Modern Physics

Thornton and Rex, Chapter 6

Quantum Theory

SCHRÖDINGER'S EQUATION

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

A GENERAL FORM OF THE SOLUTION IS

$$\psi = A e^{i(kx-\omega t)} = A\left(\cos(kx-\omega t) + i \sin(kx-\omega t)\right)$$

$$\frac{\partial \psi}{\partial t} = -i\omega\psi \qquad \frac{\partial \psi}{\partial x} = ik\psi \qquad \frac{\partial^2 \psi}{\partial x^2} = -k^2\psi$$

SUBSTITUTING $\Rightarrow$

$$i\hbar(-i\omega\psi) = -\frac{\hbar^2}{2m}(-k^2\psi) + V\psi$$

$$\therefore \left(\hbar\omega - \frac{\hbar^2 k^2}{2m} - V\right)\psi = 0$$

$$E = h\nu = \hbar\omega \quad \text{AND} \quad p = \frac{h}{\lambda} = \hbar k$$

$$\Rightarrow \left(E - \frac{p^2}{2m} - V\right)\psi$$ WHICH $= 0$ (NON-RELATIVISTICALLY)

k = WAVENUMBER

$$= \frac{2\pi}{\lambda}$$

ω = ANGULAR FREQUENCY

$$= 2\pi\nu$$

PROBABILITY AND NORMALIZATION

THE PROBABILITY OF A PARTICLE BEING BETWEEN X AND $X + dX$ IS

$$P(x)\,dx = \psi^*(x,t)\,\psi(x,t)\,dx$$

WHERE ψ^* IS THE COMPLEX CONJUGATE.

THE PROBABILITY OF BEING BETWEEN X_1 AND X_2 IS

$$P = \int_{x_1}^{x_2} \psi^* \psi \, dx$$

THE WAVEFUNCTION MUST BE <u>NORMALIZED</u> SO THAT THE PROBABILITY OF BEING SOMEWHERE ON THE X AXIS IS UNITY.

$$\int_{-\infty}^{\infty} \psi^* \psi \, dx = 1$$

PROPERTIES OF VALID WAVE FUNCTIONS

1. ψ MUST BE FINITE EVERYWHERE
2. ψ MUST BE SINGLE VALUED
3. ψ AND $\frac{\partial \psi}{\delta x}$ MUST BE CONTINUOUS

   FOR FINITE POTENTIALS (SO THAT $\frac{\partial^2 \psi}{\delta x^2}$ REMAINS SINGLE VALUED).

4. $\psi \rightarrow 0$ AS $x \longrightarrow \pm \infty$

THESE PROPERTIES ARE SOMETIMES CALLED <u>BOUNDARY CONDITIONS</u> AND MUST BE SATISFIED IF THE ψs CORRESPOND TO REALITY.

TIME-INDEPENDENT SCHRÖDINGER EQUATION

IN MANY (MOST) CASES, THE POTENTIAL WILL NOT DEPEND ON TIME. THEN THE WAVE FUNCTION'S DEPENDENCES ON TIME AND SPACE CAN BE SEPARATED

$$\psi(x,t) \longrightarrow \psi(x)e^{-i\omega t}$$

$$i\hbar\frac{\partial\psi}{\partial t} \rightarrow i\hbar(-i\omega)\psi = \hbar\omega\psi = E\psi$$

SO S's EQT $\Rightarrow$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

THE PROBABILITY DENSITY

$$\psi^*(x,t)\psi(x,t) \longrightarrow \psi^*(x)\psi(x)e^{i\omega t}e^{-i\omega t}$$

$$= \psi^*(x)\psi(x)$$

AND SO THE PROBABILITY DISTRIBUTIONS ARE CONSTANT IN TIME.

EXPECTATION VALUES

CONSIDER THE MEASUREMENT OF A QUANTITY EG POSITION, X. THE AVERAGE VALUE OF MANY MEASUREMENTS IS

$$\bar{x} = \frac{\sum_i N_i x_i}{\sum_i N_i}$$

CHANGING FROM DISCRETE TO CONTINUOUS VARIABLES

$\Rightarrow$

$$\bar{x} = \frac{\int_{-\infty}^{+\infty} x \, P(x) \, dx}{\int_{-\infty}^{+\infty} P(x) \, dx}$$

WHERE $P(x)$ IS THE PROB. OF OBSERVING THE PARTICLE AT A PARTICULAR x

IN QUANTUM MECHANICS WE USE THE WAVE FUNCTIONS TO CALCULATE THIS AVERAGE WHICH WE CALL <u>THE EXPECTATION VALUE</u>.

$$\langle x \rangle = \frac{\int_{-\infty}^{+\infty} x \, \psi^* \psi \, dx}{\int_{-\infty}^{+\infty} \psi^* \psi \, dx}$$

NOTE THE SYMBOL

IF NORMALIZED $\Rightarrow$ THE DENOMINATOR $= 1$

THE SAME PROCEDURE CAN BE USED TO FIND THE EXPECTATION OF ANY FUNCTION $g(x)$.

$$\langle g(x) \rangle = \int_{-\infty}^{+\infty} \psi^* \, g(x) \, \psi \, dx$$

WHAT ARE THE EXPECTATION VALUES OF p OR E?

