Modern Physics

Thornton and Rex, Chapter 12

Nuclear Physics

A PRE-HISTORY

1896 RADIOACTIVITY WAS FIRST OBSERVED BY HENRI BECQUEREL.

THERE WERE 3 KINDS:

- α RAYS + (HELIUM)
- γ RAYS 0 (EM WAVES)
- β RAYS - (ELECTRONS)

1902 RUTHERFORD AND SODDY SHOWED THAT, IN RADIOACTIVE DECAY, AN ATOM CHANGED INTO A DIFFERENT ELEMENT. SODDY CALLED THIS TRANSMUTATION.

1909 RUTHERFORD AND ROYDS SHOWED THAT α RAYS WERE HELIUM NUCLEI.

1912 RUTHERFORD (AND GEIGER AND MARSDEN) SHOWED THAT THE NUCLEUS WAS CONFINED TO A VERY SMALL RADIUS (A FEW x 10^{-15} METERS) AT THE CENTER OF THE ATOM (WHICH WAS A FEW x 10^{-11} METERS).

ARTIFICIAL TRANSMUTATION

RUTHERFORD, HAVING POSTULATED THE NUCLEUS AND DEFINED ITS MAIN PROPERTIES OF MASS AND CHARGE, WAS EAGER TO EXAMINE ITS STRUCTURE.

IN 1919 HE SUCCEEDED J.J. THOMSON AS DIRECTOR OF THE CAVENDISH LABORATORY IN CAMBRIDGE.

BOMBARDMENT OF A VARIETY OF ELEMENTS WITH α RAYS CONTINUED INTENSIVELY AND PRODUCED MUCH NEW INFORMATION.

MARSDEN HAD OBSERVED THE PRESENCE OF SOME PARTICLES WITH LONG RANGES WHEN HE FIRED α RAYS INTO AIR.

RUTHERFORD INVESTIGATED AND, AFTER 3 YEARS OF CAREFUL EXPERIMENTATION, SUCCEEDED IN OBSERVING THE FIRST CONTROLLED TRANSMUTATION IN THE LABORATORY.

BY COLLIDING NITROGEN WITH α RAYS RUTHERFORD SUCCEEDED IN CREATING HYDROGEN AND OXYGEN.

WE WOULD WRITE THIS AS:-

$$^{14}N_7 + {}^{4}\alpha_2 \rightarrow {}^{1}H_1 + {}^{17}O_8$$

HERE THE SUPERSCRIPT REFERS TO THE NUCLEAR MASS OF THE ELEMENTS (THE ATOMIC WEIGHT) AND THE SUBSCRIPT REFERS TO THE NUCLEAR ELECTRIC CHARGE (THE ATOMIC NUMBER). AS YOU CAN SEE, BOTH ARE "CONSERVED".

SOMETIMES THIS REACTION CAN BE WRITTEN IN A FORM OF SHORTHAND AS:-

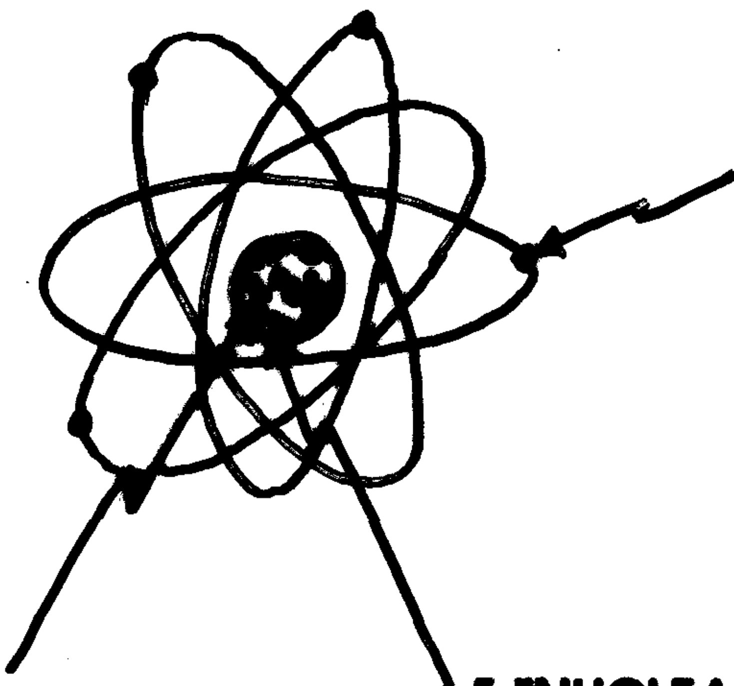
$$^{14}N_7\ (\alpha,p)\ {}^{17}O_8$$

(NOTE THAT IN HIS 1919 PUBLICATION, RUTHERFORD NAMED THE HYDROGEN NUCLEUS THE PROTON (SYMBOL p) FROM THE GREEK FOR "FIRST" BECAUSE IT IS THE FIRST IDENTIFIED BUILDING BLOCK FOR ALL THE NUCLEI.)

THE STRUCTURE OF THE NUCLEUS?

AN EARLY THEORY OF THE STRUCTURE GUESSED THAT THE NUCLEUS WAS COMPOSED OF BOTH PROTONS AND ELECTRONS; PROTONS TO GET THE MASS RIGHT AND JUST ENOUGH ELECTRONS TO COUNTER-BALANCE THE PROTONS TO GET THE CHARGE RIGHT.

FOR EXAMPLE, BERYLLIUM WHICH HAS AN ATOMIC WEIGHT OF 9 AND AN ATOMIC NUMBER OF 4 WOULD BE EXPECTED TO HAVE A NUCLEUS CONSISTING OF NINE PROTONS AND FIVE "NUCLEAR" ELECTRONS (WITH, OF COURSE, FOUR ELECTRONS ORBITING AROUND THE NUCLEUS).



THIS MODEL SOON RAN INTO SEVERE DIFFICULTIES. CALCULATIONS (USING HEISENBERG'S UNCERTAINTY PRINCIPLE) SHOWED THAT IT WAS IMPOSSIBLE TO CONFINE A LIGHT PARTICLE SUCH AS THE ELECTRON IN A SPACE AS SMALL AS THE NUCLEUS. (SEE EXAMPLE 6.8)

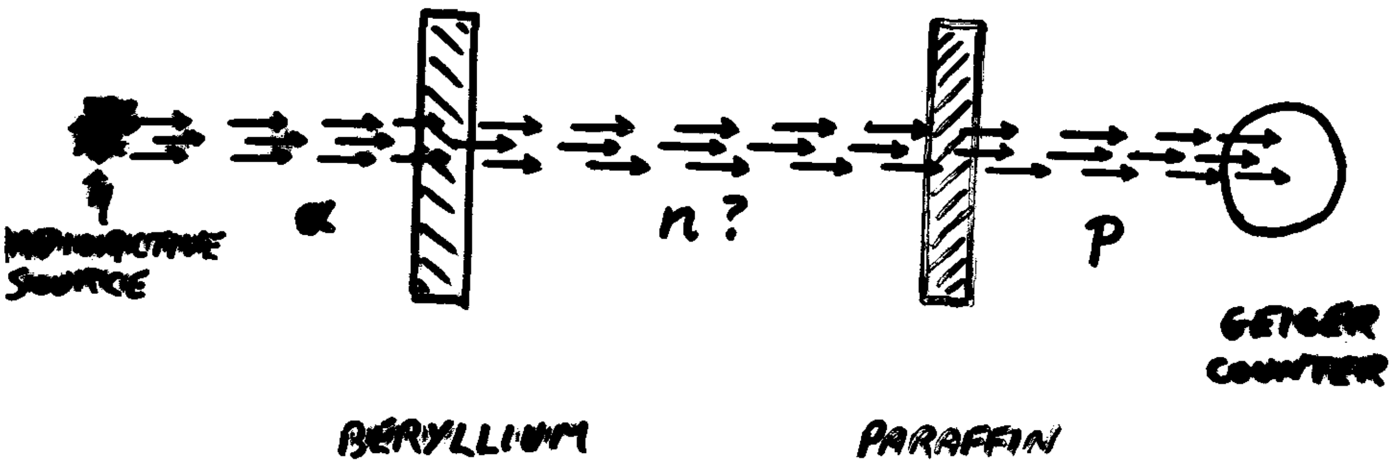
THE PROBLEM WAS SOLVED BY THE REALIZATION OF THE EXISTENCE OF ANOTHER "ELEMENTARY" PARTICLE, THE NEUTRON.

