Matter Waves

Thornton and Rex, Ch. 5

MATTER WAVES

IN 1924, A STUDENT (AND ARISTOCRAT!) AT THE UNIVERSITY OF PARIS, PRINCE LOUIS DE BROGLIE (1892 - 1987) SUGGESTED THAT IF PHOTONS (WAVES) COULD BEHAVE LIKE PARTICLES THEN PERHAPS PARTICLES COULD BEHAVE LIKE WAVES!

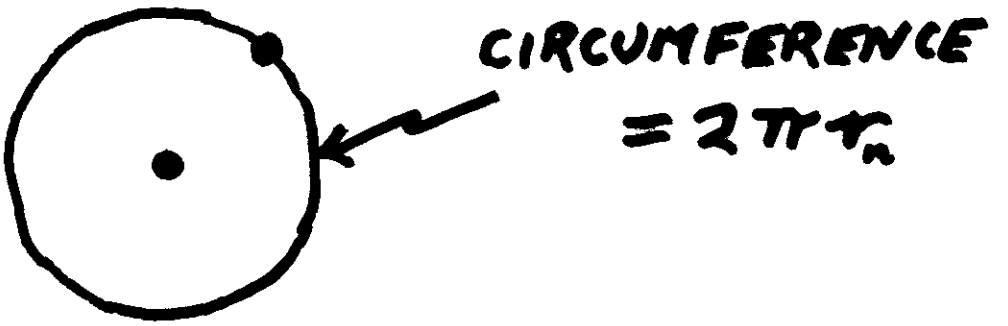
IT SEEMED UNLIKELY. THERE WAS CERTAINLY NO EXPERIMENTAL EVIDENCE FOR SUCH A THING. BUT DE BROGLIE PERSISTED AND, USING SOME ANALOGIES FROM THE PHOTOELECTRIC EFFECT, AND FROM COMPTON SCATTERING, HE SUGGESTED THAT THE "WAVELENGTH" OF AN OBJECT COULD BE RELATED TO ITS MOMENTUM IN THE FOLLOWING WAY:-

$$\lambda = h/p = h/mv$$

h OF COURSE IS PLANCK'S CONSTANT.

ONE OF THE REASONS THAT DE BROGLIE SUSPECTED THAT HE WAS ON TO SOMETHING WAS THAT, IF HE CALCULATED THE "WAVELENGTHS" OF THE ELECTRONS IN A BOHR ATOM, HE NOTICED THAT HE COULD DERIVE THE ORIGINAL BOHR CRITERION FOR STATIONARY STATES:-

CONSIDER AN ELECTRON ORBITING AN ATOM AT A RADIUS, r_n.



WE MIGHT EXPECT SOMETHING SPECIAL FOR RADII WHERE THE DE BROGLIE WAVELENGTH FITTED INTO THE CIRCUMFERENCE AN INTEGRAL NUMBER OF TIMES.

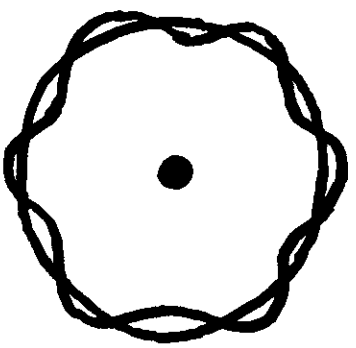
$$2 \pi r_n = n \lambda = nh/mv$$

mvr_n = ANGULAR MOMENTUM $nh/2\pi = n\hbar$

WHICH WAS EXACTLY BOHR'S ORIGINAL GUESS.

FOR EXAMPLE, THIS

WOULD BE THE n = 6 ORBIT.



SO n IS THE NUMBER OF DE BROGLIE WAVELENGTHS THAT CAN BE FITTED INTO THE CIRCUMFERENCE OF AN ORBIT.