WE FIRST HAVE TO REPRESENT THEM IN TERMS OF x AND t. FOR EXAMPLE:-

$$\frac{\partial \psi}{\partial x} = \frac{\partial}{\partial x}\left(A e^{+i(kx-\omega t)}\right) = ik\psi$$

$$\therefore \frac{\partial \psi}{\partial x} = \frac{ip}{\hbar}\psi$$

$$\therefore p\psi = -i\hbar \frac{\partial}{\partial x}\psi$$

WE DEFINE THE <u>MOMENTUM</u> <u>OPERATOR</u>

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

THE SYMBOL ^ DENOTES ANY OPERATOR WHICH CAN ACT ON THE WAVE FUNCTION ψ.

THEN THE EXPECTATION VALUE OF MOMENTUM BECOMES

$$\langle p \rangle = \int_{-\infty}^{+\infty} \psi^* \left(-i\hbar \frac{\partial}{\partial x} \right) \psi \, dx = -i\hbar \int_{-\infty}^{+\infty} \psi^* \frac{\partial \psi}{\partial x} \, dx$$

SIMILARLY

$$\frac{\partial \psi}{\partial t} = -i\omega \psi = -i \frac{E}{\hbar} \psi$$

$$\therefore \quad E\psi = i\hbar \frac{\partial \psi}{\partial t}$$

SO THE <u>ENERGY</u> <u>OPERATOR</u> IS

$$\hat{E} = i\hbar \frac{\partial}{\partial t}$$

AND THE EXPECTATION VALUE OF ENERGY IS

$$\langle E \rangle = i\hbar \int_{-\infty}^{+\infty} \psi^* \frac{\partial \psi}{\partial t} \, dx$$

A SELF-CONSISTENCY CHECK

NON-RELATIVISTICALLY WE HAVE

$$E = K + V = \frac{p^2}{2m} + V$$

$$\therefore \quad \hat{E}\psi = \left(\frac{\hat{p}^2}{2m} + V\right)\psi$$

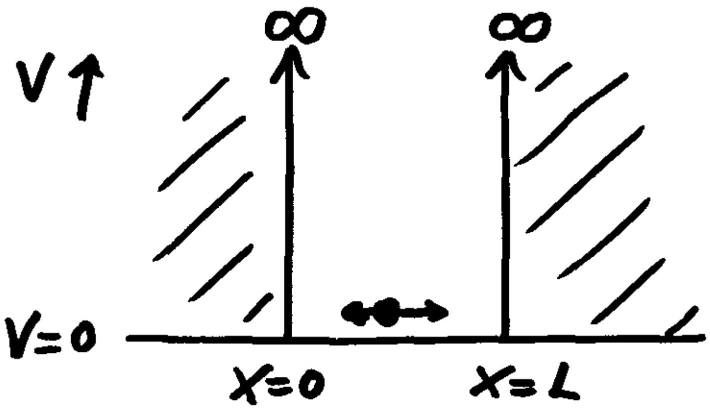
$$\therefore \quad i\hbar\frac{\partial\psi}{\partial t} = \frac{1}{2m}\left(-i\hbar\frac{\partial}{\partial x}\right)^2\psi + V\psi$$

$$\therefore \quad i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

THE SCHRÖDINGER EQUATION!

WE ARE GOING TO LOOK AT SCHRÖDINGER'S EQUATION AND THE RESULTING WAVE FUNCTIONS FOR VARIOUS POSSIBLE POTENTIALS.

INFINITE SQUARE WELL POTENTIAL



$V(x) = \infty$
FOR $x < 0$
$x > L$

$V(x) = 0$ FOR
$0 < x < L$

THE PARTICLE IS CONSTRAINED TO MOVE ONLY BETWEEN $X=0$ AND $X=L$

OUTSIDE THIS "WELL" $V=\infty$ AND HERE THE ONLY POSSIBLE SOLUTION TO S's EQT IS $\psi = 0$.

THEREFORE THERE IS ZERO PROBABILITY FOR THE PARTICLE TO BE LOCATED OUTSIDE OF THE WELL.

INSIDE THE WELL, S's TIME-INDEPENDENT EQUATION IS

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = E\psi$$

$$\therefore \frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi = -\frac{2m\,p^2}{\hbar^2\,2m}\psi$$

$$\Updownarrow$$

$$\therefore \frac{d^2\psi}{dx^2} = -k^2\psi$$

A GENERAL SOLUTION TO THIS WAVE EQUATION IS

$$\psi(x) = A\sin kx + B\cos kx$$

BUT THE WAVE FUNCTION MUST BE CONTINUOUS SO $\psi(x=0) = 0$ $\therefore B = 0$ AND $\psi(x=L) = 0$ $\Rightarrow$

$$\psi(x) = A\sin kx \quad \text{WITH} \quad kL = n\pi$$

$$\left[\, k = \frac{2\pi}{\lambda} = \frac{2\pi\,p}{h} = \frac{p}{\hbar} = \frac{\sqrt{2mE}}{\hbar} \,\right]$$

THE NORMALIZATION CONDITION $\Rightarrow$

$$\int_{-\infty}^{+\infty} \psi^* \psi \, dx = 1 \quad \Longrightarrow \quad A^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

