Relativity

1905 - Albert Einstein:

- Brownian motion

  □ atoms.

- Photoelectric effect.

  □ Quantum Theory

- "On the Electrodynamics of Moving Bodies"

  □ The Special Theory of Relativity

The Luminiferous Ether

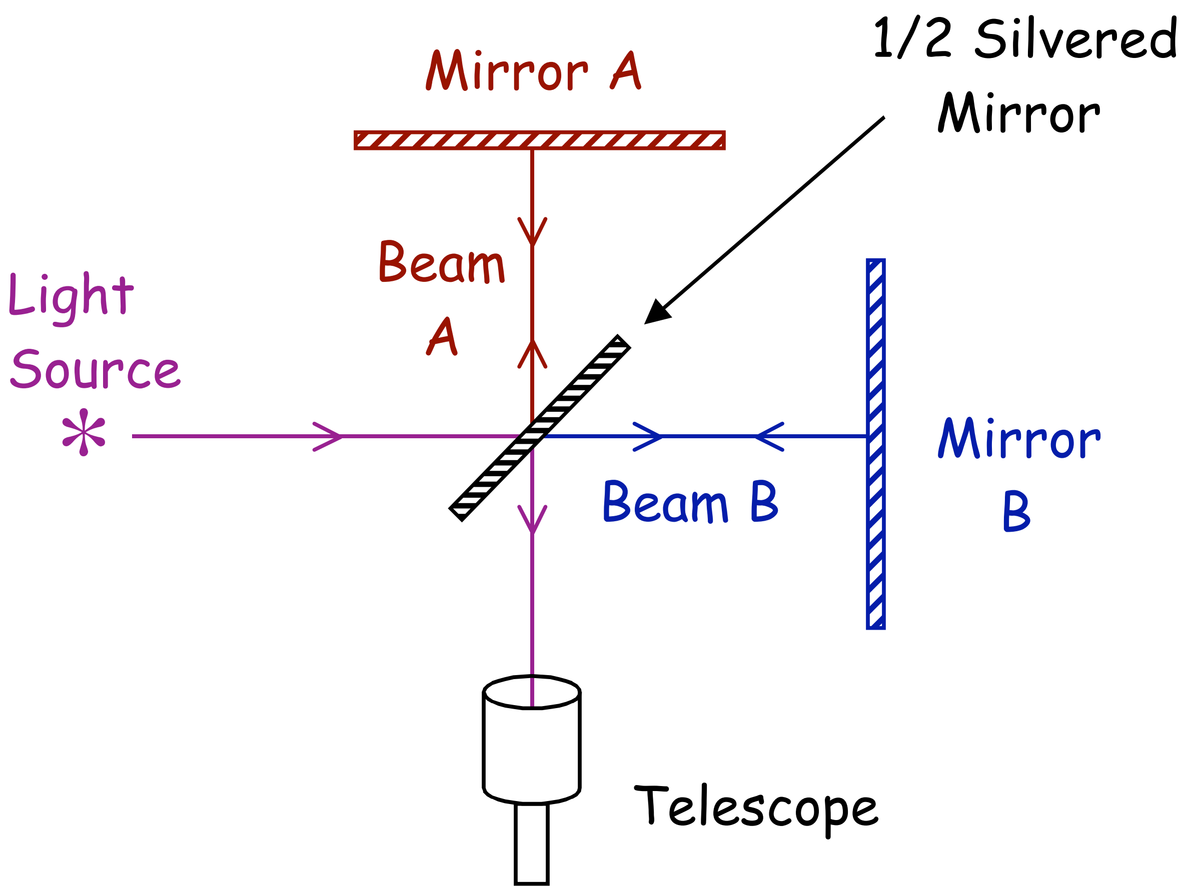
Hypothesis: EM waves (light) travel through some medium - The Ether

Speed of light: $c = 3 \times 10^8$ m/s
w.r.t fixed ether.

The earth moves at $v = 3 \times 10^4$ m/s
w.r.t fixed ether.