RUTHERFORD HAD OFTEN THOUGHT OF THE POSSIBILITY OF A NEUTRAL PARTICLE WITH THE SAME MASS AS THE PROTON. IN THE SAME 1920 TALK IN WHICH HE COINED THE NAME "PROTON" HE SPECULATED ON THE EXISTENCE OF A "NEUTRON" AND, IT IS IMPORTANT TO NOTE, KEPT ALIVE THE POSSIBILITY OF JUST SUCH A PARTICLE IN THE MINDS OF HIS PUPILS.

THE DISCOVERY OF [redacted]

IN 1930, W. BOTHE AND H. BECKER DISCOVERED A PUZZLING, PENETRATING RADIATION FROM BERYLLIUM BOMBARDED BY α PARTICLES. THIS RADIATION WAS NOT AFFECTED BY ELECTRIC OR MAGNETIC FIELDS <u>BUT</u> IT DID NOT APPEAR TO BE EM RADIATION.

IN 1932, THE ENGLISH PHYSICIST, JAMES CHADWICK, (A FORMER STUDENT OF RUTHERFORD, WORKING AT THE CAVENDISH) IDENTIFIED THE RADIATION BY ALLOWING IT TO STRIKE A PARAFFIN TARGET. PROTONS WERE KNOCKED OUT OF THE PARAFFIN AND WERE THEN DETECTED BY A GEIGER COUNTER.



FROM ENERGY AND MOMENTUM MEASUREMENTS, CHADWICK WAS ABLE TO SHOW THAT THE PROTONS IN THE PARAFFIN MUST HAVE BEEN HIT BY AN UNCHARGED PARTICLE OF APPROXIMATELY THE SAME MASS AS THE PROTON.

CHADWICK OFFICIALLY NAMED IT THE NEUTRON.
(SYMBOL = n).

$$^{9}Be_{4} + {}^{4}\alpha_{2} \rightarrow {}^{1}n_{0} + {}^{12}C_{6}$$

OR:-

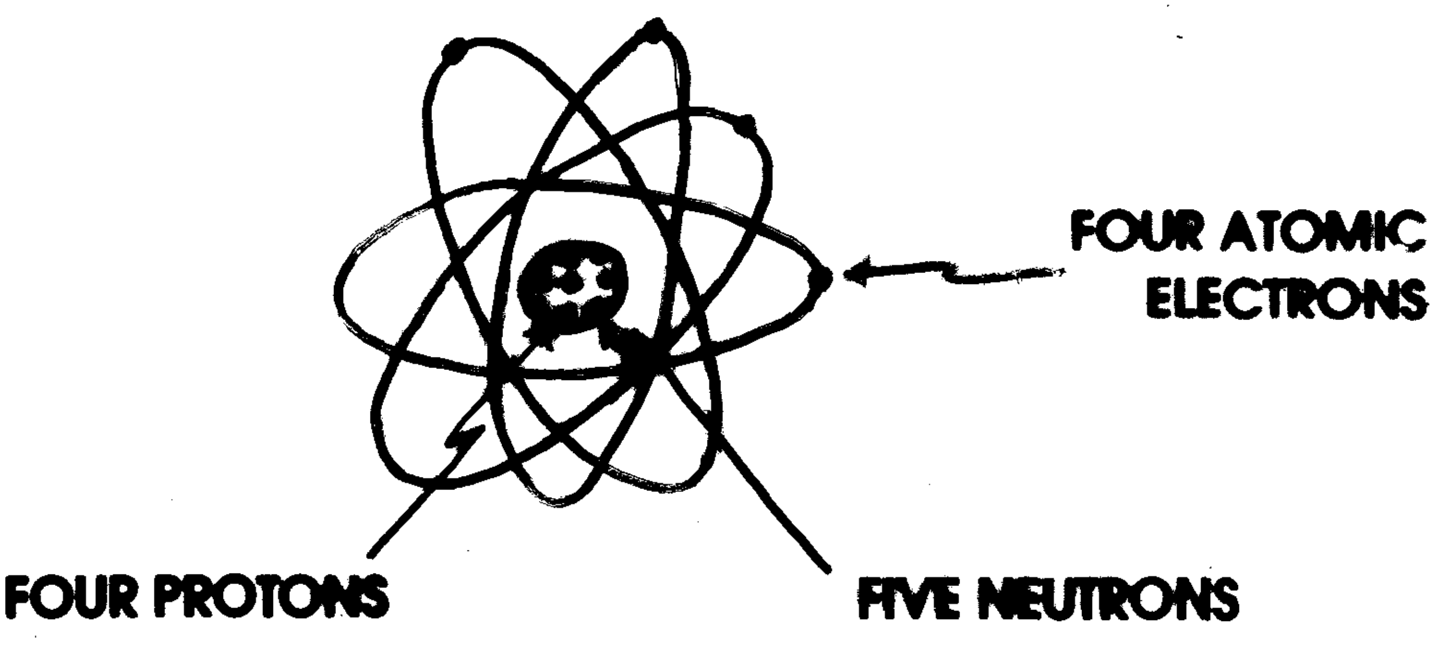
$$^{9}Be_{4}\ (\alpha,n)\ {}^{12}C_{6}$$

THE GENERIC NAME FOR A PROTON OR A NEUTRON IS A <u>NUCLEON</u>.

THE PROTON-NEUTRON MODEL OF THE NUCLEUS

SOON AFTER CHADWICK'S DISCOVERY, WERNER HEISENBERG SUGGESTED A MODEL WHERE THE NUCLEUS CONTAINS NO ELECTRONS BUT ONLY PROTONS AND NEUTRONS.

THE CHARGE ON THE NUCLEUS IS THEN GIVEN BY THE NUMBER OF PROTONS (WHICH IS EQUAL TO THE NUMBER OF ORBITING ELECTRONS) AND WHICH IS THE ATOMIC NUMBER. AND THE MASS OF THE NUCLEUS IS THE SUM OF THE MASSES OF THE PROTONS AND NEUTRONS. SO, USING MY PREVIOUS EXAMPLE OF BERYLLIUM (ATOMIC WEIGHT OF 9 AND ATOMIC NUMBER OF 4) WOULD HAVE FOUR PROTONS, FIVE NEUTRONS AND FOUR ELECTRONS ORBITING AROUND.



PROPERTIES OF ELEMENTARY PARTICLES (CIRCA 1932)

PARTICLE	SYMBOL	CHARGE (C)	MASS (kg)
ELECTRON	$^{0}e_{-1}$	-1.6×10^{-19}	9.1×10^{-31}
PROTON	$^{1}p_{1}$	$+1.6\times10^{-19}$	1.7×10^{-27}
NEUTRON	$^{1}n_{0}$	0	1.7×10^{-27}

WE WRITE:-

A = ATOMIC WEIGHT = $N_{PROTONS} + N_{NEUTRONS}$

Z = ATOMIC NUMBER = $N_{PROTONS} = N_{ELECTRONS}$

N = NEUTRON NUMBER = $N_{NEUTRONS} = A - Z$

SOME EXAMPLES:-

	A	Z	N
$^{1}H_{1}$	1	1	0
$^{4}He_{2}$	4	2	2

	A	Z	N
$^{7}Li_{3}$	7	3	4
$^{9}Be_{4}$	9	4	5
$^{12}C_{6}$	12	6	6
$^{16}O_{8}$	16	8	8
$^{23}Na_{11}$	23	11	12
$^{27}Al_{13}$	27	13	14
$^{35}Cl_{17}$	35	17	18
$^{56}Fe_{26}$	56	26	30
$^{63}Cu_{29}$	63	29	34
$^{108}Ag_{47}$	108	47	61
$^{197}Au_{79}$	197	79	118
$^{208}Pb_{82}$	208	82	126
$^{238}U_{92}$	238	92	146

ISOTOPES

THE "NAME" OF THE ELEMENT, HYDROGEN, IRON, LEAD, ETC. IS DETERMINED BY Z (THE NUMBER OF ELECTRONS) i.e. THE NAME OF AN ELEMENT IS A CHEMICAL PROPERTY.