INTEGRAL TABLES $\Rightarrow$ $A^2 \dfrac{L}{2} = 1$

$$\therefore \ A = \sqrt{\frac{2}{L}}$$

SO THE NORMALIZED WAVE FUNCTIONS ARE

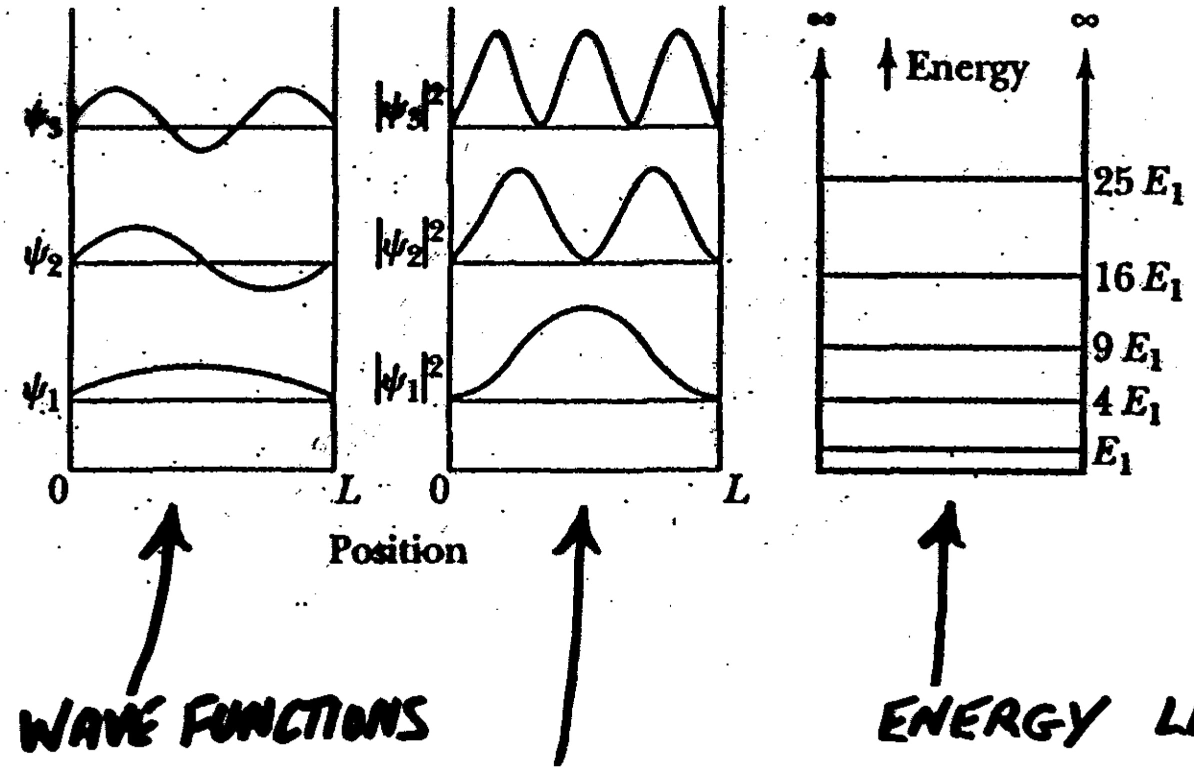
$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \qquad n = 1, 2, 3, \dots$$

WITH $k = \dfrac{n\pi}{L} = \sqrt{\dfrac{2mE_n}{\hbar^2}}$

$$\therefore \ E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

THE POSSIBLE ENERGIES (CALLED <u>ENERGY LEVELS</u>) ARE QUANTIZED WITH n BEING THE <u>QUANTUM NUMBER</u>.

Fig 6.3



(WITH $E_1 = \dfrac{\pi^2 \hbar^2}{2mL^2}$

BEING CALLED

THE <u>GROUND</u> <u>STATE</u>)

3-DIMENSIONAL WELL (OR BOX)

SCHRÖDINGER'S TIME-INDEPENDENT EQUATION IS

$$\frac{-\hbar^2}{2m}\underbrace{\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right)}_{\text{THE LAPLACIAN OPERATOR}} + V\psi = E\psi$$

$$\Rightarrow \qquad \frac{-\hbar^2}{2m}\nabla^2\psi + V\psi = E\psi$$

A PARTICLE INSIDE AN INFINITE WELL OF SIDES L_1 L_2 AND L_3 HAS A WAVE FUNCTION OF

BOX

$$\psi = A \sin k_1 x \sin k_2 y \sin k_3 z$$

WITH $k_1 = \dfrac{n_1 \pi}{L_1}$ $\quad k_2 = \dfrac{n_2\pi}{L_2}$ $\quad k_3 = \dfrac{n_3\pi}{L_3}$

THE ALLOWED ENERGIES DEPEND ON THE 3 QUANTUM NUMBERS n_1 n_2 AND n_3

$$E = \frac{\pi^2\hbar^2}{2m}\left(\frac{n_1^2}{L_1^2}+\frac{n_2^2}{L_2^2}+\frac{n_3^2}{L_3^2}\right)$$

IF WE HAVE A CUBICAL BOX WITH $L_1 = L_2 = L_3 = L$
THEN THE ENERGY VALUES BECOME

$$E = \frac{\pi^2 \hbar^2}{2mL^2}\left(n_1^2 + n_2^2 + n_3^2\right)$$

FOR THE GROUND STATE WE HAVE $n_1 = n_2 = n_3 = 1$
AND SO THE GROUND STATE ENERGY IS

$$E_0 = \frac{3\pi^2 \hbar^2}{2mL^2}$$

THE 1ST EXCITED STATE HAS

$n_1 = 2 \quad n_2 = 1 \quad n_3 = 1$ OR

$n_1 = 1 \quad n_2 = 2 \quad n_3 = 1$ OR

$n_1 = 1 \quad n_2 = 1 \quad n_3 = 2$

THESE ARE 3 DIFFERENT WAVE FUNCTIONS
BUT EACH WITH AN ENERGY OF

$$E_1 = \frac{6\pi^2 \hbar^2}{2mL^2}$$

WE SAY THAT THESE 3 STATES ARE <u>DEGENERATE</u>.

