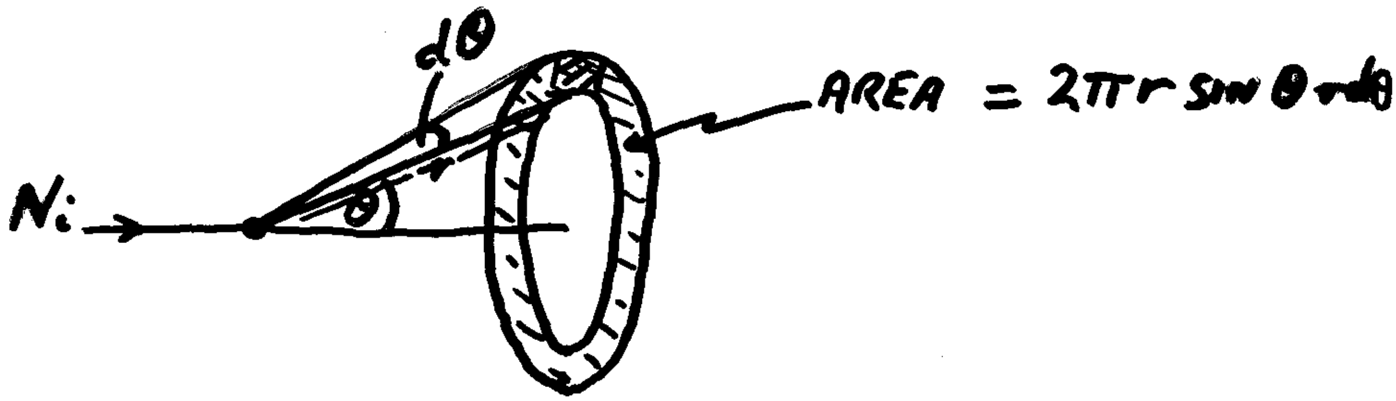
$$\sigma = \pi b^2$$

THIS IS RELATED TO THE PROBABILITY OF BEING SCATTERED BY A NUCLEUS.

IN PRACTICE WE USUALLY SET UP A DETECTOR (THE SCINTILLATION SCREEN OR A GEIGER COUNTER) AT A CERTAIN ANGLE OR OVER AN ANGULAR RANGE $\theta \rightarrow \theta + d\theta$ SO WE WANT A DIFFERENTIAL SCATTERING PROBABILITY.

$$df = -\pi n t \left(\frac{Z_1 Z_2 e^2}{8\pi \varepsilon_0 K}\right)^2 \cot\frac{\theta}{2} \operatorname{cosec}^2\frac{\theta}{2}\, d\theta$$

IF THERE ARE N_i INCIDENT PARTICLES THEN THE NUMBER SCATTERED INTO THE RING ABOUT $d\theta$ IS $N_i |df|$



PER UNIT AREA BETWEEN θ AND $\theta + d\theta$ $= \dfrac{N_i |df|}{2\pi r^2 \sin\theta\, d\theta}$

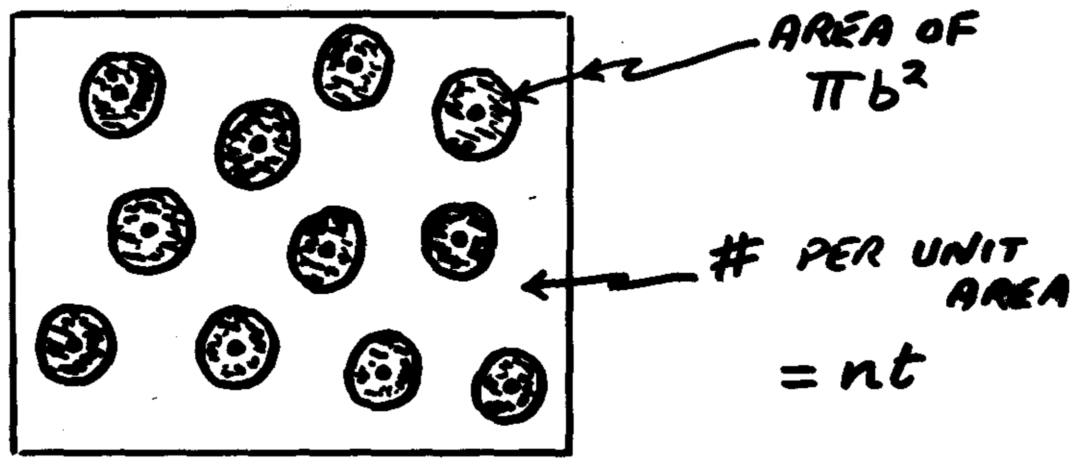
IF WE HAVE n ATOMS PER UNIT VOLUME AND A TARGET THICKNESS t THEN THE NUMBER OF NUCLEI PER UNIT AREA

$$= nt.$$

$$n = N_A \frac{\text{ATOMS}}{\text{MOLE}} \times \frac{1}{A} \frac{\text{MOLE}}{\text{GM}} \times \rho \frac{\text{GM}}{\text{CM}^3}$$

$$\therefore n = \frac{\rho N_A}{A}$$

<u>END-ON VIEW</u>:



THE PROBABILITY OF BEING SCATTERED IS THE AREA OF THE SCATTERING CROSS SECTION DIVIDED BY THE TOTAL AREA :-

$$f = nt\pi b^2 = \pi nt \left(\frac{Z_1 Z_2 e^2}{8\pi \varepsilon_0 K}\right)^2 \cot^2 \frac{\theta}{2}$$

$$\Rightarrow \boxed{N(\theta) = \frac{N_i n t}{16} \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \frac{Z_1^2 Z_2^2}{r^2 K^2 \sin^4\left(\frac{\theta}{2}\right)}}$$

THE RUTHERFORD SCATTERING EQUATION

IMPORTANT FEATURES:

SCATTERING IS:-

(A) PROPORTIONAL TO THE SQUARE OF THE CHARGE (ATOMIC NUMBER) OF BOTH THE INCIDENT PARTICLE AND THE SCATTERING NUCLEUS,

(B) PROPORTIONAL TO K^{-2},

(C) INVERSELY PROPORTIONAL TO THE 4^{TH} POWER OF $\sin\frac{\theta}{2}$,

(D) PROPORTIONAL TO THE TARGET THICKNESS (FOR THIN TARGETS).