SOME ELEMENTS EXIST IN DIFFERENT FORMS WITH DIFFERENT NUMBERS OF NEUTRONS.

FOR EXAMPLE:-

OXYGEN	Z = 8	A = 15	N = 7
OXYGEN	Z = 8	A = 16	N = 8
OXYGEN	Z = 8	A = 17	N = 9

THESE ARE CALLED <u>ISOTOPES</u>.

SIZES AND SHAPES OF NUCLEI

NUCLEI ARE MOST OFTEN ASSUMED TO BE SPHERES OF RADIUS R, WHERE:-

$$R = r_o A^{1/3}$$
$$\text{AND } r_o = 1.2\text{E-}15 \text{ m.}$$

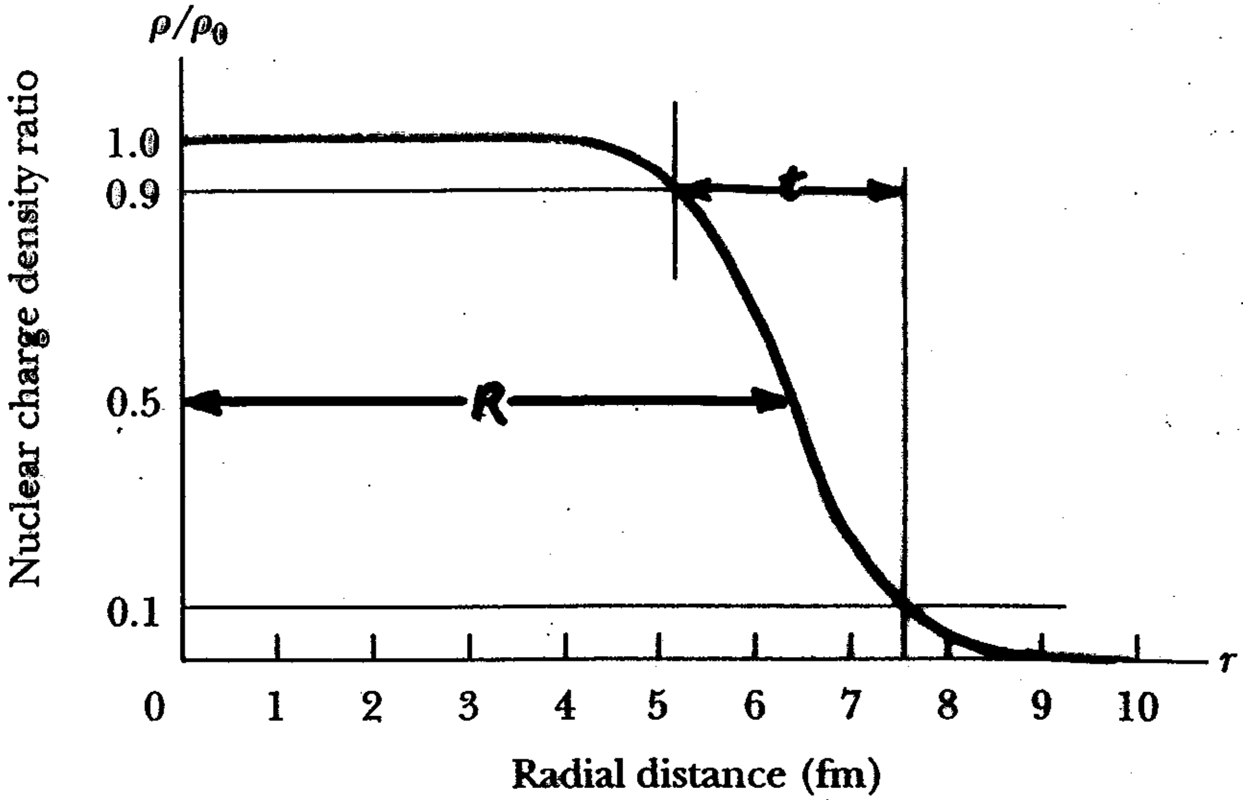
10^{-15} M = 1 FEMTOMETER (fm) BUT, BY COINCIDENCE, IT IS ALSO USED TO HONOR ENRICO FERMI, ONE OF THE PIONEERS OF NUCLEAR PHYSICS, SO WE SAY:-

1 fm = 1 Fermi

THE ACTUAL SHAPES OF NUCLEI WERE FOUND IN THE 1950s BY ROBERT HOFSTADTER AT STANFORD USING 500 MeV ELECTRONS ($\lambda_{deBroglie}$ ~ 2.5 fm). THEY ARE CHARACTERIZED BY A "FERMI" DISTRIBUTION:-

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-R)/a}}$$

R IS THE DISTANCE THAT THE NUCLEAR DENSITY HAS FALLEN TO 50% OF ITS CENTRAL VALUE.
t = 4.4a IS THE SURFACE THICKNESS (90% → 10%).



ρ/ρ0
Nuclear charge density ratio
1.0
0.9
0.5
0.1
t
R
r
0
1
2
3
4
5
6
7
8
9
10
Radial distance (fm)

NUCLEAR FORCES

IF NUCLEI ARE COMPOSED OF ONLY PROTONS AND NEUTRONS, WHAT HOLDS THEM TOGETHER?

THE PROTONS MUST REPEL EACH OTHER WITH ENORMOUS FORCES DUE TO THEIR POSITIVE CHARGES AND THE VERY SMALL DISTANCE SEPARATING THEM.

IN THE NUCLEUS THERE MUST EXIST SOME NEW KIND OF ATTRACTIVE FORCE BETWEEN THE PROTONS (AND NEUTRONS) WHICH IS STRONGER THAN THE ELECTRICAL REPULSION BETWEEN THE PROTONS.

THIS FORCE IS <u>NOT</u> THE FORCE OF GRAVITY (WHICH IS ABOUT 10^{35} TIMES WEAKER THAN THE ELECTRICAL REPULSION.

THE NEW FORCE IS CALLED THE STRONG NUCLEAR FORCE.

WITHIN THE NUCLEUS IT IS SEVERAL HUNDRED TIMES STRONGER THAN THE ELECTROMAGNETIC FORCE.

THE STRONG NUCLEAR FORCE HAS A SHORT RANGE. OUTSIDE OF THE NUCLEUS THE EM FORCE DOMINATES AND THE NUCLEUS BEHAVES JUST LIKE A POSITIVE CHARGE.

ARTIFICIAL RADIOACTIVITY

THIS WAS DISCOVERED BY IRENE JOLIOT-CURIE (THE DAUGHTER OF MME. MARIE CURIE) AND FREDERIC JOLIOT. THEY WERE AWARDED THE NOBEL PRIZE FOR CHEMISTRY IN 1935.

THEY BOMBARDED MANY MATERIALS WITH ENERGETIC PARTICLES SUCH AS PROTONS AND α PARTICLES. THESE CAN FORCE THEIR WAY INTO A STABLE NUCLEUS AND CREATE A NEW, UNSTABLE NUCLEUS, WHICH WILL THEN DECAY INTO ANOTHER NUCLEUS AND OTHER LIGHTER PARTICLES. THIS IS THE PHENOMENON OF ARTIFICIAL (OR INDUCED) RADIOACTIVITY.

FOR EXAMPLE, RUTHERFORD'S EXPERIMENT OF 1919 WAS ACTUALLY AN EXAMPLE OF ARTIFICIAL RADIOACTIVITY.