THE DEGENERACY CAN BE REMOVED IF THE SIDES HAVE DIFFERENT LENGTHS, OR BY ADDING AN EXTERNAL FIELD OR POTENTIAL THAT IS NOT SYMMETRIC IN THE 3 DIRECTIONS.

IF THE SIDES ARE L, $2L$, $3L$

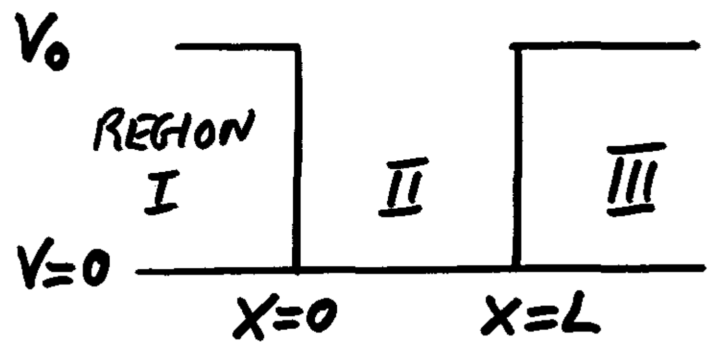
$$\Rightarrow E = \frac{\pi^2 \hbar^2}{2mL^2}\left(\frac{n_1^2}{1} + \frac{n_2^2}{4} + \frac{n_3^2}{9}\right)$$

G.S. OBVIOUSLY IS $n_1 = n_2 = n_3 = 1$

1ST EXCITED STATE IS $n_1 = n_2 = 1 \quad n_3 = 2$

ETC.

FINITE SQUARE WELL POTENTIAL



$V(x) = V_0$
FOR $x < 0$
$x > L$

$V(x) = 0$ FOR
$0 < x < L$

CONSIDER A PARTICLE OF ENERGY $E < V_0$.

CLASSICALLY THIS WILL BE BOUND INSIDE THE WELL

IN QUANTUM MECHANICS THERE IS A FINITE PROBABILITY OF IT BEING OUTSIDE OF THE WELL (IN REGIONS I AND III).

S's EQT $\Rightarrow$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = (E - V_0)\psi$$

FOR REGIONS I AND III

$$\Rightarrow \frac{d^2\psi}{dx^2} = \alpha^2\psi$$

WHERE $\alpha^2 = \frac{2m(V_0 - E)}{\hbar^2}$

IS A POSITIVE CONSTANT

THE SOLUTIONS TO THIS EQUATION ARE EXPONENTIAL DECAYS :-

$$\psi_{I}(x) = A e^{\alpha x} \qquad \text{REGION I } (x<0)$$

$$\psi_{III}(x) = B e^{-\alpha x} \qquad \text{REGION III } (x>L)$$

(THE POSITIVE EXPONENT WOULDN'T HAVE BEEN CORRECT FOR REGION III BECAUSE IT $\rightarrow \infty$.)

(DITTO THE NEGATIVE EXPONENT FOR REGION I.)