 Speed of light w.r.t earth should depend on direction.

The Michelson-Morley Experiment



An interferometer

The interference fringes should shift.

But no effect was observed!

What was wrong?

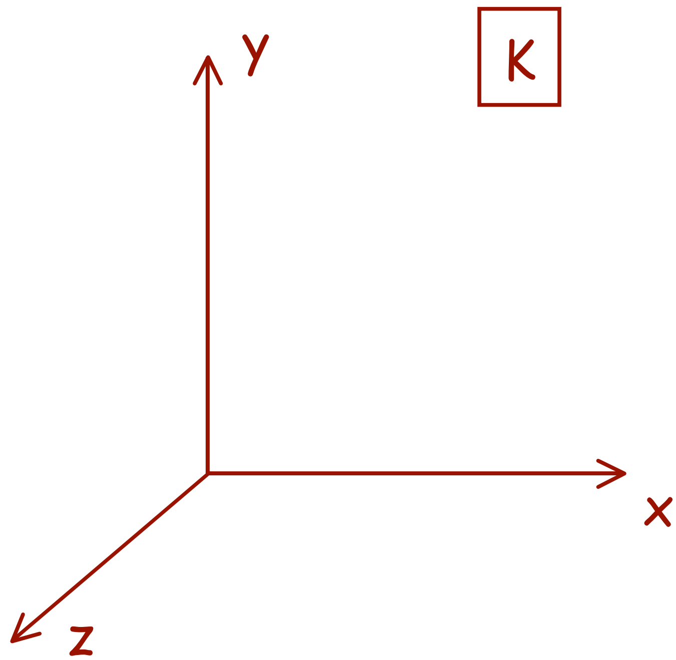
The Lorentz-Fitzgerald Contraction

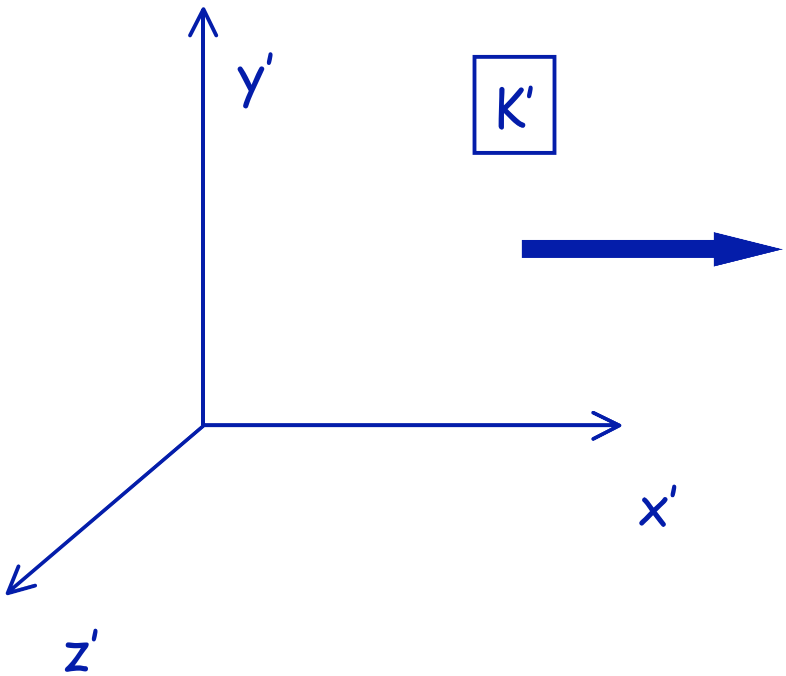
Suppose that the ether squashes any object moving through it?