THE PROCESS REALLY WAS:-

$$^{14}N_7 + {}^{4}\alpha_2 \rightarrow {}^{18}F_9 \rightarrow {}^{1}p_1 + {}^{17}O_8$$

THE FLUORINE NUCLEUS WAS AN UNSTABLE ISOTOPE (STABLE FLUORINE IS $^{19}F_9$) AND IT DECAYS INTO OXYGEN AND A PROTON. RUTHERFORD DID NOT HAVE THE INSTRUMENTATION TO OBSERVE THE TWO-STEP PROCESS BUT, 15 YEARS LATER, THE JOLIOT-CURIES DID.

ACCELERATORS

MANY OTHER POSSIBILITIES EXIST TO INITIATE NUCLEAR REACTIONS (AND SO STUDY NUCLEAR FORCES). BUT THEY REQUIRE COLLIDING PARTICLES OF SUFFICIENT ENERGY TO OVERCOME THE COULOMB REPULSION AND SO GET CLOSE ENOUGH TO THE TARGET NUCLEUS.

THIS ENERGY COULD BE OBTAINED WITH RADIOACTIVE SOURCES (WHICH RUTHERFORD, CHADWICK AND THE JOLIOT-CURIES HAD USED) BUT THEY WERE OFTEN NOT INTENSE ENOUGH TO OBSERVE THE INFREQUENT REACTIONS.

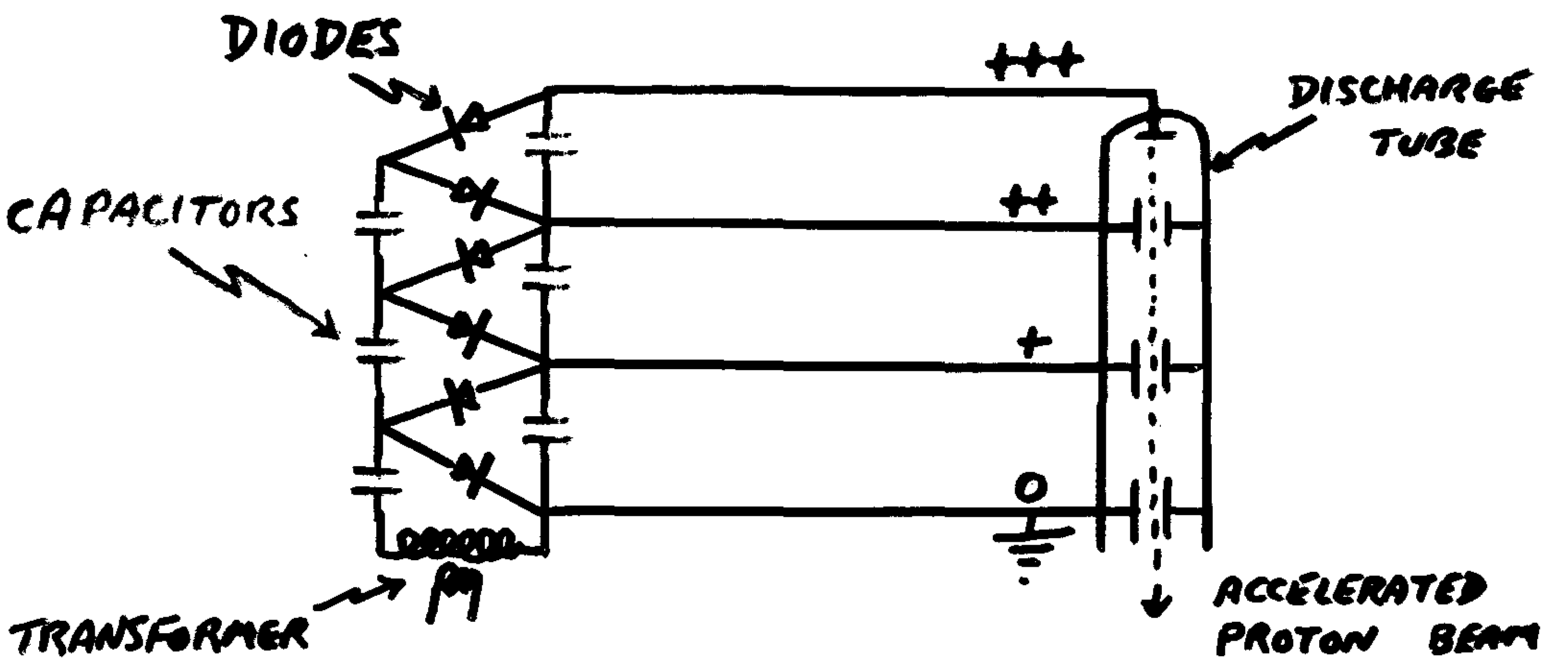
ON THE OTHER HAND, VERY INTENSE SOURCES OF PROTONS OR α PARTICLES COULD BE MADE AVAILABLE WITH DISCHARGE TUBES. IF SOME METHOD COULD BE USED TO INCREASE THE VOLTAGE OF THE DISCHARGE TUBE OR SOMEHOW ACCELERATE THE PARTICLE TO HIGH ENERGY WITH ELECTRIC FIELDS THEN NUCLEAR REACTIONS COULD BE STUDIED MORE EASILY.

A "RACE" WAS ON TO "SPLIT THE ATOM" BY ARTIFICIAL MEANS.

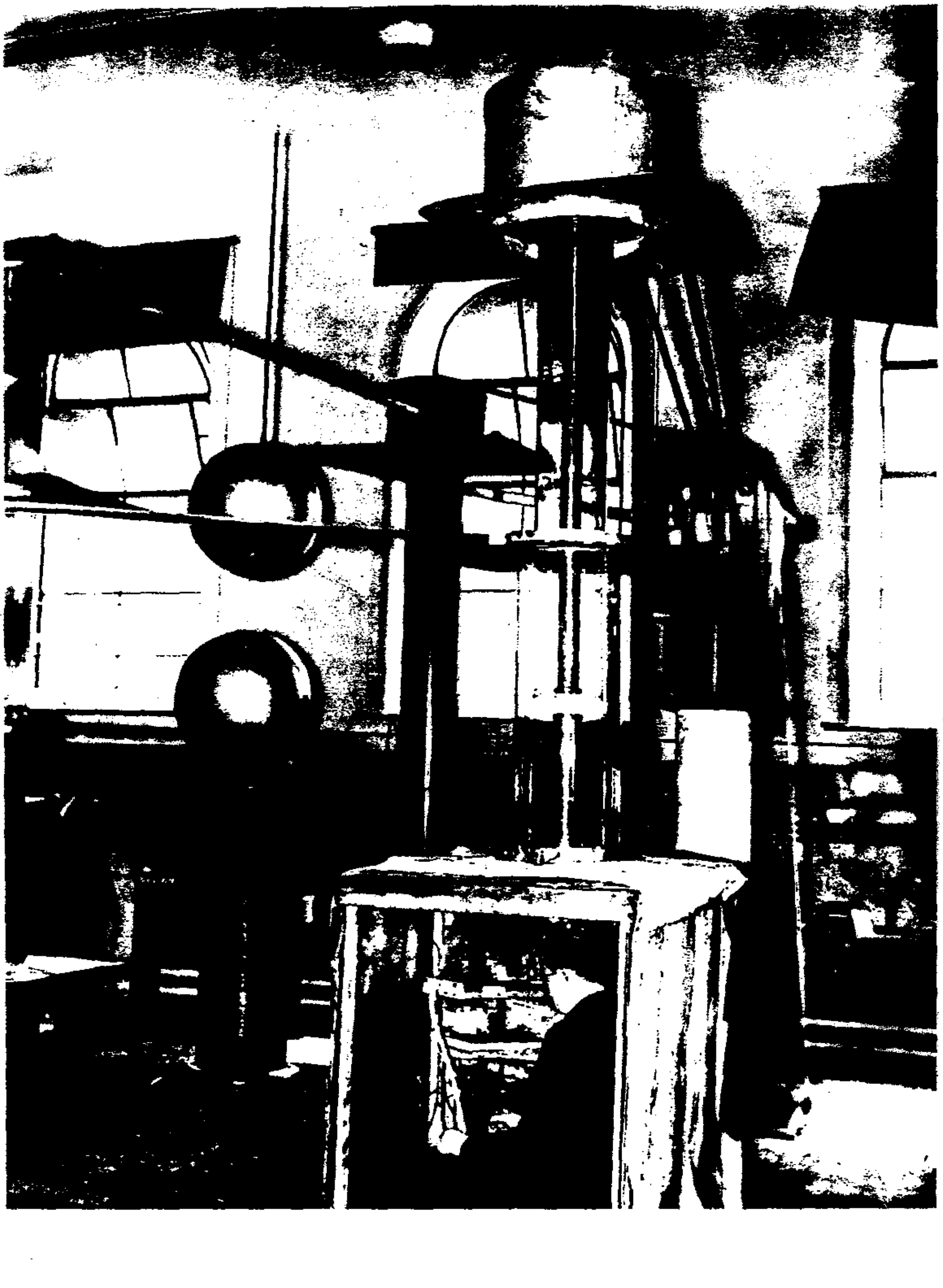
ENTRANTS TO THIS RACE INCLUDED THE VAN DER GRAAFF GENERATOR, THE CYCLOTRON, AND EVEN NATURAL LIGHTNING.