THE MINIMUM SEPARATION OF THE INCIDENT PARTICLE AND THE SCATTERING NUCLEUS IS GIVEN BY:-

ENERGY $= K + \sim 0$

ENERGY $= 0 + \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{R_{MIN}}$

CONSERVATION OF ENERGY $\Rightarrow$

$$K = \frac{1}{4\pi\varepsilon_0} \frac{Z_1 Z_2 e^2}{R_{MIN}}$$

$$\therefore \quad R_{MIN} = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 K}$$

RUTHERFORD USED 7·7 MeV α-RAYS SCATTERED FROM GOLD

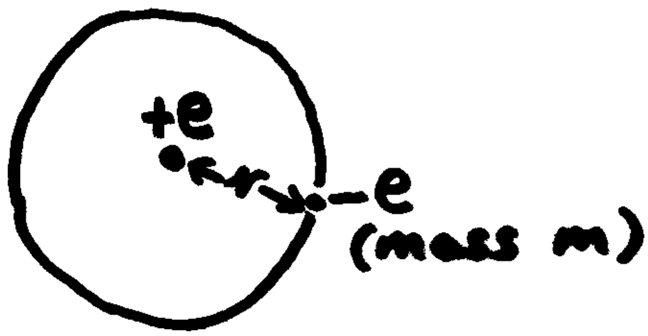
$$\therefore R_{MIN} = \frac{(8{\cdot}99E9).2.79.(1{\cdot}6E{-}19)^2}{(7{\cdot}7E6)(1.6E{-}19)}$$

$$= 2{\cdot}95E{-}14\ m$$

RUTHERFORD'S MODEL OF THE ATOM

[TAKE THE SIMPLEST CASE OF HYDROGEN; ONE ELECTRON AND ONE UNIT OF CENTRAL CHARGE.]

[AND ALSO ASSUME CIRCULAR ORBITS.]



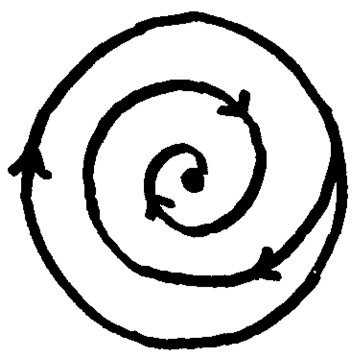
THE SINGLE ELECTRON CIRCLES THE POSITIVE NUCLEUS JUST AS A PLANET CIRCLES THE SUN.

COULOMB'S FORCE OF ATTRACTION BETWEEN THE POSITIVE NUCLEUS AND THE NEGATIVE ELECTRON SUPPLIES THE CENTRIPETAL FORCE THAT IS ALWAYS REQUIRED FOR MOTION IN A CIRCLE.

$$\frac{Ke^2}{r^2} = \frac{mv^2}{r} \qquad \therefore\ v^2 = \frac{Ke^2}{mr}$$

PROBLEMS

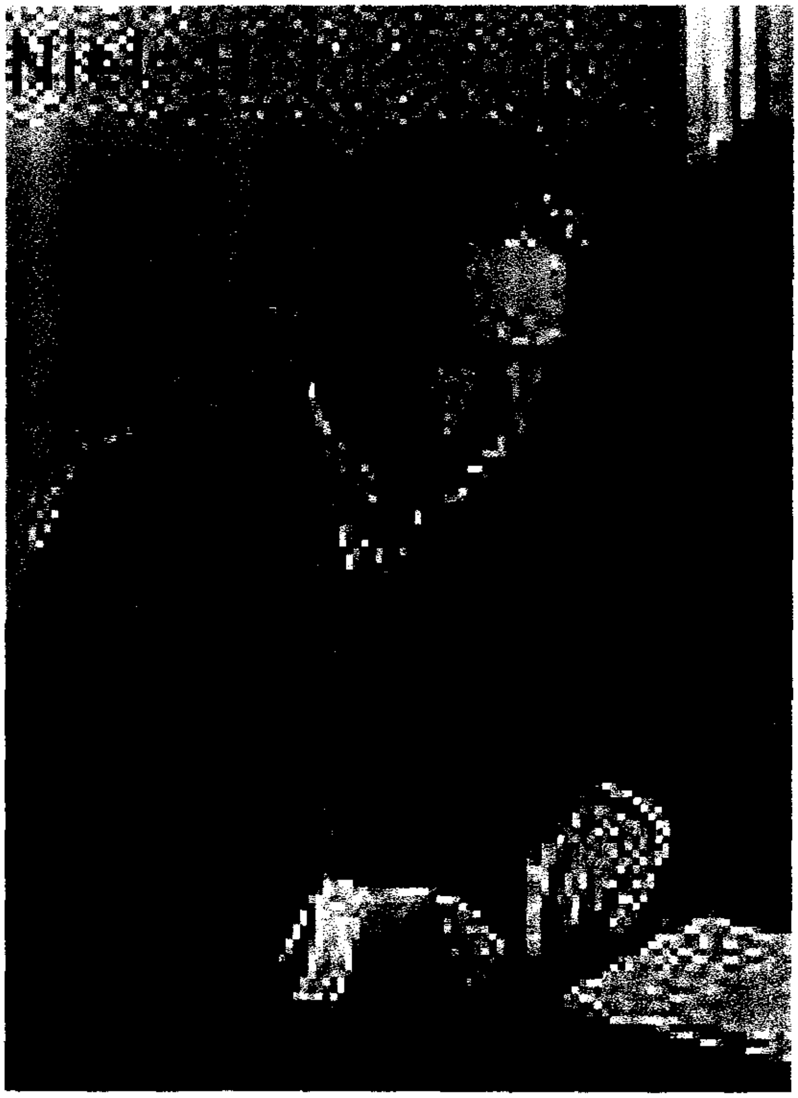
THERE WERE SOME SERIOUS DIFFICULTIES WITH THIS MODEL. IN PARTICULAR IT WAS NOTED THAT THE ELECTRON IS ACCELERATING AND, ACCORDING TO MAXWELL'S EQUATIONS, SHOULD RADIATE ENERGY AND THUS SPIRAL DOWN TOWARDS THE NUCLEUS. IT IS POSSIBLE TO CALCULATE THAT THIS WOULD OCCUR IN ABOUT 1E-9 SEC!