To counteract the change in light speed, we need:

$$d' = d\sqrt{1 - v^2/c^2}$$

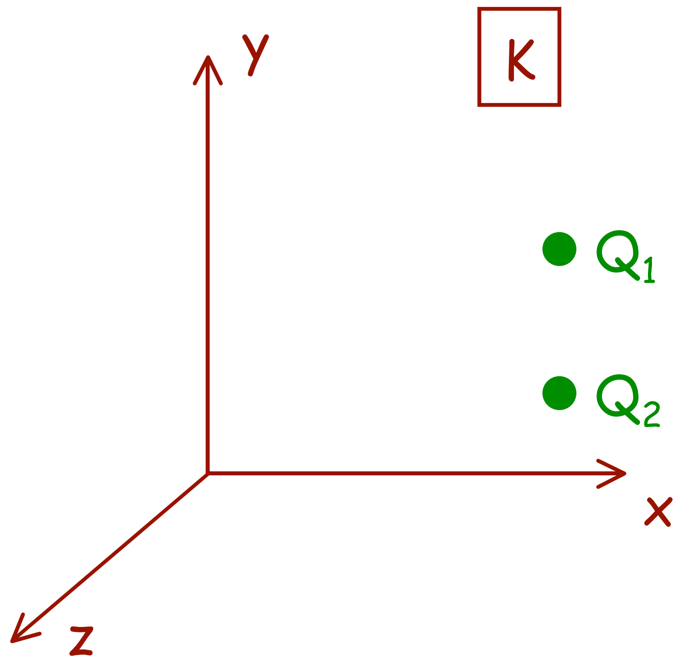
Galilean Transformations.

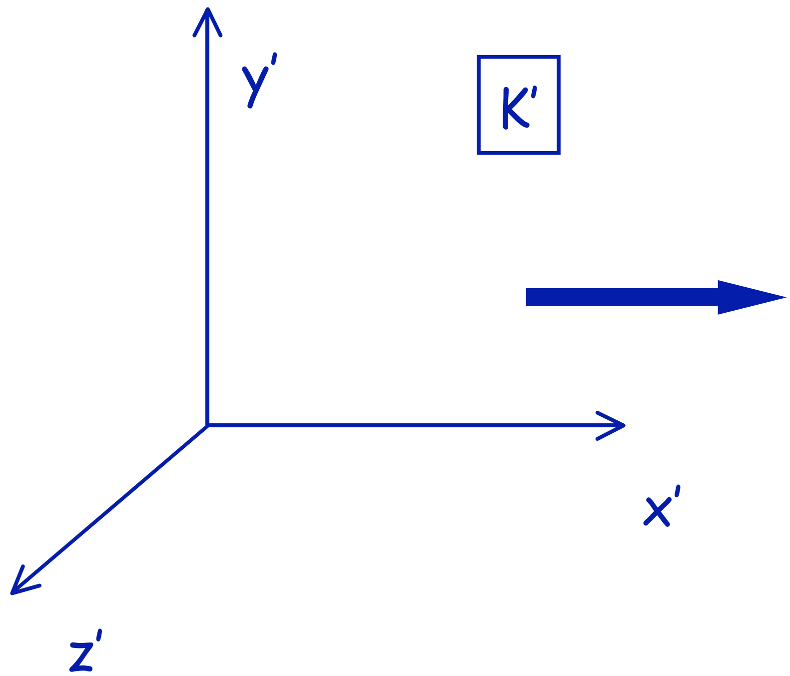




$$t' = t$$
$$z' = z$$
$$y' = y$$
$$x' = x - vt$$

In frame K, two charges at rest. Force is given by Coulomb's law.





In moving frame K', two charges are moving.
Since moving charges are currents,
Force is Coulomb + Magnetism.

Principle of relativity:

"The laws of nature are the same in all inertial reference frames"

Something is wrong!

- Maxwell's Equations?
- The Principle of Relativity?
- Gallilean Transformations?