THE WINNER WAS A COMPLICATED ARRANGEMENT OF TRANSFORMERS, DIODE RECTIFIERS, AND CAPACITORS ASSEMBLED AT THE CAVENDISH BY JOHN COCKCROFT AND ERNEST WALTON.

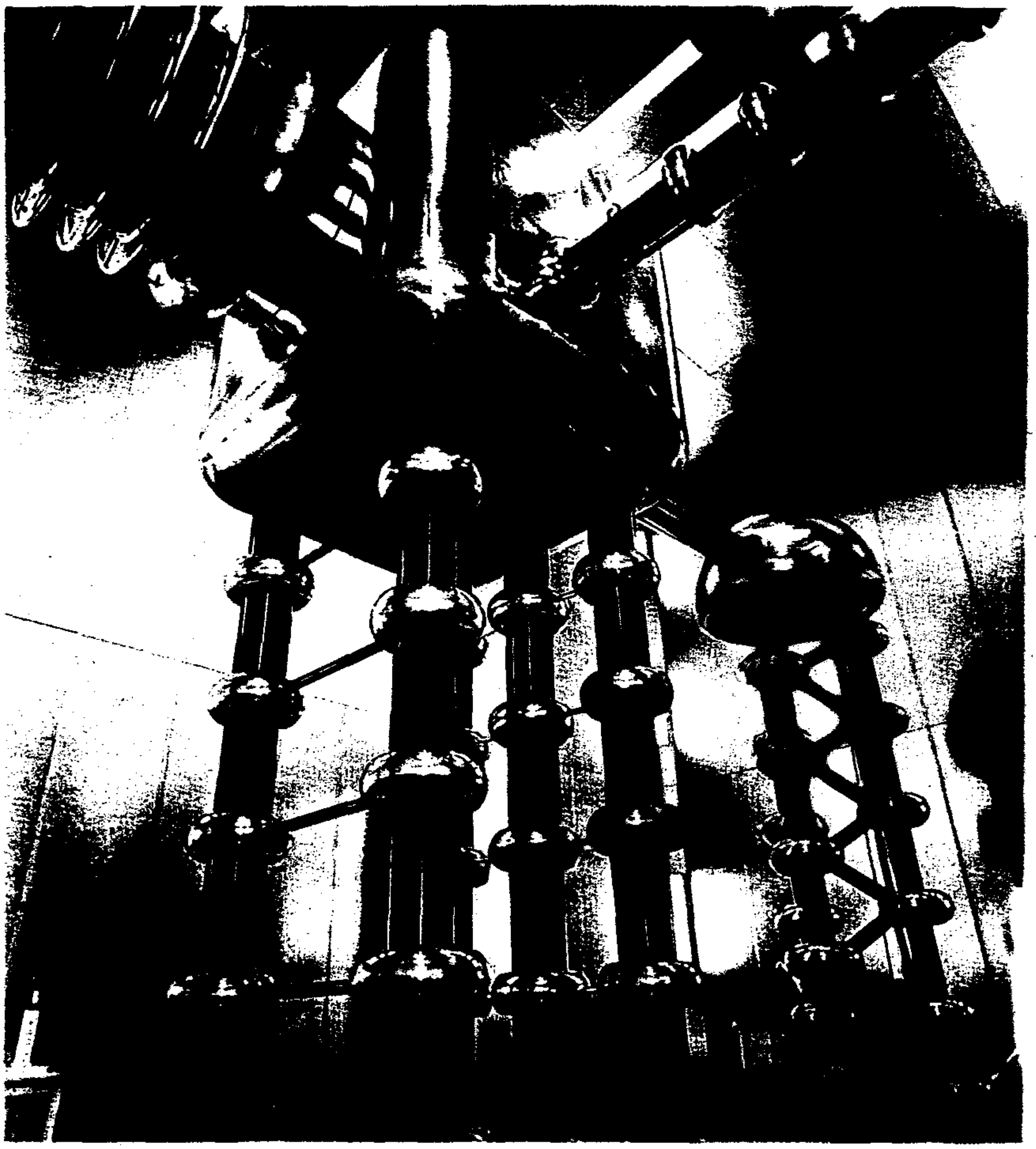
THE COCKCROFT-WALTON GENERATOR



I'VE ILLUSTRATED A 3-STAGE STACK. THE DISCHARGE TUBE WOULD BE CHARGED UP TO 600 KV, THE FIRST ELECTRODE WOULD BE AT 400 KV, THE NEXT AT 200 KV AND THE BOTTOM ONE AT GROUND. A PROTON WOULD GAIN 200 keV AT EACH STAGE.



A View of the Cockcroft-Walton Apparatus in late 1931



A Modern Cockcroft-Walton Machine (at Fermilab)

COCKCROFT AND WALTON WERE ABLE TO ACCELERATE PROTONS TO ENERGIES OF ABOUT 800 keV (800,000 VOLTS).

THEY ACCOMPLISHED THE TRANSMUTATION OF LITHIUM BY BOMBARDING IT WITH PROTONS OF THIS ENERGY.

THE REACTION WAS:-

$$^{7}Li_{3} + {}^{1}p_{1} \rightarrow {}^{4}He_{2} + {}^{4}He_{2}$$

AFTER THIS SUCCESS, ACCELERATORS WERE OFTEN CALLED ATOM SMASHERS.

NEUTRON ACTIVATION

THERE IS A METHOD OF INITIATING NUCLEAR REACTIONS WITHOUT THE NEED FOR AN ACCELERATOR. THIS IS THE NEUTRON, WHICH OF COURSE HAS THE DISADVANTAGE THAT, BEING UNCHARGED, IT CAN NOT BE ACCELERATED BY ELECTRIC FIELDS (OR BENT BY MAGNETIC FIELDS). ON THE OTHER HAND IT HAS THE GREAT ADVANTAGE THAT IT CAN PENETRATE CLOSE TO THE NUCLEUS WITHOUT BEING REPELLED BY THE COULOMB FORCE.

IN THE EARLY 1930'S THE GREAT ITALIAN PHYSICIST ENRICO FERMI (1901 - 1954) REALIZED THE IMPORTANCE OF A CAREFUL STUDY OF THE NEUTRON BOMBARDMENT OF VARIOUS NUCLEI. HE BOMBARDED A SERIES OF ELEMENTS FROM HYDROGEN TO URANIUM WITH NEUTRONS AND STUDIED THE RADIOACTIVE TRANSFORMATIONS PRODUCED.

WHEN HE CAME TO URANIUM FERMI WAS UNABLE TO IDENTIFY THE FINAL PRODUCTS OF THE REACTIONS. AT ABOUT THIS TIME, HE WAS FORCED TO LEAVE ITALY BECAUSE OF THE POLITICAL SITUATION UNDER MUSSOLINI. HE AND HIS FAMILY EMIGRATED TO THE UNITED STATES; ACCEPTING A POSITION FIRST AT COLUMBIA UNIVERSITY AND LATER MOVING TO THE UNIVERSITY OF CHICAGO.