ALSO, CALCULATIONS FOR ATOMS WITH MORE THAN ONE ELECTRON SHOW ONE SIGNIFICANT DIFFERENCE COMPARED WITH THE "PLANETARY" ANALOGY. ORBITING PLANETS ATTRACT EACH OTHER WHEREAS THE ELECTRONS WOULD REPEL EACH OTHER. RUTHERFORD'S MODEL OF THE ATOM WAS INHERENTLY UNSTABLE --- IT WOULD FALL APART.

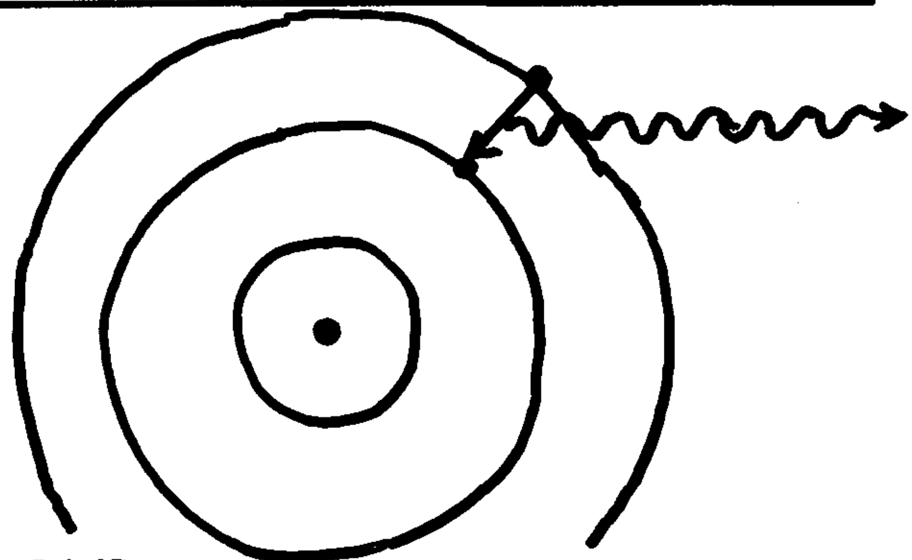
NIELS BOHR

OCTOBER 7 1885



Happy 122th Birthday!!

BOHR'S MODEL OF THE ATOM



NIELS BOHR (DANISH, 1885 - 1962) TIDIED UP RUTHERFORD'S MODEL WITH SOME *AD HOC* ASSUMPTIONS:-

1. THE ELECTRON COULD BE FOUND ONLY IN SPECIAL ORBITS, WHICH HE CALLED STATIONARY STATES.
2. AN ELECTRON IN A STATIONARY STATE OBEYS THE LAWS OF CLASSICAL PHYSICS.
3. TRANSITIONS BETWEEN STATIONARY STATES ARE ACCOMPANIED BY THE EMISSION OR ABSORPTION OF RADIATION WITH AN ENERGY EXACTLY CORRESPONDING TO THE DIFFERENCE IN ENERGY BETWEEN THE TWO ORBITS.

BOHR'S MODEL OF THE ATOM

BOHR'S MODEL ACCOUNTS FOR THE STABILITY OF THE ATOM BECAUSE THE ELECTRON COULD NOT SPIRAL CLOSER TO THE NUCLEUS THAN THE FIRST ORBIT.

BOHR GUESSED THAT EACH ORBIT WAS CHARACTERIZED BY A DIFFERENT VALUE OF THE "ANGULAR MOMENTUM":-

$$L = mvr$$

HE DIDN'T WANT TO ALLOW EVERY POSSIBLE VALUE OF ANGULAR MOMENTUM (THERE WERE ONLY CERTAIN VALUES FOR THE STATIONARY STATES) SO HE SET THE ANGULAR MOMENTUM TO BE AN INTEGRAL NUMBER TIMES PLANCK'S CONSTANT, WHICH JUST HAPPENS TO HAVE THE DIMENSIONS OF ANGULAR MOMENTUM. (ACTUALLY HE USED PLANCK'S REDUCED CONSTANT, $\hbar = h/2\pi$.)

$$L = mvr = n\hbar$$

WHERE n IS AN INTEGER, 1, 2, 3, ETC.