<u>THE ONLY ORBITS THAT ARE ALLOWED ARE THOSE THAT HAVE AN INTEGRAL NUMBER OF DE BROGLIE WAVELENGTHS.</u>

DE BROGLIE PRESENTED HIS IDEAS IN HIS PH.D. THESIS. BUT THERE WAS NO EXPERIMENTAL EVIDENCE TO SUPPORT HIM AND HIS PROFESSORS WERE SKEPTICAL. ONE SENT A COPY OF THE THESIS TO ALBERT EINSTEIN. THE GREAT MAN REPLIED THAT THE IDEAS CERTAINLY APPEARED TO BE CRAZY, BUT THEY WERE IMPORTANT, AND THE WORK WAS "SOUND". DE BROGLIE OBTAINED HIS PH.D. DEGREE IN 1924 AND, IN 1929, HE WAS AWARDED THE NOBEL PRIZE.

BEFORE DE BROGLIE'S THESIS WAS PUBLISHED, CLINTON DAVISSON (BELL LABS IN THE U.S.) WAS STUDYING VACUUM TUBES FOR RADIOS. HE AIMED ELECTRONS AT METAL TARGETS AND MONITORED THEIR ANGLES AND DIRECTIONS AFTER SCATTERING. DAVISSON FOUND A STRANGE PATTERN IN THE SCATTERED ELECTRONS SIMILAR TO THAT SEEN IN X-RAY SCATTERING.

AFTER DE BROGLIE'S WORK, IT WAS POINTED OUT THAT THIS PATTERN CORRESPONDED TO AN INTERFERENCE PATTERN, AS IF THE ELECTRON WAS BEHAVING LIKE A WAVE.

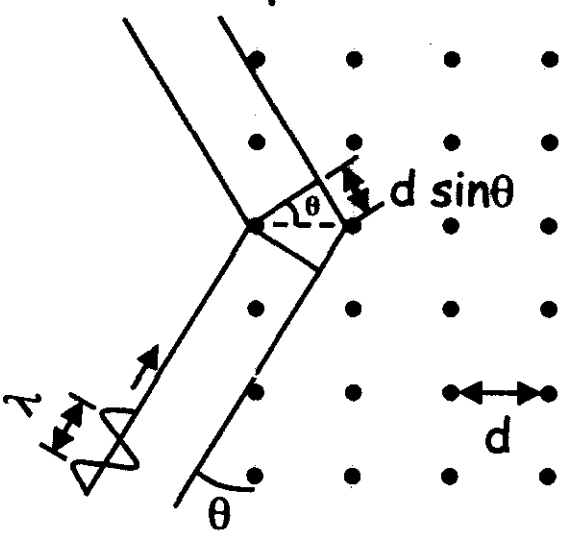
DAVISSON CAREFULLY REPEATED HIS EXPERIMENT; MEASURING THE MOMENTUM OF THE ELECTRONS AND THE APPARENT WAVELENGTH OF THE INTERFERENCE PATTERN. THE EXPERIMENT WAS THE FIRST TO PROVE THE EXISTENCE OF THE WAVE NATURE OF MATTER. DAVISSON AND G.P. THOMSON (THE SON OF J.J. THOMSON) WHO HAD PERFORMED A SIMILAR EXPERIMENT IN BRITAIN WERE ALSO AWARDED THE NOBEL PRIZE (IN 1937).

Discovery of Electron Waves

1925 - Davisson and Germer were scattering electrons from metals.

On scattering electrons off crystallized Nickel, they saw peaks at certain angles.

⇒ Interference spectra!