BINDING ENERGY

BECAUSE THE STRONG NUCLEAR FORCE BINDS NUCLEONS (PROTONS AND NEUTRONS) TOGETHER, WORK MUST BE DONE TO SEPARATE THEM. CONVERSELY ENERGY IS RELEASED WHEN NUCLEONS JOIN TOGETHER TO FORM A STABLE NUCLEUS.

THE ENERGY DIFFERENCE BETWEEN A NUCLEUS AND ITS SEPARATE CONSTITUENT NUCLEONS IS CALLED THE <u>BINDING ENERGY</u>.

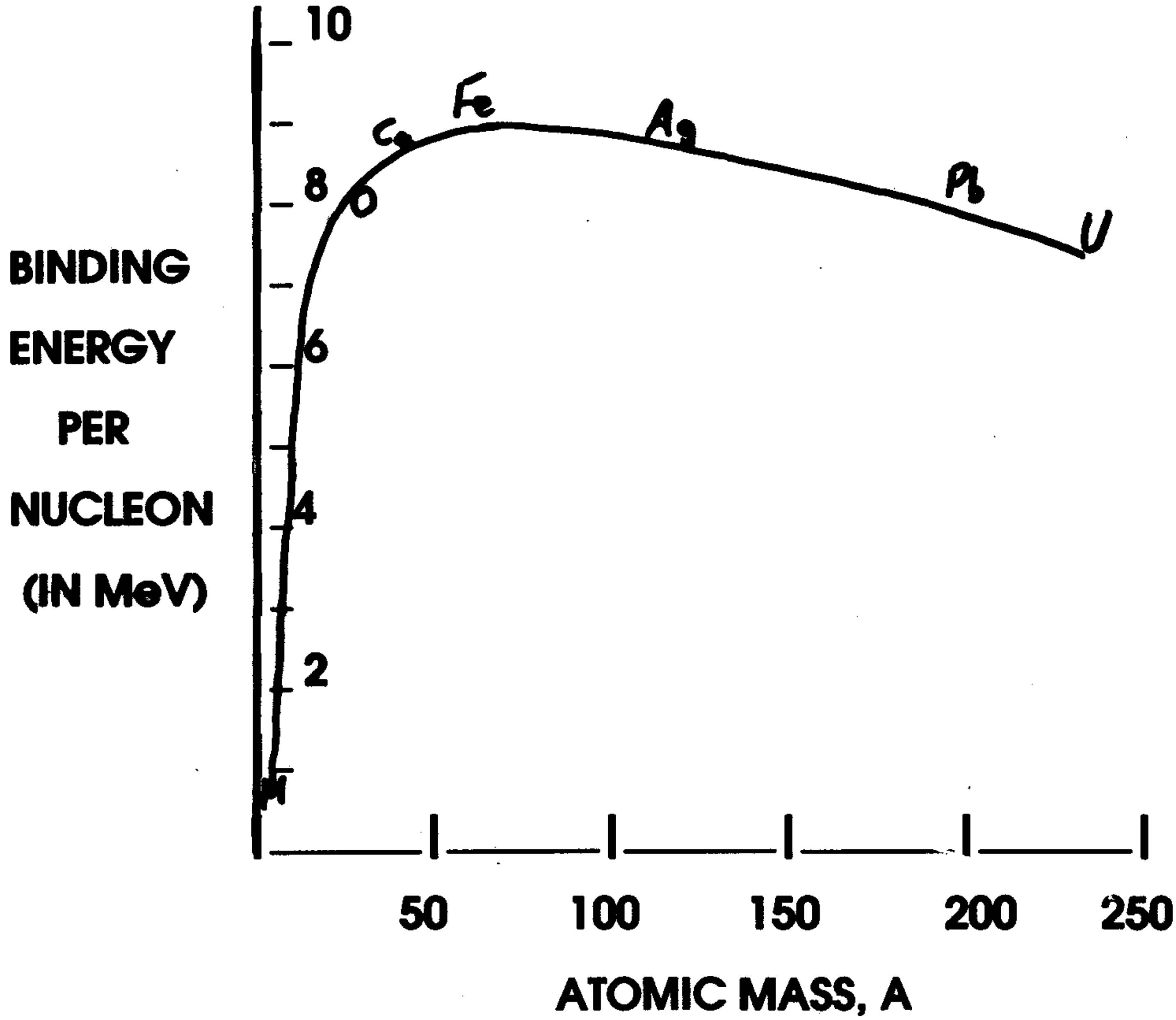
BY THE SPECIAL THEORY OF RELATIVITY ($E = mc^2$), THE TOTAL MASS OF THE NUCLEUS WILL BE LESS THAN THE SUM OF THE MASSES OF THE CONSTITUENT PROTONS AND NEUTRONS.

$$\text{BINDING ENERGY} = (Z \times m_p + N \times m_n - {}^{A}M_{Z}) \times c^2$$

- $Z \times m_p$: MASS OF Z PROTONS
- $N \times m_n$: MASS OF N NEUTRONS
- ${}^{A}M_{Z}$: MASS OF THE NUCLEUS

A VARIABLE THAT IS OFTEN USED IS THE BINDING ENERGY PER NUCLEON:-

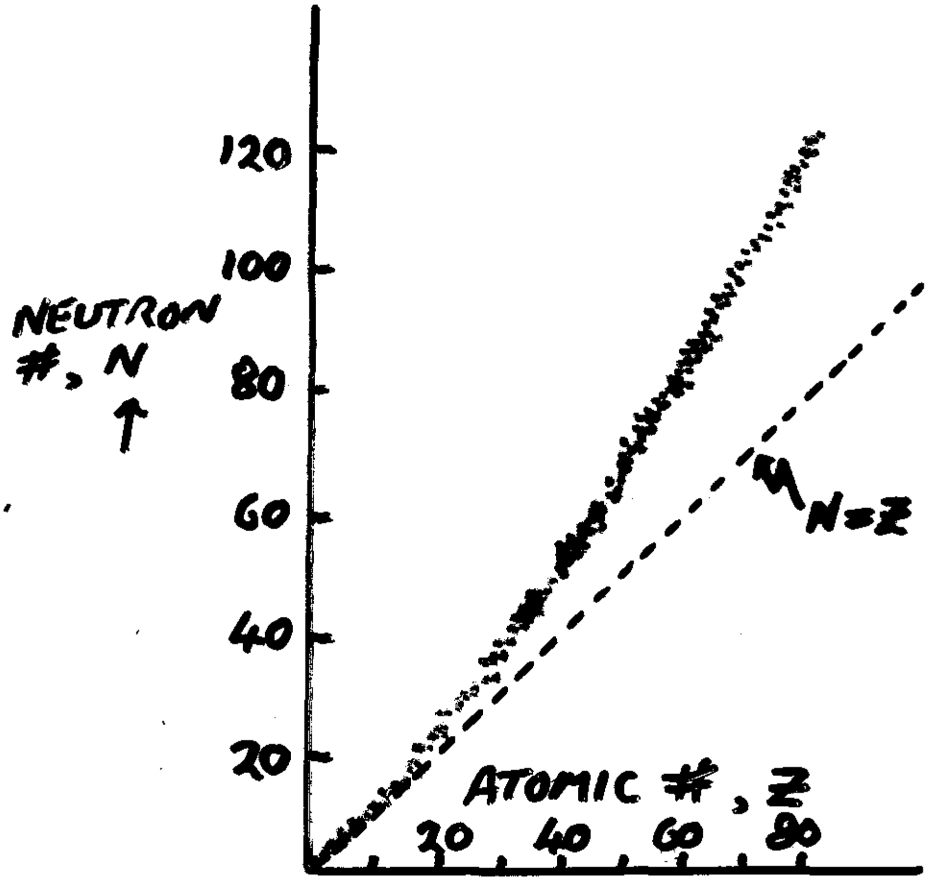
$$= \frac{\text{BINDING ENERGY}}{A}$$



10
8
6
4
2
BINDING
ENERGY
PER
NUCLEON
(IN MeV)
Fe
Ag
Pb
U
50
100
150
200
250
ATOMIC MASS, A

THE STABILITY PLOT

IN A COMMON WAY OF VIEWING NUCLEI, INVENTED BY EMILIO SEGRE, THE NUMBER OF NEUTRONS IS PLOTTED AS A FUNCTION OF THE NUMBER OF PROTONS.