IN REGION II (INSIDE THE WELL) WE HAVE

$$\frac{d^2\psi}{dx^2} = -k^2\psi$$

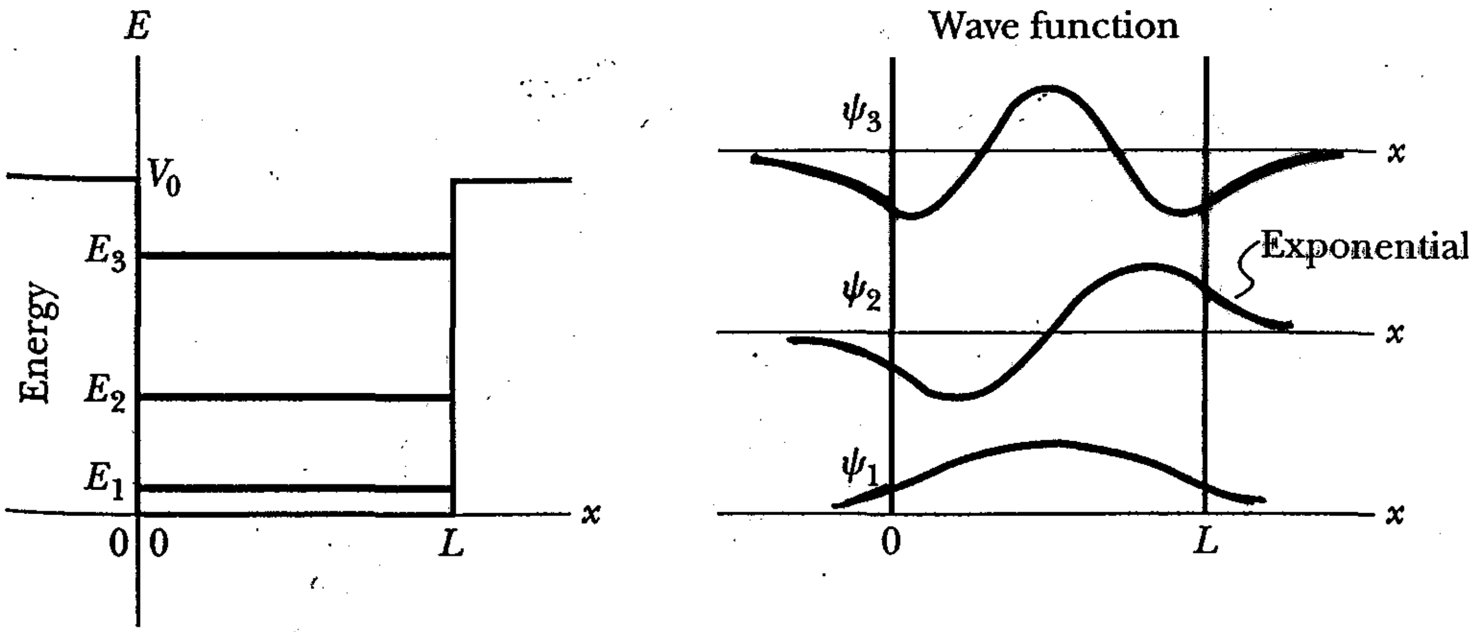
AS BEFORE WITH $k^2 = \frac{2mE}{\hbar^2}$

THE SOLUTION IS

$$\psi_{II}(x) = Ce^{ikx} + De^{-ikx} \qquad \text{REGION II } \; 0<x<L$$

THIS IS A WAVE WHICH HAS TO MATCH (BE CONTINUOUS WITH) THE EXPONENTIAL DECAYS FOUND IN REGIONS I AND III.

Fig. 6.5



- THE 'PENETRATION DISTANCE', WHICH VIOLATES CLASSICAL PHYSICS, DEPENDS ON $\frac{1}{\alpha}$ AND SO $= \frac{\hbar}{\sqrt{2m(V_0 - E)}}$ i.e. IS PROPORTIONAL TO PLANCK'S CONSTANT.

- WAVELENGTHS ARE LONGER THAN IN INFINITE WELL CASE (BECAUSE THEY EXTEND PAST THE WELL BOUNDARY) $\Rightarrow$ LOWER ENERGY LEVELS.

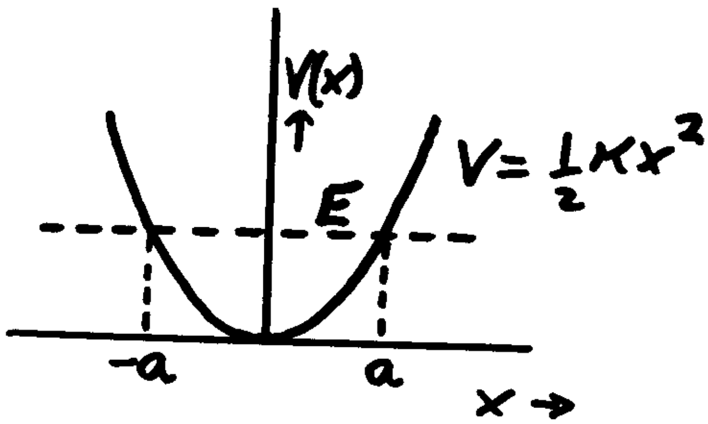
- # OF ENERGY LEVELS IS LIMITED BY THE REQUIREMENT THAT $E_n < V_0$

SIMPLE HARMONIC OSCILLATOR

SIMPLE HARMONIC MOTION IS VERY COMMON IN NATURE. IT IS PRODUCED BY A FORCE THAT IS PROPORTIONAL TO (AND IN THE OPPOSITE DIRECTION TO) ANY DISPLACEMENT IN POSITION e.g. A SPRING

$$F = -Kx \quad \Rightarrow \quad V(x) = \frac{1}{2}Kx^2$$

CONSIDER A PARTICLE WITH ENERGY E IN SUCH A POTENTIAL



SCHRÖDINGER'S EQUATION IS

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}\left(E - \frac{1}{2}Kx^2\right)\psi$$

$$= \left(-\frac{2mE}{\hbar^2} + \frac{mK}{\hbar^2}x^2\right)\psi$$

SUBSTITUTING $\beta = \frac{2mE}{\hbar^2}$ AND $\alpha^2 = \frac{mK}{\hbar^2}$ GIVES

$$\frac{d^2\psi}{dx^2} = (\alpha^2 x^2 - \beta)\psi$$

THE SOLUTIONS TO THIS EQUATION WILL BE WAVE FUNCTIONS THAT SHOW OSCILLATING BEHAVIOR INSIDE THE WELL AND EXPONENTIAL BEHAVIOR OUTSIDE OF THE WELL.