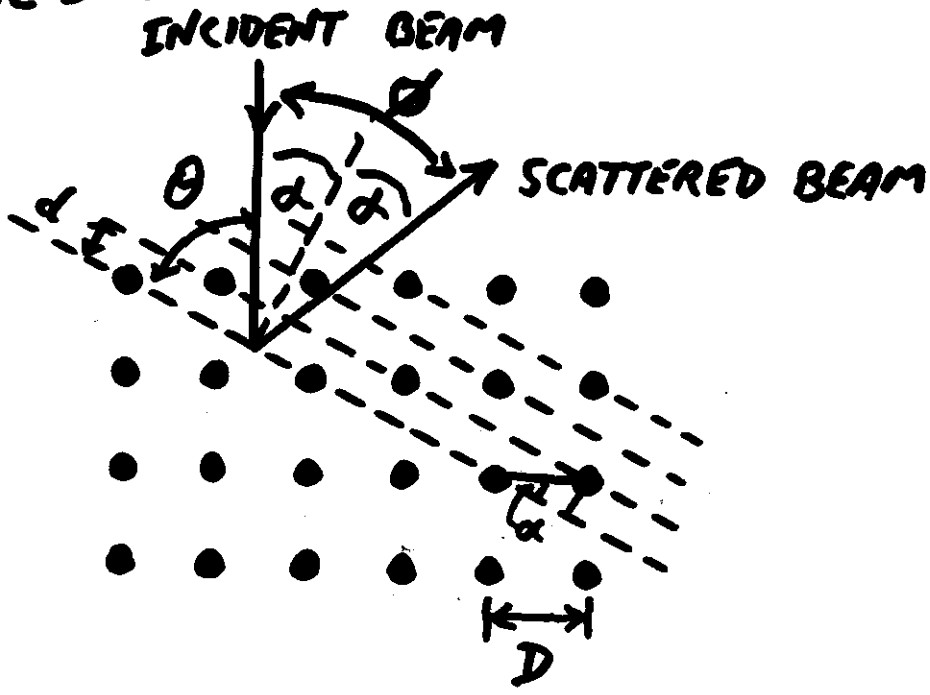
Interference is constructive (i.e. peaks) when

$$2\,d\sin\theta = n\lambda$$

for integer n. (Bragg's law)

ALL OF THE FEATURES OF X-RAY SCATTERING CAN BE SEEN IN ELECTRON SCATTERING.

NOTE, JUST TO COMPLICATE THINGS, THERE ARE 3 DIFFERENT ANGLES THAT CAN BE USED :-



CONSIDER A PARTICULAR BRAGG PLANE
INCIDENT ANGLE IS θ RELATIVE TO THIS PLANE OR α RELATIVE TO THE PERPENDICULAR TO THIS PLANE :- $\alpha = 90^\circ - \theta$