IT CAN BE SEEN THAT THE LIGHTER NUCLEI CONTAIN ALMOST EQUAL NUMBERS OF PROTONS AND NEUTRONS, WHILE HEAVIER NUCLEI SHOW A DISTINCT EXCESS OF NEUTRONS.

THIS HAS BEEN ATTRIBUTED TO THE FACT THAT, WITH INCREASING NUMBER OF PROTONS, THE COULOMB'S FORCE OF REPULSION REQUIRES MORE AND MORE NEUTRONS (ATTRACTIVE) TO OVERCOME IT.

Radioactive Decay

In any nuclear reaction, the following quantities are conserved:

1. Nucleon Number, A
2. Charge
3. Energy
4. Momentum

α Decay

$$^{227}Th_{90} \rightarrow {}^{223}Ra_{88} + {}^{4}\alpha_{2}$$

The masses of the two nuclei on the RHS add up to about 1.1×10^{-29} kg less than the mass on the LHS. The difference is made up in Kinetic Energy (using $E=mc^2$).

Most of the energy (about 6 MeV) is taken by the α particle.

β Decay

$$^{24}Na_{11} \rightarrow {}^{24}Mg_{12} + {}^{0}e_{-1}$$

The electron takes away 1 unit of electric charge, increasing Z of the nucleus by 1. Atomic mass A is unchanged.

(It's as if a neutron changes into a proton and an electron.)

Positive β Decay

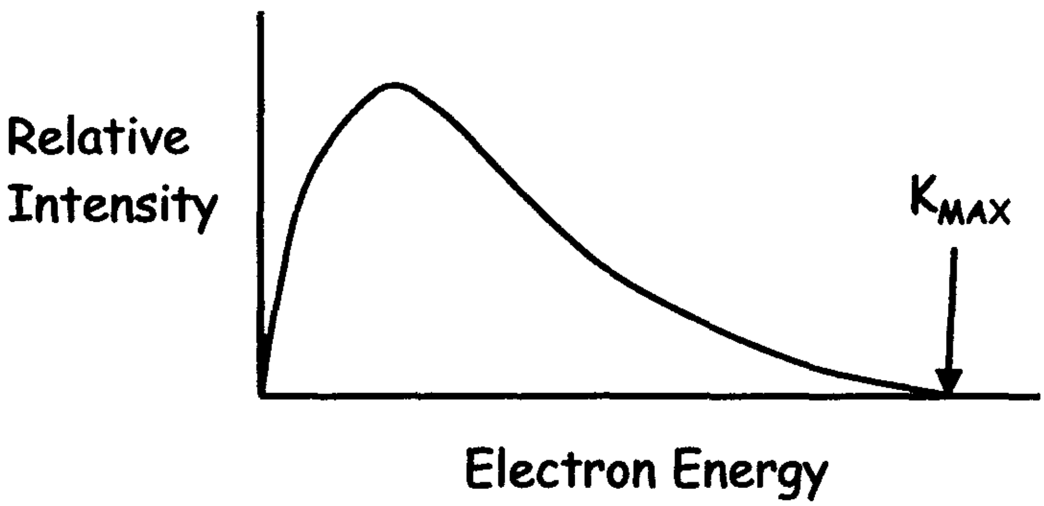
The anti-particle to the electron (the positron, predicted by the Dirac Equation) can also take part in β decay.

$$^{13}N_{7} \rightarrow {}^{13}C_{6} + {}^{0}e_{+1}$$

(It's as if a proton changes into a neutron and a positron.)

For β decay (positive or negative), one would expect the electron to come out with a single energy, just as for α decay.

However, experiment showed a continuous spectrum of energies:



In 1930 Wolfgang Pauli solved this by suggesting that the electron energy was shared with a new particle, the neutrino.

The neutrino is chargeless and (almost) massless.

1956 - neutrino finally detected.

Difficult to detect because it does not interact through EM or strong force, only through the weak force.

1998 - evidence of a nonzero mass confirmed.

Its mass is of order 10^{-3} eV/c^2 or less (compared to an electron mass of $m_e = 5.11 \times 10^5$ eV/c^2).

We can now write the β decays as:

$$^{24}Na_{11} \rightarrow {}^{24}Mg_{12} + {}^{0}e_{-1} + \bar{\nu}$$

$$^{13}N_{7} \rightarrow {}^{13}C_{6} + {}^{0}e_{+1} + \nu$$

Electron Capture

$$^{55}Fe_{26} + {}^{0}e_{-1} \rightarrow {}^{55}Mn_{25} + \nu$$

For higher-Z nuclides, it is possible for electron capture to occur.

The effect is the same as for positive β decay: a proton is converted to a neutron.

When electron capture occurs, the hole left by the captured inner electron will be filled by an outer electron which drops down, while emitting an X-ray of characteristic wavelength.

γ Decay

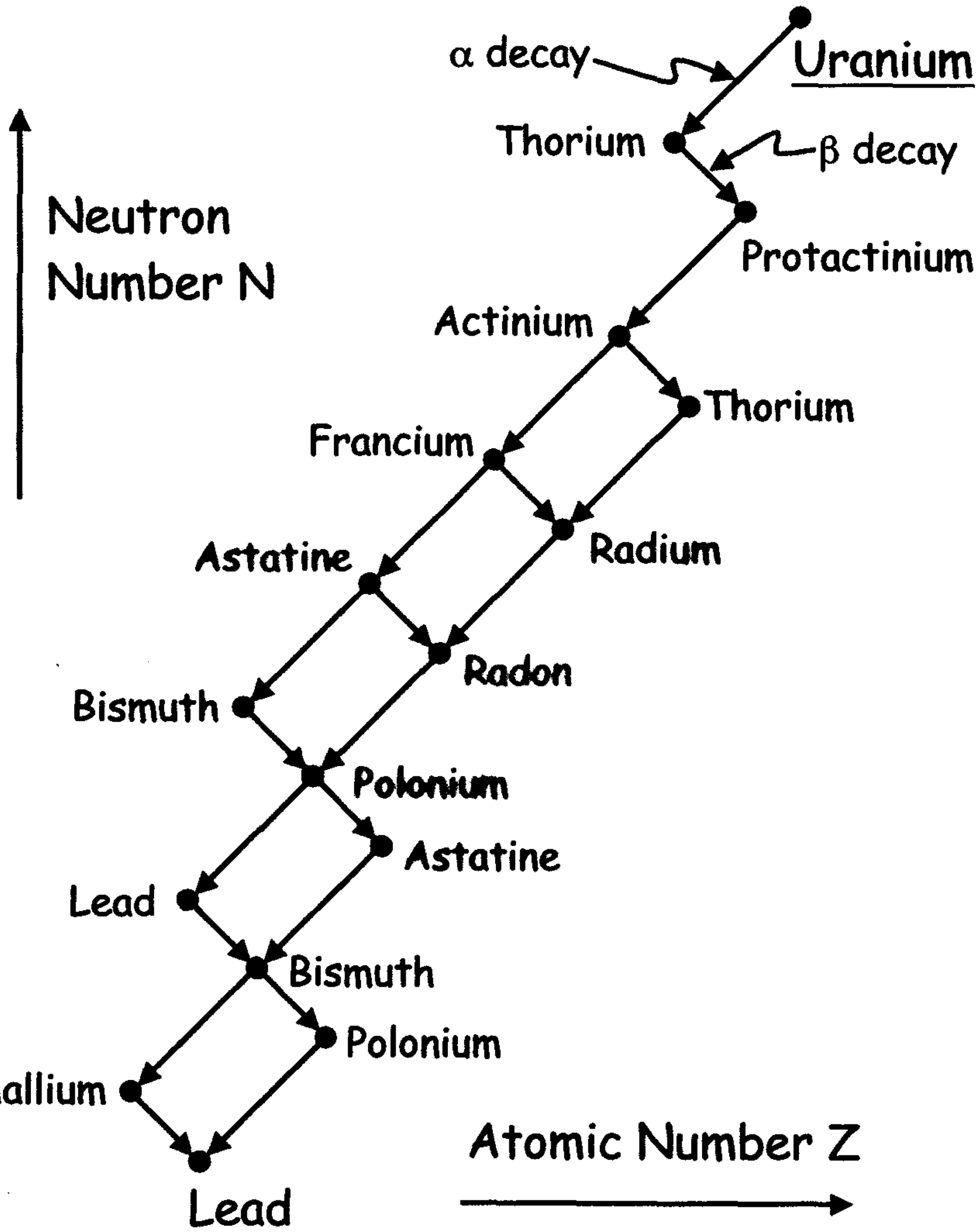
Just like an atom, a nucleus can also have excited states. Often, an α or β decay of a radioactive nucleus will leave the daughter nucleus in an excited state.