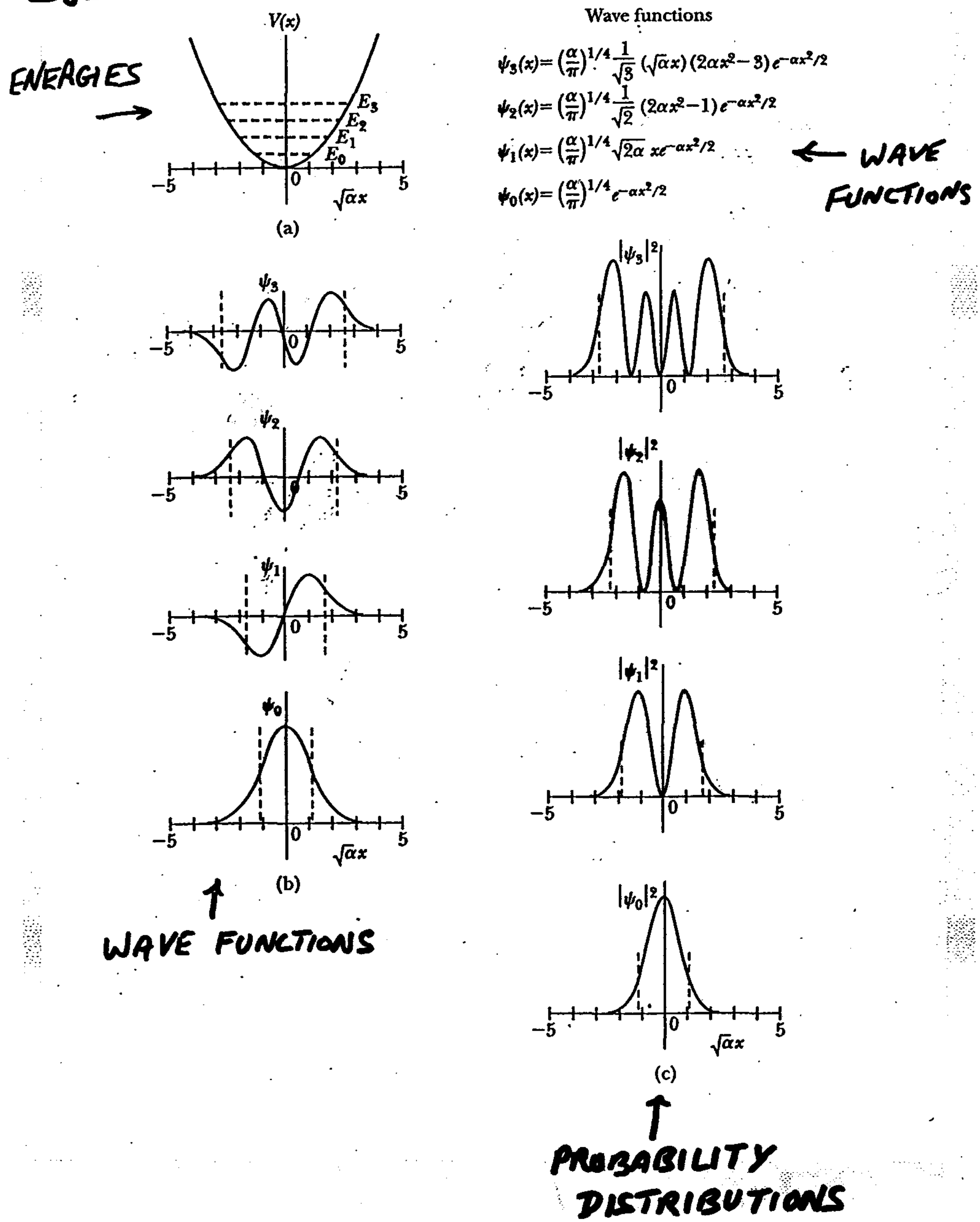
IN FACT THE WAVE FUNCTION SOLUTIONS ARE

$$\psi_n = H_n(x) e^{-\frac{\alpha x^2}{2}}$$

POLYNOMIALS OF ORDER n
(HERMITE POLYNOMIALS)
⇒ THE OSCILLATORY BEHAVIOR AT SMALL X

GAUSSIAN
⇒ THE EXPONENTIAL TAIL AT LARGE X

Fig. 6.10


ENERGIES
$V(x)$
E_3
E_2
E_1
E_0
-5 0 5
$\sqrt{\alpha}x$
(a)
Wave functions
$\psi_3(x) = \left(\frac{\alpha}{\pi}\right)^{1/4}\frac{1}{\sqrt{3}}(\sqrt{\alpha}x)(2\alpha x^2 - 3)e^{-\alpha x^2/2}$
$\psi_2(x) = \left(\frac{\alpha}{\pi}\right)^{1/4}\frac{1}{\sqrt{2}}(2\alpha x^2 - 1)e^{-\alpha x^2/2}$
$\psi_1(x) = \left(\frac{\alpha}{\pi}\right)^{1/4}\sqrt{2\alpha}\, x e^{-\alpha x^2/2}$
$\psi_0(x) = \left(\frac{\alpha}{\pi}\right)^{1/4}e^{-\alpha x^2/2}$
WAVE FUNCTIONS
ψ_3
ψ_2
ψ_1
ψ_0
$\sqrt{\alpha}x$
(b)
WAVE FUNCTIONS
$|\psi_3|^2$
$|\psi_2|^2$
$|\psi_1|^2$
$|\psi_0|^2$
$\sqrt{\alpha}x$
(c)
PROBABILITY DISTRIBUTIONS

THE ENERGY LEVELS ARE

$$E_n = \left(n+\frac{1}{2}\right)\hbar\sqrt{\frac{K}{m}} = \left(n+\frac{1}{2}\right)\hbar\omega$$

WHERE $\omega^2 = \frac{K}{m}$ IS THE CLASSICAL ANGULAR FREQ.