ALSO $d = D \sin\alpha$ $\qquad \phi = 2\alpha$

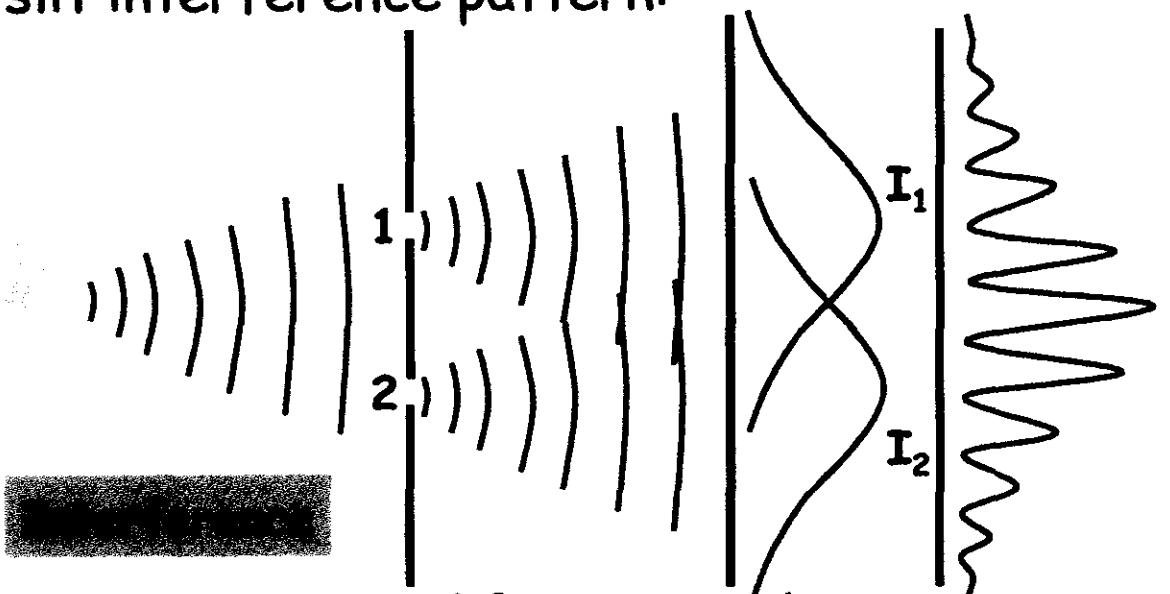
BRAGG'S LAW =>

$$n\lambda = 2d\sin\theta = 2d\cos\alpha = 2D\sin\alpha\cos\alpha$$

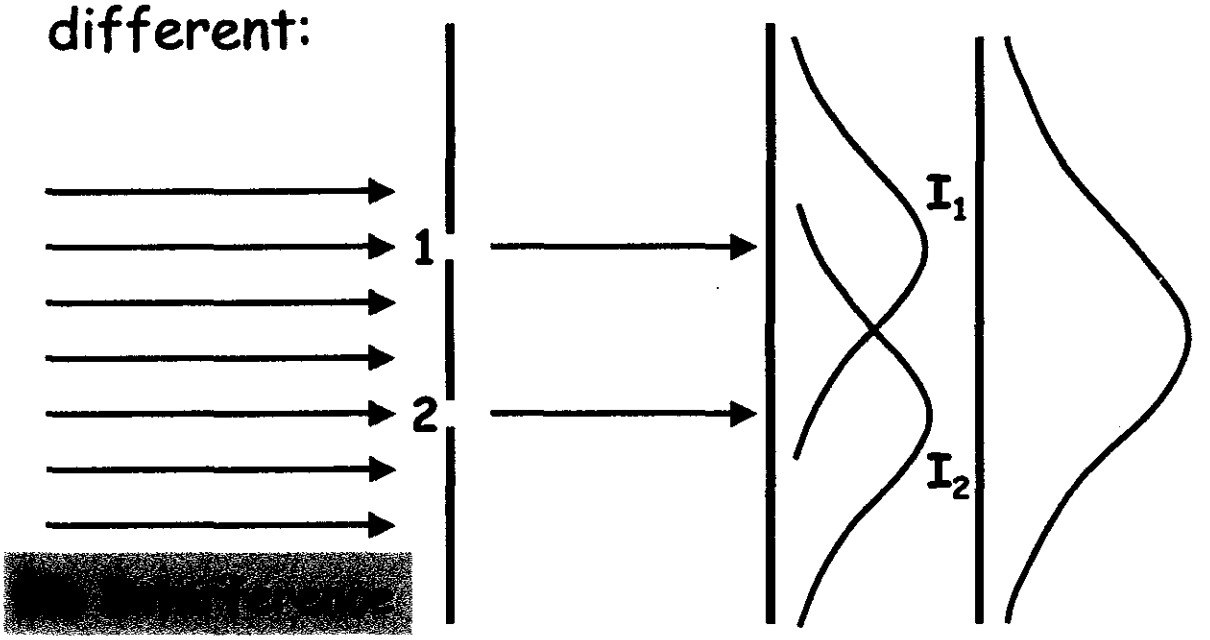
$$\therefore n\lambda = D\sin 2\alpha = D\sin\phi$$

Wave/Particle Duality

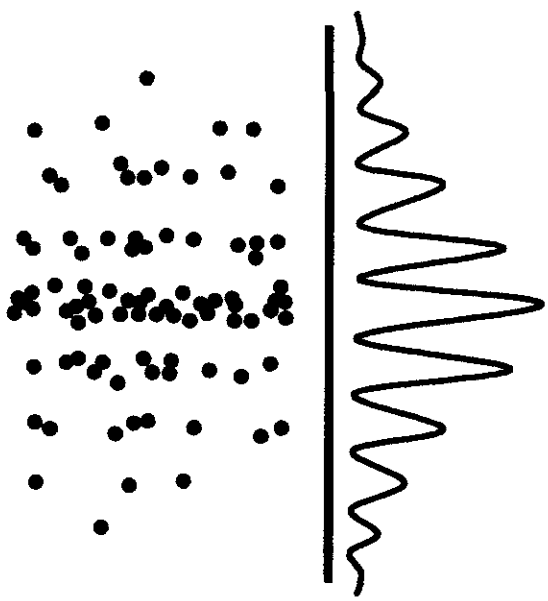
Wave nature of light from the double-slit interference pattern:



Pattern expected from particles is very different:



What happens at very low intensities?