BOHR'S MODEL OF THE ATOM

[AGAIN TAKING THE SIMPLEST CASE OF HYDROGEN]

WE DESCRIBE BOHR'S ASSUMPTIONS AS SAYING *"THAT BOHR QUANTIZED ANGULAR MOMENTUM."*

WE HAVE:- $v = n\hbar/mr$

OF COURSE THE CENTRIPETAL FORCE IS STILL SUPPLIED BY THE COULOMB ATTRACTION BETWEEN THE NUCLEUS (CHARGE = +e) AND THE ELECTRON (CHARGE = -e).

$$\frac{Ke^2}{r^2} = \frac{mv^2}{r}$$

$$\therefore v^2 = \frac{Ke^2}{mr}$$

NOW, SUBSTITUTING FROM BOHR'S ANGULAR MOMENTUM REQUIREMENT GIVES:-

$$\left(\frac{n\hbar}{mr}\right)^2 = \frac{Ke^2}{mr}$$

$$\therefore r = \frac{n^2\hbar^2}{Kme^2}$$

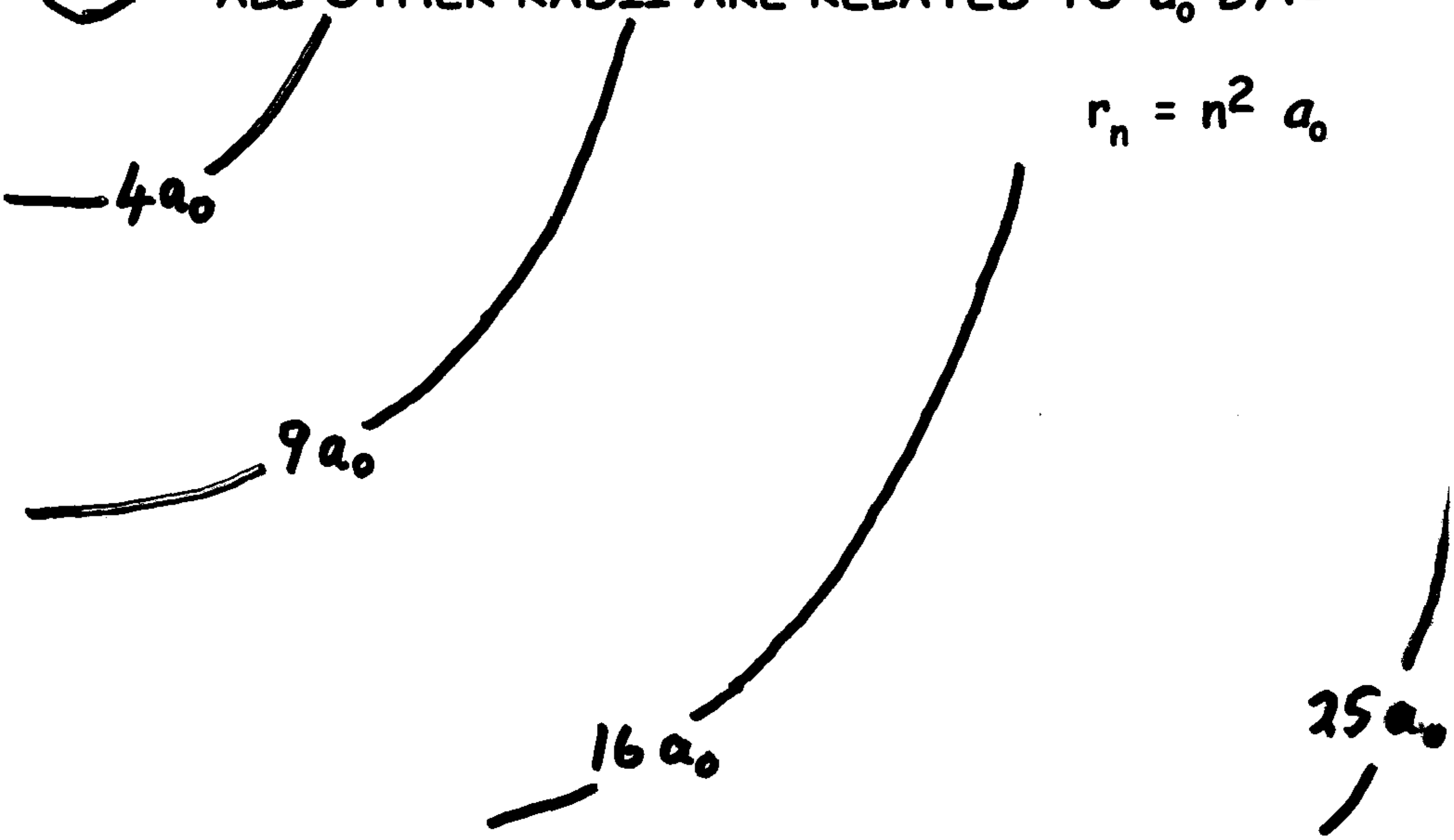
ACCORDING TO BOHR THESE ARE THE ONLY POSSIBLE RADII FOR AN ELECTRON ORBITING A HYDROGEN ATOM. EACH INTEGER, n, IDENTIFIES A PARTICULAR ORBIT (OR STATIONARY STATE).

NOTICE THAT THE SMALLEST RADIUS ORBIT CORRESPONDS TO n = 1. THIS RADIUS IS CALLED THE BOHR RADIUS AND IT IS OFTEN DENOTED BY A SPECIAL SYMBOL, a_0. IT HAS A VALUE OF:-

$$a_0 = 1^2\hbar^2/kme^2 = 5.3E\text{-}11 \text{ m}$$

ALL OTHER RADII ARE RELATED TO a_0 BY:-

$$r_n = n^2 a_0$$



CALCULATION OF THE ENERGY OF EACH ORBIT

KINETIC ENERGY = $\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{Ke^2}{mr}\right) = \frac{1}{2}\frac{Ke^2}{r}$