THE MINIMUM ENERGY (GIVEN BY $n=0$) IS

$$E_0 = \frac{1}{2}\hbar\omega$$ <u>THE ZERO-POINT ENERGY</u>

THIS MINIMUM VALUE CAN BE CALCULATED FROM HEISENBERG'S UNCERTAINTY PRINCIPLE.

IN SHM THE AVERAGE PE = THE ~~AVERAGE~~ KE AND THEY EACH EQUAL $\frac{1}{2}$ TOTAL ENERGY

$$\therefore \frac{1}{2}E = \frac{1}{2}K\overline{X^2} = \frac{\overline{p^2}}{2m}$$

Ex 6.11

$$\therefore E = K\Delta x^2 = \frac{\Delta p^2}{m}$$

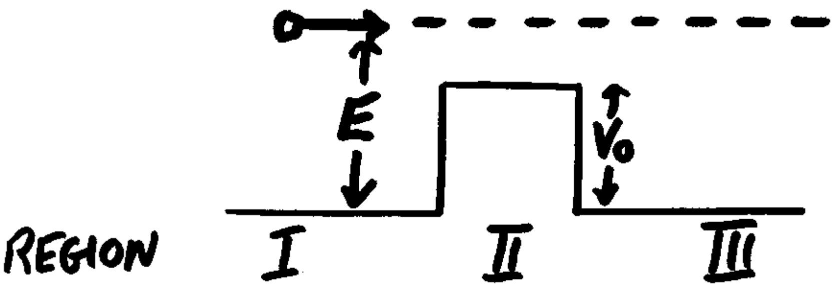
$$\therefore \Delta x = \frac{\Delta p}{\sqrt{mK}}$$

ALSO, FROM UNCERTAINTY PRINCIPLE $\Delta x = \frac{\hbar}{2\Delta p}$

$$E = K\Delta x\Delta x = K\frac{\Delta p}{\sqrt{mK}}\frac{\hbar}{2\Delta p} = \frac{\hbar}{2}\sqrt{\frac{K}{m}} = \frac{1}{2}\hbar\omega$$

BARRIERS AND TUNNELING

CONSIDER A PARTICLE OF TOTAL ENERGY E APPROACHING A CHANGE IN POTENTIAL V_0



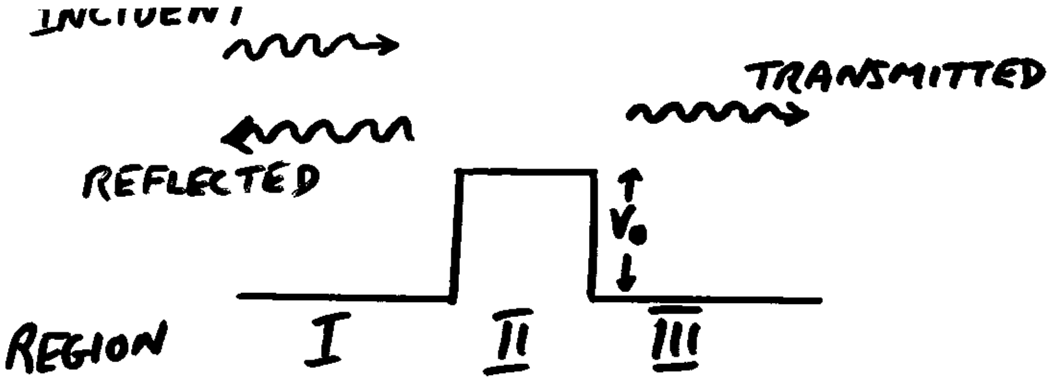
AT THE BARRIER THE KINETIC ENERGY BECOMES $K = E - V_0$ AND THE MOMENTUM AND VELOCITY ARE SIMILARLY REDUCED

$$v' = \sqrt{\frac{2(E-V_0)}{m}} \qquad p' = \sqrt{2m(E-V_0)}$$

IN QUANTUM MECHANICS THE WAVELENGTH ALSO CHANGES

$$k' = \frac{\sqrt{2m(E-V_0)}}{\hbar}$$

AND WE FIND THAT THERE ARE FINITE PROBABILITIES OF THE PARTICLE BEING REFLECTED AS WELL AS TRANSMITTED.



SCHRÖDINGER'S EQUATION FOR THE 3 REGIONS

I $$\frac{d^2\psi_I}{dx^2} + \frac{2m}{\hbar^2}E\psi_I = 0$$

II $$\frac{d^2\psi_{II}}{dx^2} + \frac{2m}{\hbar^2}(E-V_0)\psi_{II} = 0$$

III $$\frac{d^2\psi_{III}}{dx^2} + \frac{2m}{\hbar^2}E\psi_{III} = 0$$

SOLUTIONS ARE :-

INCIDENT REFLECTED

I $$\psi_I = Ae^{ikx} + Be^{-ikx}$$

II $$\psi_{II} = Ce^{ik'x} + De^{-ik'x}$$ ← INTERMEDIATE

III $$\psi_{III} = Fe^{ikx}$$ ← TRANSMITTED

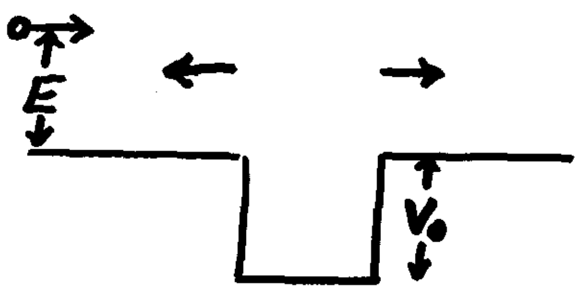
THE IDEA IS TO FIND THESE COEFFICIENTS BY APPLYING THE BOUNDARY CONDITIONS (CONTINUITY ETC.) AT THE INTERFACES.