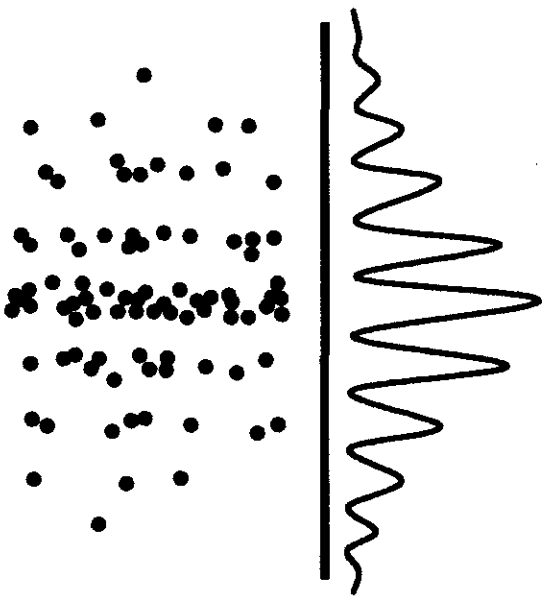


Photons hit at discrete points, gradually building up the interference pattern.

Does the photon go through slit 1 or slit 2?

Neither! (or rather, both!)

What about electrons?



They exhibit the same interference pattern (although at smaller wavelengths than for visible light.)

But the important feature of EM waves is that the wavefront is broad, and it goes through both slits to produce the interference pattern.

But surely the electrons don't go through both slits - they go through one or the other. You can turn the beam down so that there's a very low rate (say 1 per minute). However the distribution gradually builds up and eventually shows the interference pattern. (See Fig. 5.18)

However, if we devise a method of observing which slit the electron goes through, 1 or 2, then there is no interference.

The Complementarity Principle

In trying to observe which slit the electron went through we are examining the particle-like behavior of the electron. When we look at the interference pattern we are observing its wave-like behavior.

Niels Bohr resolved the dilemma by pointing out that the particle-like and wave-like aspects of nature are complementary.

It is not possible to describe simultaneously physical observables in terms of both particles and waves.

THE Schrödinger EQUATION

WHAT ARE THESE DE BROGLIE MATTER WAVES? WHAT DO THEY MEAN?

EARLY IN 1925, ERWIN SCHRODINGER, AN AUSTRIAN PHYSICIST, BEGAN TO SEARCH FOR A MATHEMATICAL EQUATION THAT WOULD DESCRIBE THE AMPLITUDE OF THE WAVE OF A PARTICLE. WITHIN A YEAR HE HAD FOUND JUST SUCH AN EQUATION, A PARTIAL DIFFERENTIAL EQUATION:-

$$ih\frac{\partial\psi}{\partial t} = -\frac{h^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

THIS IS SCHRODINGER'S EQUATION. IT DESCRIBES THE BEHAVIOR OF PARTICLE WAVES IN JUST THE SAME WAY THAT MAXWELL'S EQUATIONS DESCRIBE THE BEHAVIOR OF ELECTROMAGNETIC RADIATION.

SCHRODINGER NOTED THAT THE WAVES WERE 3-DIMENSIONAL WAVES, NOT CONFINED TO BOHR'S ORBITS. BUT EACH WAVE HAD A LARGE AMPLITUDE AT THE POSITION OF THE CORRESPONDING BOHR ORBIT IN THE ATOM. AND THE ENERGIES ASSOCIATED WITH THE WAVES WERE EXACTLY THE SAME AS THE ENERGY OF THE ELECTRONS IN THE BOHR ORBITS.

SCHRODINGER CALLED HIS EQUATION A WAVE EQUATION, A DESCRIPTION OF WHAT HAPPENED TO THE WAVELENGTHS AND AMPLITUDES OF MATTER WAVES UNDER THE INFLUENCE OF FORCES OR POTENTIALS. THE STUDY OF THE BEHAVIOR OF MATTER WAVES IS KNOWN AS <u>WAVE MECHANICS</u>.

IT WAS THE GERMAN PHYSICIST, MAX BORN, WHO FINALLY SUGGESTED HOW SCHRODINGER'S WAVE AMPLITUDES COULD BE INTERPRETED. THE AMPLITUDE, ψ, CAN BE INTERPRETED AS A PROBABILITY DISTRIBUTION. IT REPRESENTS THE PROBABILITY OF FINDING PARTICLES (SUCH AS ELECTRONS) IN VARIOUS PLACES.