POTENTIAL ENERGY = $(-e)V$

$= -e\frac{Ke}{r} = -\frac{Ke^2}{r}$

TOTAL ENERGY = KIN. ENERGY + POT. ENERGY

$$E = -\frac{1}{2}\frac{Ke^2}{r}$$

SUBSTITUTING FOR $r = \frac{n^2\hbar^2}{Kme^2}$

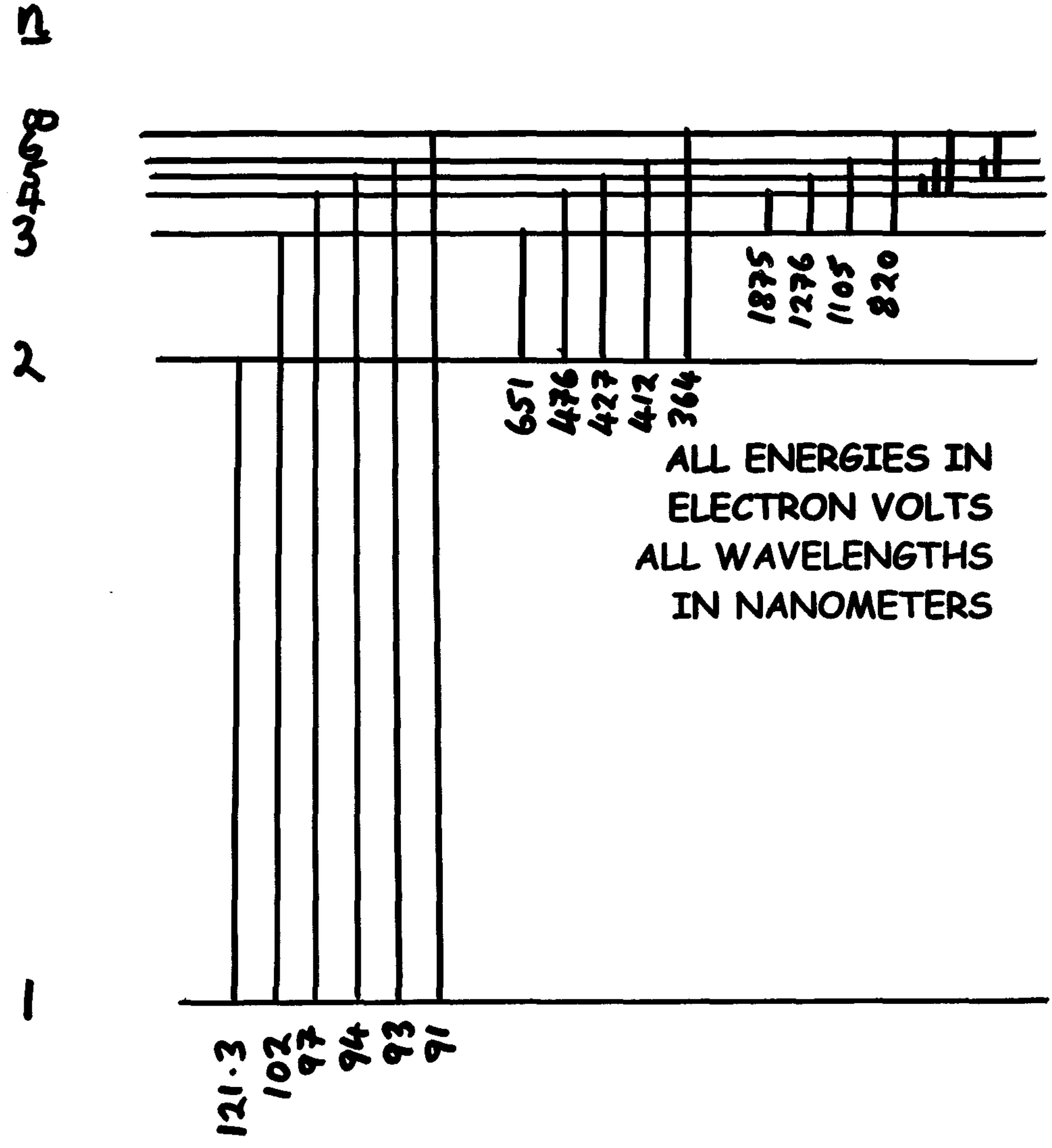
GIVES TOTAL ENERGY =

$$E = -\frac{1}{2}\frac{Ke^2\,Kme^2}{n^2\hbar^2} = -\frac{1}{2}\frac{K^2me^4}{n^2\hbar^2}$$

$\therefore E = -\frac{E_0}{n^2}$ WHERE $E_0 = \frac{1}{2}\frac{K^2me^4}{\hbar^2}$

$\longrightarrow E_0 = 2.2E\text{-}18\ J = 13.6\ eV$

THE ENERGY LEVELS



EMISSION AND ABSORPTION OF RADIATION

BOHR'S POSTULATE WAS THAT IF AN ELECTRON CHANGED FROM ONE ORBIT TO ANOTHER, ENERGY IS EITHER EMITTED OR ABSORBED, WITH THE RESULTING RADIATION HAVING AN ENERGY CORRESPONDING TO THE DIFFERENCE IN THE ENERGY LEVELS.

BOHR ALSO USED ONE MORE IDEA OF MAX PLANCK; IF THE ENERGY CHANGE IS E THEN THE FREQUENCY OF THE LIGHT IS GIVEN BY:-

$$E = hf = hc/\lambda$$

h = PLANCK'S CONSTANT

$= 6.6 \times 10^{-34}$ J.s

WHILE BOHR WAS DEVELOPING HIS MODEL HE REVIEWED THE FORMULAE OF BALMER AND RYDBERG.