IT'S VERY TEDIOUS AND WE WON'T DO IT.

YOU CAN CALCULATE THE PROBABILITY OF REFLECTION (B^2/A^2) AND TRANSMISSION (F^2/A^2).

DEPENDING ON THE WIDTH AND HEIGHT OF THE BARRIER WE CAN GET EITHER COMPLETE REFLECTION OR COMPLETE TRANSMISSION. [SIMILAR TO OPTICS]

ALL CALCULATIONS CAN BE REDONE FOR THE CASE OF A PARTICLE INCIDENT ON A POTENTIAL WELL.



AGAIN, IT IS POSSIBLE TO HAVE REFLECTION AND/OR TRANSMISSION

$$T = \frac{1}{\left(1 + \dfrac{V_0^2 \sin^2(k'L)}{4E(E \mp V_0)}\right)}$$

− FOR BARRIER
+ FOR WELL

NOTE THAT IN EITHER CASE, THERE IS A POSSIBILITY OF $T = 1$.

WHEN $\sin^2 k'L = 0 \Rightarrow k'L = n\pi$

THIS CORRESPONDS TO THE CASE WHEN THE REFLECTED WAVE AT THE FIRST INTERFACE AND THE REFLECTED WAVE AT THE SECOND INTERFACE ARE OUT OF PHASE AND SO THEY DESTRUCTIVELY INTERFERE.

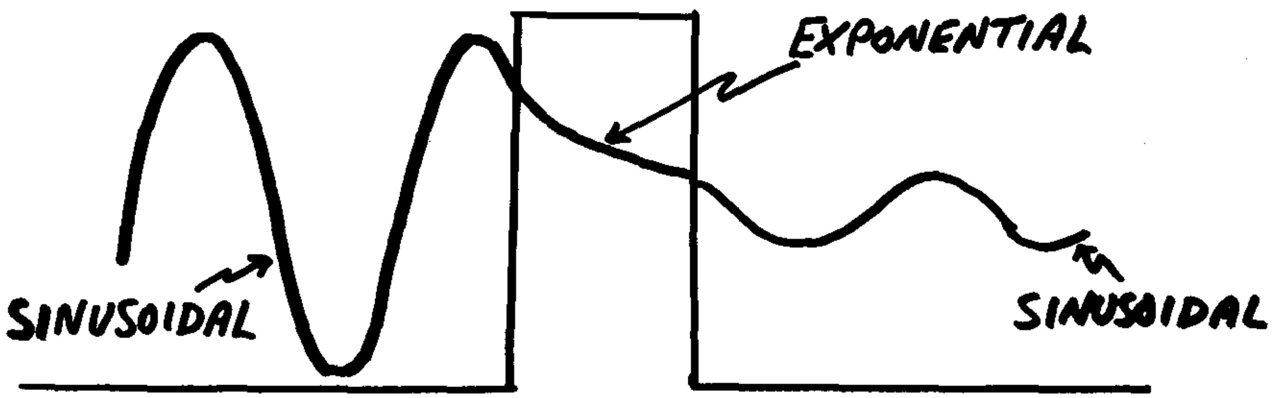
A TOTALLY NON-CLASSICAL RESULT OCCURS WHEN A PARTICLE IS INCIDENT ON A POTENTIAL BARRIER WITH $E < V_0$.

THE INTERMEDIATE WAVEFUNCTIONS BECOME

$$\psi_{II} = C e^{Kx} + D e^{-Kx}$$

WHERE $K = \dfrac{\sqrt{2m(V_0 - E)}}{\hbar}$ THIS IS A REAL POSITIVE NUMBER

THIS, OF COURSE, CORRESPONDS TO EXPONENTIAL DECAY AND (DEPENDING ON THE HEIGHT AND WIDTH OF THE BARRIER) IT IS POSSIBLE FOR THE PARTICLE TO EMERGE ON THE OTHER SIDE.



THIS PROCESS IS CALLED <u>TUNNELING</u>.

NOW THE PROBABILITY OF TRANSMISSION BECOMES

$$T = \frac{1}{\left(1 + \frac{V_0^2 \sinh^2 KL}{4E(V_0 - E)}\right)}$$

NOTE THAT THE $\sin^2$ TERM $\rightarrow$ $\sinh^2$

$$\sinh KL = \frac{e^{KL} - e^{-KL}}{2}$$

FOR $KL \gg 1$ $\quad \sinh KL \rightarrow \frac{e^{KL}}{2}$

$$\Rightarrow T = 16 \frac{E}{V_0}\left(1 - \frac{E}{V_0}\right) e^{-2KL}$$