IN BORN'S VIEW THE ELECTRON HAD A GOOD CHANCE OF BEING FOUND IN A REGION ONLY IF THE SQUARE OF ITS WAVE AMPLITUDE, ψ, WAS LARGE THERE. THIS INTERPRETATION GIVES SOME INHERENT UNCERTAINTY IN THE POSITION OF OBJECTS.

NIELS BOHR BECAME CONVINCED OF THE USEFULNESS OF SCHRODINGER'S WAVE EQUATION AND OF THE PROBABILISTIC INTERPRETATION OF MAX BORN. HIS INSTITUTE IN COPENHAGEN BECAME THE CENTER FOR THE INTERPRETATION OF THE NEW WAVE MECHANICS.

BOHR STATED THAT THE AMPLITUDE AS CALCULATED BY SCHRODINGER'S EQUATION CONTAINS <u>ALL</u> OF THE INFORMATION (ITS POSITION, ENERGY, MOMENTUM ETC.) WE CAN OBTAIN ABOUT THE BEHAVIOR OF AN ELECTRON OR ATOM. AND THAT THIS INFORMATION CONTAINS SOME INHERENT UNCERTAINTY.

TO THIS ASSERTION, EINSTEIN, PLANCK AND EVEN SCHRODINGER AND DE BROGLIE ALL OBJECTED. THEY COULD NOT BELIEVE THAT IT WAS NOT POSSIBLE TO PREDICT PRECISELY WHERE TO FIND AN ELECTRON IN AN ATOM.

SCHRODINGER WAS PARTICULARLY UNHAPPY. HE HAD DEVELOPED HIS EQUATION IN THE HOPE OF EXPLANING THE BOHR ATOM WITHOUT THE DISCRETE JUMPS OF ENERGY THAT BOHR HAD USED. NOW THE DISCRETE JUMPS WERE STILL THERE AND AN ADDITIONAL UNCERTAINTY BESIDES!

HEISENBERG'S UNCERTAINTY PRINCIPLE

WORKING CLOSELY WITH THE COPENHAGEN PHYSICISTS, WERNER HEISENBERG, A POSTDOCTORAL FELLOW AT GOTTINGEN, HAD DEVELOPED HIS OWN APPROACH TO THE QUANTUM THEORY. HE NEVER MENTIONED PARTICLES OR WAVES, BUT SPOKE IN TERMS OF AN ABSTRACT MATHEMATICAL QUANTITY, QUANTUM STATES. THESE STATES WERE BASED ON GENERAL PROPERTIES OF MATRICES.

THE APPROACHES OF SCHRODINGER AND HEISENBERG APPEARED TO HAVE NOTHING TO DO WITH EACH OTHER WHEN THEY WERE FIRST INTRODUCED IN 1926 BUT SOON IT WAS SHOWN THAT HEISENBERG'S STATES WERE SOLUTIONS TO SCHRODINGER'S EQUATION.

HEISENBERG'S METHOD WAS PARTICULARLY APPROPRIATE FOR CONSIDERING THE INHERENT UNCERTAINTY REQUIRED BY THE PROBABILITY DESCRIPTION. HE DEVELOPED A SET OF RULES KNOWN AS UNCERTAINTY RELATIONS. THERE ARE CERTAIN PAIRS OF PHYSICAL QUANTITIES THAT CANNOT BE DETERMINED SIMULTANEOUSLY TO ANY DESIRED ACCURACY.

ONE SUCH PAIR OF VARIABLES IS ENERGY AND TIME.

ANOTHER IS POSITION AND MOMENTUM.

THUS, IT IS IMPOSSIBLE TO SPECIFY SIMULTANEOUSLY BOTH THE POSITION AND MOMENTUM OF A PARTICLE.

IF THERE IS AN UNCERTAINTY IN POSITION EQUAL TO Δx AND AN UNCERTAINTY IN MOMENTUM Δp THEN;-

$\Delta x \Delta p$ MUST BE GREATER THAN $\sim h/4\pi$

SO AS Δx GETS SMALLER, Δp GETS BIGGER

[AND VICE VERSA!]

THE DIRAC EQUATION

SCHRODINGER'S EQUATION DID NOT MEET THE REQUIREMENTS OF THE THEORY OF RELATIVITY. IT WAS NON-RELATIVISTIC.