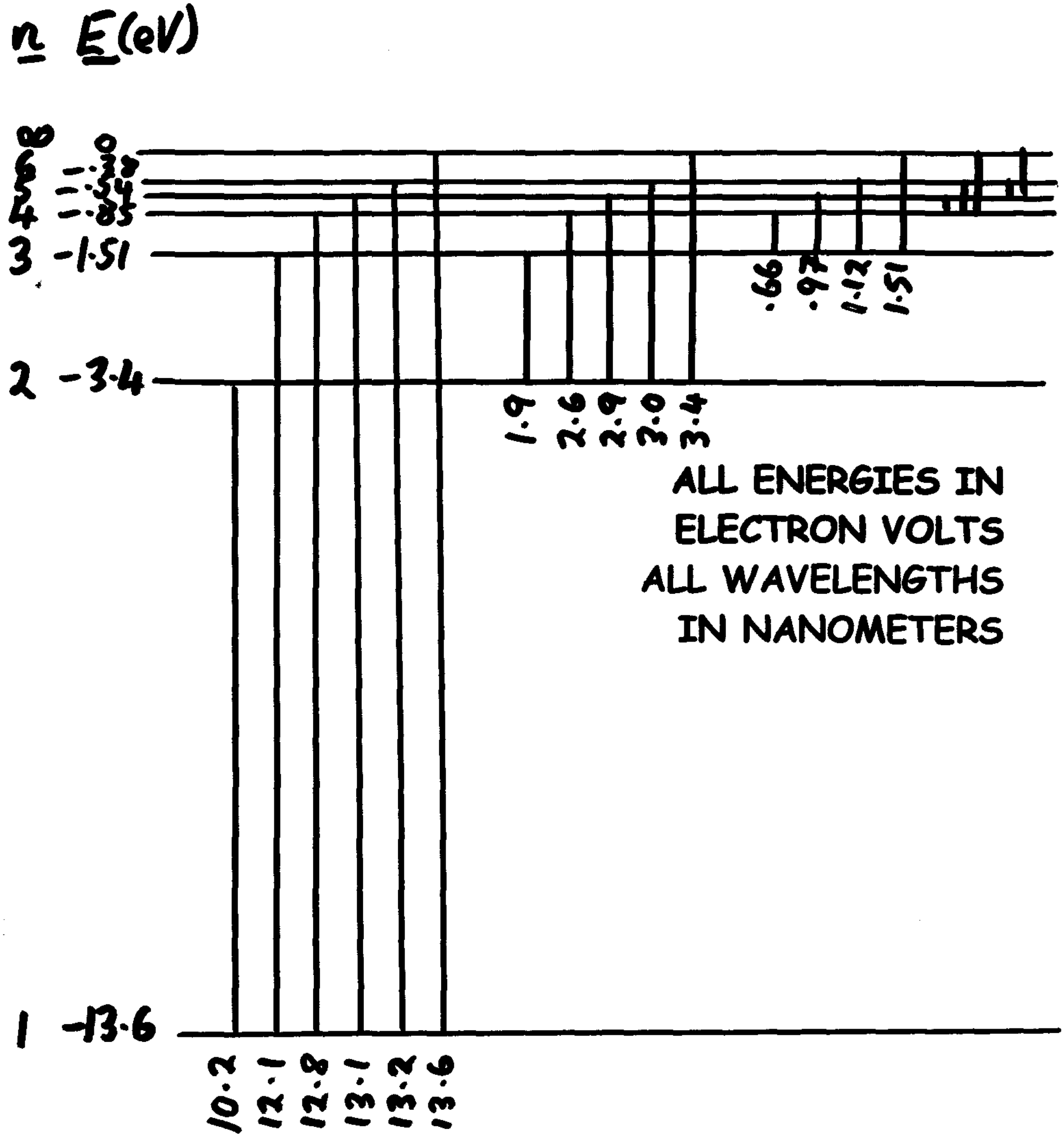
$$1/\lambda = R_H(1/m^2 - 1/n^2)$$

THE RYDBERG CONSTANT HAD A PRECISELY MEASURED VALUE OF R_H = <u>10,967,760</u> m^{-1}

"AS SOON AS I SAW BALMER'S FORMULA," HE SAID AFTERWARD, *"THE WHOLE THING WAS IMMEDIATELY CLEAR TO ME."*

HE CALCULATED THAT THE FREQUENCIES OF LIGHT CORRESPONDING TO TRANSITIONS BETWEEN HIS ORBITS COULD BE CALCULATED AS FOLLOWS:-

THE ENERGY LEVELS



ALL ENERGIES IN
ELECTRON VOLTS
ALL WAVELENGTHS
IN NANOMETERS

$\lambda(nm) = 1240/E_{phot}$ OR $\lambda(m) = 1240E\text{-}9/E_{phot}$

$1/\lambda(m) = E_{phot}/1240E\text{-}9 = 13.6/1240E\text{-}9\ (1/m^2 - 1/n^2)$

$1/\lambda(m) = \underline{10{,}967{,}742}\ (1/m^2 - 1/n^2)$

A PRETTY GOOD AGREEMENT!!

BOHR HAD SUCCESSFULLY CALCULATED THE RYDBERG CONSTANT STARTING FROM A TOTALLY NEW THEORY.

"THERE IS NOTHING IN THE WORLD WHICH IMPRESSES A PHYSICIST MORE," AN AMERICAN PHYSICIST COMMENTED, *"THAN A NUMERICAL AGREEMENT BETWEEN EXPERIMENT AND THEORY, AND I DO NOT THINK THAT THERE CAN EVER HAVE BEEN A NUMERICAL AGREEMENT MORE IMPRESSIVE THAN THIS ONE."*

GENERALIZATION OF THE BOHR MODEL

IF THE ATOMIC NUMBER OF THE ATOM IS Z, THEN THERE ARE Z ELECTRONS (EACH OF CHARGE = -e) SURROUNDING A NUCLEUS OF CHARGE = +Ze.

EACH ELECTRON NOW HAS AN ORBIT OF RADIUS

$$r_n = \frac{n^2 \times a_0}{Z}$$

AND AN ENERGY OF

$$E_n = \frac{Z^2 \times E_0}{n^2}$$

SO THE RADII ARE DECREASED BY A FACTOR OF Z AND THE ENERGIES ARE INCREASED BY A FACTOR OF Z^2.