The excited nucleus will then decay to the ground state with the emission of a high energy photon (γ ray).

$$^{225}Th^{*}{}_{90} \rightarrow {}^{225}Th_{90} + \gamma$$

Obviously, this leaves A and Z unchanged.

A radioactive decay "chain" (Segre chart)



Decay times

Radioactive decay is a probabilistic event.

For large numbers of nuclei, the number that decay in a short time will be proportional to the total number N and the time Δt:

$$\Delta N = - \lambda N \Delta t$$

The solution to this differential equation is

$$N(t) = N(0)\, e^{-\lambda t}$$

Activity

The Activity of a radioactive substance is defined as the number of decays per unit time:

Activity: $R = -dN/dt = \lambda N(t)$

The SI unit is the Becquerel:

1 Bq = 1 decay/second

An older, but still used, unit is the Curie:

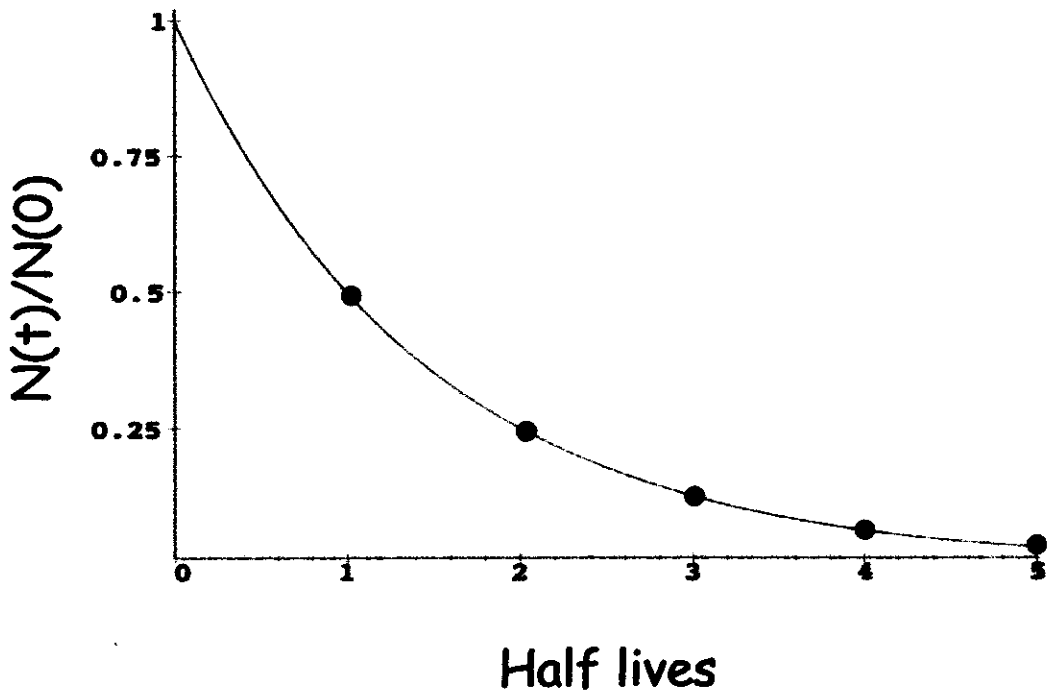
1 Ci = 3.7×10^{10} decays/second

The activity falls also falls off exponentially:

$R(t) = R(0)\, e^{-\lambda t}$

λ is the decay constant.

Half-life



The half-life ($t_{1/2}$) is the time for the number of radioactive nuclei to drop by a factor of 2. It is easy to show

$$t_{1/2} = \ln(2)/\lambda = 0.693/\lambda$$

12.26 A radioactive sample decreases in activity by a factor of 5 in 1 hour. What is its half-life?

$$R = R_0 e^{-\lambda t}$$

$$\Rightarrow \quad \frac{1}{5} = e^{-\lambda 1}$$

$$\therefore \quad \ln \frac{1}{5} = -\lambda \qquad \Rightarrow \lambda = 1.609$$

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{1.609} = \boxed{0.431 \text{ hr}}$$

OR

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^n \quad \text{where } n = \text{\# of } \tfrac{1}{2}\text{-lives}$$

$$\therefore \quad \frac{1}{5} = \left(\frac{1}{2}\right)^n$$

$$\ln \frac{1}{5} = n \ln \frac{1}{2} \qquad \Rightarrow n = 2.32$$

So, if 1 hour = 2.32 $\frac{1}{2}$-lives

then 1 $\frac{1}{2}$-life $= \dfrac{1}{2.32} = \boxed{0.431 \text{ hr}}$

Carbon Dating

Radioactive ^{14}C is continually produced in the atmosphere by bombardment of ^{14}N by neutrons produced by cosmic rays:

$$n + {}^{14}N \rightarrow {}^{14}C + p$$

There is a natural equilibrium ratio of ^{14}C to ^{12}C of $R_0 = 1.2\times10^{-12}$. This ratio occurs in the carbon in the CO_2 taken up by living organisms.

But when a living organism dies, the ^{14}C decays and the ratio of $^{14}C/^{12}C$ decreases, allowing us to date the time of death of the organism by

$$R(t) = R_0 e^{-\lambda t}$$

Example 12.18

A bone is found to have a ratio of $^{14}C/^{12}C = 1.10E-12$. How old is it?

The $\frac{1}{2}$-life of ^{14}C is 5730 years

$$\Rightarrow \lambda = \frac{0.693}{\tau_{\frac{1}{2}}} = \frac{0.693}{5730} = 1.21E-4$$

$$R = R_0 e^{-\lambda t}$$

$$\Rightarrow \frac{R}{R_0} = \frac{1.1E-12}{1.2E-12} = 0.917 = e^{-\lambda t}$$

$$\therefore \ln 0.917 = -\lambda t$$

$$\therefore t = -\frac{\ln 0.917}{\lambda} = -\frac{-0.0870}{1.21E-4}$$

$$= \boxed{719 \text{ years}}$$

A TIME-DATING QUESTION (SEE Ex 12.17)

$$^{238}U \longrightarrow {}^{206}Pb$$

ASSUME ORIGINALLY N_0 U AND 0 Pb

AFTER TIME t $\quad N_0 e^{-\lambda t} \quad N_0(1-e^{-\lambda t})$

RATIO OF Pb/U $= \dfrac{N_0(1-e^{-\lambda t})}{N_0 e^{-\lambda t}} = e^{\lambda t} - 1$

IF THIS RATIO IS MEASURED, AND IF WE KNOW λ, WE CAN CALCULATE t.

$$\left[\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{4.47E9\ (\text{YEARS})} = 1.55E\text{-}10\right]$$

12.47 IF RATIO IS MEASURED TO BE $R = 0.76$, WHAT IS t?

$$e^{\lambda t} = 1 + R = 1.76$$

$$\therefore \lambda t = \ln(1.76) = 0.565$$

$$\therefore t = \frac{0.565}{1.55E\text{-}10} = \boxed{3.65E9 \text{ YEARS}}$$