A RELATIVISTICALLY CORRECT QUANTUM THEORY WAS DEVELOPED BY PAUL DIRAC AT CAMBRIDGE IN 1928.

THERE IS A SQUARE ROOT IN DIRAC'S EQUATION WHICH IMPLIES TWO POSSIBLE ANSWERS FOR SOME VARIABLES SUCH AS +E OR -E.

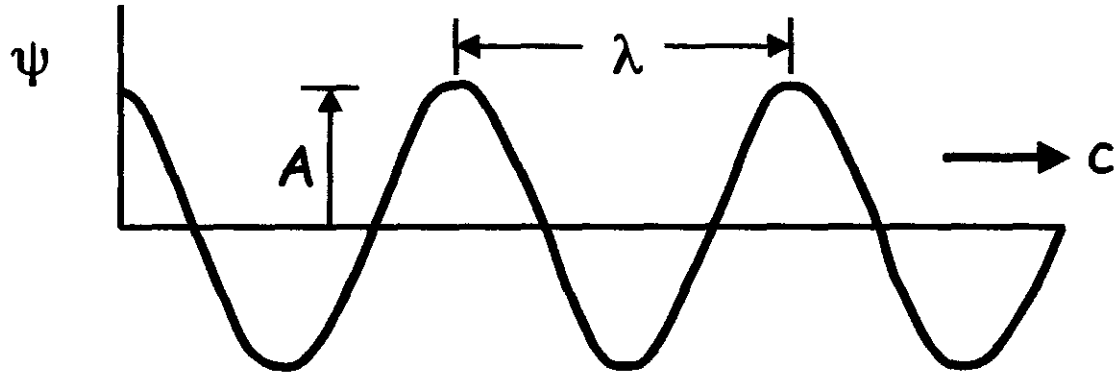
FOR A WHILE, DIRAC IGNORED THE APPARENTLY UNPHYSICAL -E SOLUTION, BUT HE DID NOTE IT IN HIS CLASSIC PAPERS. IT IS THE PREDICTION THAT EVERY PARTICLE SHOULD HAVE AN ANTIPARTICLE.

FOUR YEARS LATER, 1N 1932, THE ANTI-ELECTRON (CALLED THE POSITRON) WAS DISCOVERED.

IN 1956 THE ANTIPROTON WAS DISCOVERED.

Generalities about light waves

A plane wave:



$\psi(x,t) = A \cos[2\pi (x-ct) / \lambda]$

Amplitude: A
Wavelength: λ
Speed: c
Frequency: $\nu = c/\lambda$

It is convenient to rewrite:
$\psi(x,t) = A \cos(kx-\omega t)$

Wave number: $k = 2\pi/\lambda$
Angular frequency: $\omega = 2\pi \nu$

Wave relation: $c = \lambda\nu = \omega/k$

All light waves have same speed c in vacuum, independent of wave number k.

$\Rightarrow$ Not true for matter waves.

Planck: $E = h\nu = h\omega/2\pi$
$= \hbar\omega$

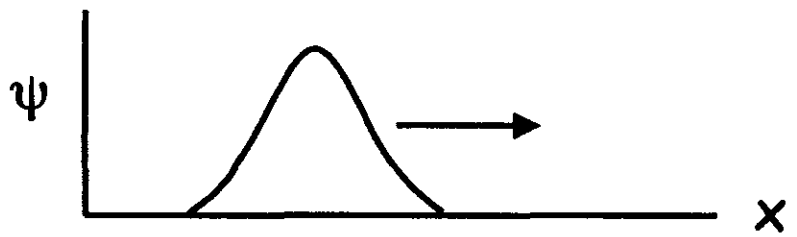
Einstein/de Broglie: $p = E/c = h/\lambda$
$= \hbar k$

A periodic wave can be constructed from a sum of plane waves:
$\Rightarrow$ Fourier Series

$$\psi(x,t) = \sum A_i \cos(k_i x - \omega_i t)$$

A wave packet can be constructed as a continuous sum (integral) of plane waves.