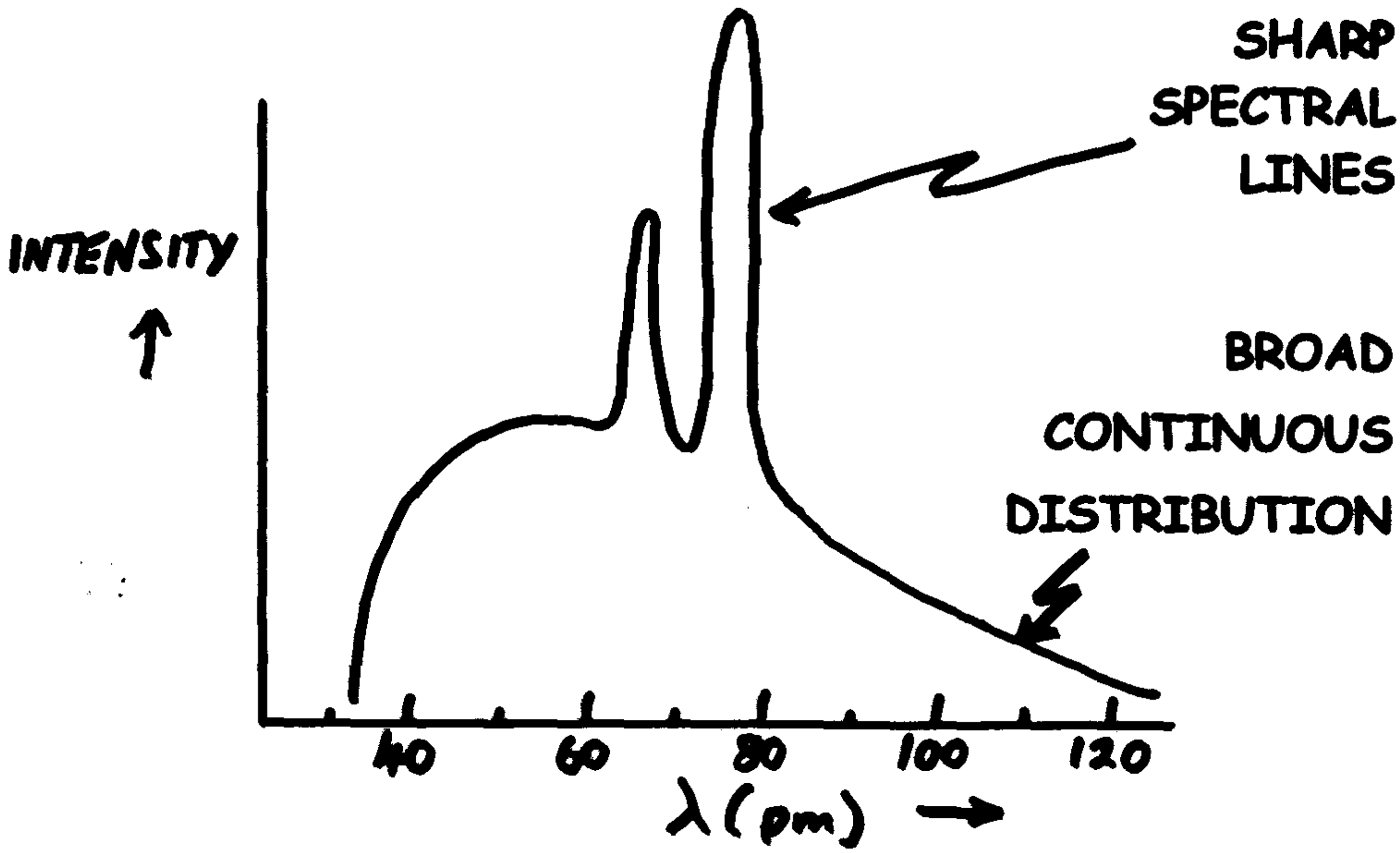
STRONGER ELECTRIC FIELDS

==> SMALLER ORBITS AND

==> MORE TIGHTLY BOUND ELECTRONS

CHARACTERISTIC X-RAY LINES

IF WE LOOK AT THE WAVELENGTH DISTRIBUTION OF X-RAYS PRODUCED BY BOMBARDING ELECTRONS ON A HIGH-Z TARGET.

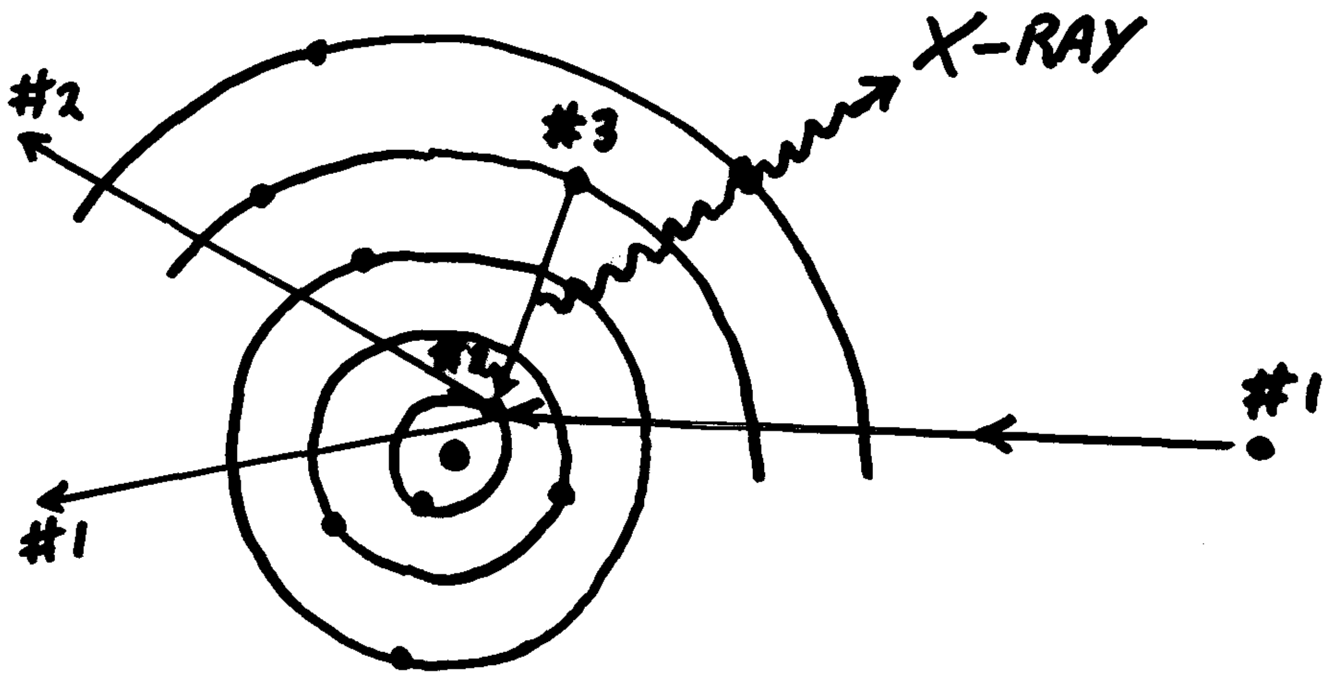


NOTE: ABOUT 1000 TIMES SHORTER THAN VISIBLE LIGHT

IF A HIGH-Z MATERIAL IS BOMBARDED BY ENERGETIC ELECTRONS THEN ONE OF THE TIGHTLY BOUND n = 1 ELECTRONS MAY BE KNOCKED OUT OF THE ATOM.

IMMEDIATELY AN ELECTRON FROM A HIGHER LEVEL ORBIT (n = 2, n = 3 OR HIGHER) WILL DROP DOWN TO THE LOWER ENERGY AND RADIATE AN X-RAY OF FREQUENCY,

$$f = (E_n - E_1)/h$$



ELECTRON #1 IS ACCELERATED WITH A HIGH VOLTAGE AND IT STRIKES THE ELECTRONS IN A HEAVY (HIGH Z) METAL TARGET.

ELECTRON #2 IS KNOCKED OUT OF THE ATOM.

ELECTRON #3 "FALLS DOWN" FROM A HIGHER ORBIT TO REPLACE #2 AND LOSES ENERGY IN THE FORM OF AN X-RAY.

IF THE FREQUENCIES OF THE X-RAYS ARE MEASURED THEN THESE CAN BE CONVERTED INTO ENERGIES AND THEN INTO ENERGY DIFFERENCIES BETWEEN THE ELECTRON ORBITS FOR THE ATOM.

BECAUSE THESE ENERGY DIFFERENCES ARE PROPORTIONAL TO THE Z^2 OF THE ATOM, A PRECISE MEASUREMENT OF THE X-RAY SPECTRUM OF A SUBSTANCE IS A GOOD WAY OF MEASURING ITS Z (i.e. THE NUMBER OF ELECTRONS AND THE SIZE OF THE CHARGE OF THE CENTRAL NUCLEUS).

THIS NUMBER BECAME KNOWN AS <u>THE ATOMIC NUMBER</u> OF THE ELEMENT.

THIS WAS FIRST REALIZED BY A YOUNG ENGLISH PHYSICIST, HENRY GWYN JEFFREYS MOSELEY. HARRY MOSELEY, AS HE WAS CALLED, PERFORMED A SERIES OF EXPERIMENTS IN 1913 AND 1914 THAT MEASURED THE FREQUENCY OF X-RAYS OF MANY ELEMENTS IN THE PERIODIC TABLE, FROM ALUMINUM (Z = 13) UP TO GOLD (Z = 79).

MOSELEY CONDUCTED HIS RESEARCH AT THE SAME TIME THAT NIELS BOHR WAS DEVELOPING HIS MODEL OF THE ATOM, AND THE TWO OF THEM CONFERRED OFTEN AND EXCHANGED INFORMATION. MOSELEY'S DATA PROVIDED A SPECTACULAR CONFIRMATION OF BOHR'S HYPOTHESES.

THE REGULARITY OF X-RAY FREQUENCIES ENABLED MOSELEY TO ORDER THE ELEMENTS BY THEIR ATOMIC NUMBER (Z). HE SAW THAT ATOMIC NUMBERS WERE ALL INTEGERS. IN SOME CASES HIS CAREFUL WORK SHOWED THAT MENDELEEV'S ORDERING (BY ATOMIC WEIGHT) WAS INCORRECT.

IN A FEW PLACES MOSELEY FOUND THAT THERE WAS MORE THAN ONE INTEGER BETWEEN THE Z NUMBERS OF APPARENTLY NEIGHBORING ELEMENTS. HE PREDICTED CORRECTLY THAT A NEW ELEMENT WOULD BE DISCOVERED.

MOSELEY'S WORK GAVE BETTER EXPERIMENTAL CONFIRMATION OF THE BOHR ATOM THAN HAD THE α-SCATTERING EXPERIMENTS OF RUTHERFORD, GEIGER AND MARSDEN. *"BECAUSE YOU SEE,"* BOHR SAID IN HIS LAST INTERVIEW, *"ACTUALLY THE RUTHERFORD WORK WAS NOT TAKEN SERIOUSLY. WE CANNOT UNDERSTAND TODAY, BUT IT WAS NOT TAKEN SERIOUSLY AT ALL. . .THE GREAT CHANGE CAME FROM MOSELEY."*

AT THE BEGINNING OF THE FIRST WORLD WAR MOSELEY ENLISTED AS A VOLUNTEER AND, IN SPITE OF ATTEMPTS BY RUTHERFORD AND OTHERS TO PROTECT HIM FROM MORTAL DANGER, HE INSISTED ON DOING COMBAT DUTY. HE WAS KILLED IN ACTION AT AGE 27 IN THE GALLIPOLI CAMPAIGN.