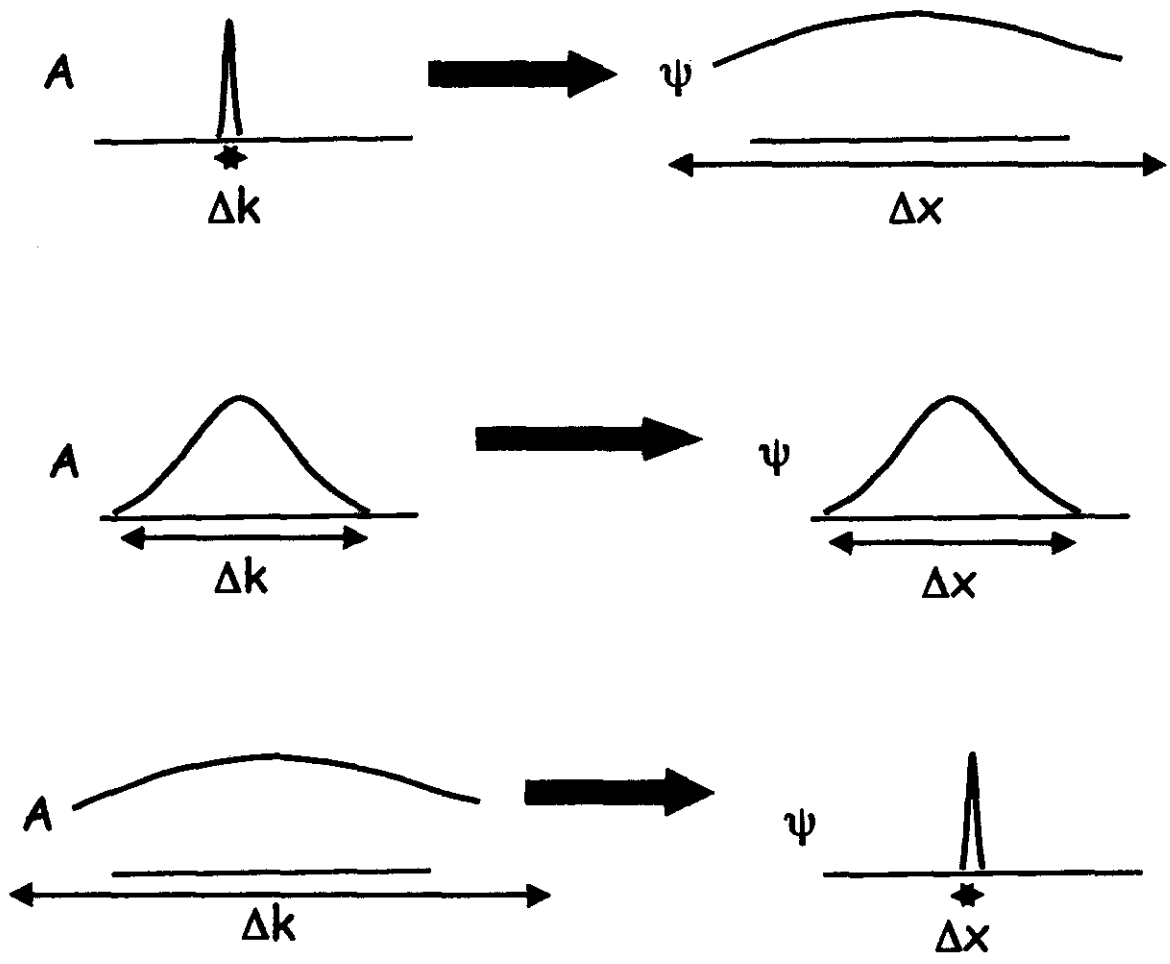
$\Rightarrow$ Fourier Transform



$$\psi(x,t) = \int A(k) \cos(kx-\omega t)\, dk$$

General fact about Fourier Transforms:

The extent Δx of the wave ψ is inversely related to the extent Δk of its Fourier Transform A.


A
Δk
ψ
Δx
A
Δk
ψ
Δx
A
Δk
ψ
Δx

We can write this as

$$\Delta x \, \Delta k \geq 1/2$$

Multiplying by $\hbar$ and using $p = \hbar k$ gives:

$$\Delta x \, \Delta p \geq \hbar/2$$

Heisenberg uncertainty principle

It is impossible to know precisely the position and the momentum of an object at the same time.