Einstein decided

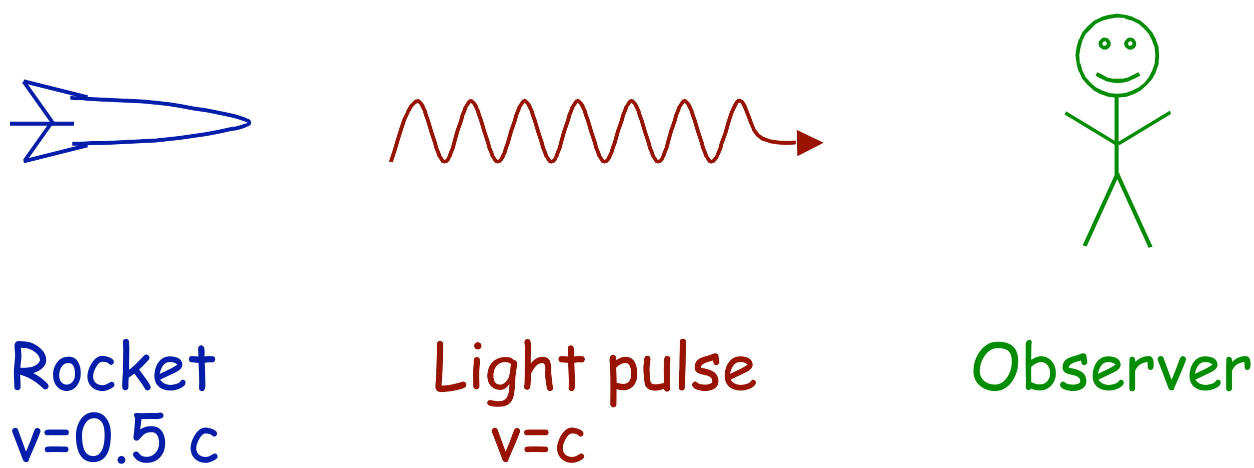
- Galilean Transformations are the problem.

Einstein's two postulates:

1. The principle of relativity is correct. The laws of physics are the same in all inertial reference frames.

2. The speed of light in vacuum is the same in all inertial reference frames ($c = 3 \times 10^8$ m/s regardless of motion of the source or observer).

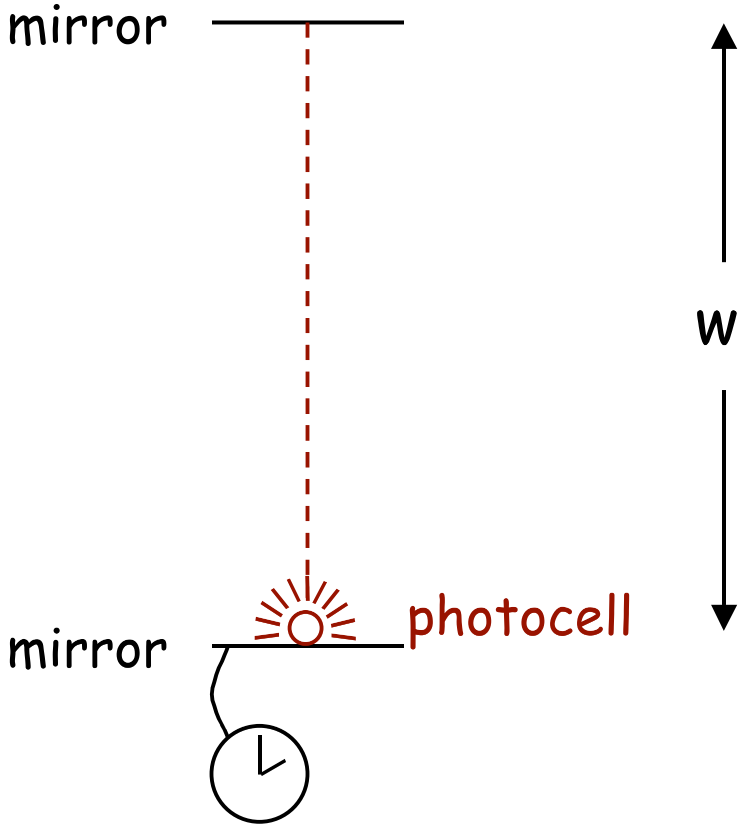
The second postulate seems to violate everyday common sense!



Einstein says: observer measures the light as traveling at speed c, not 1.5c.

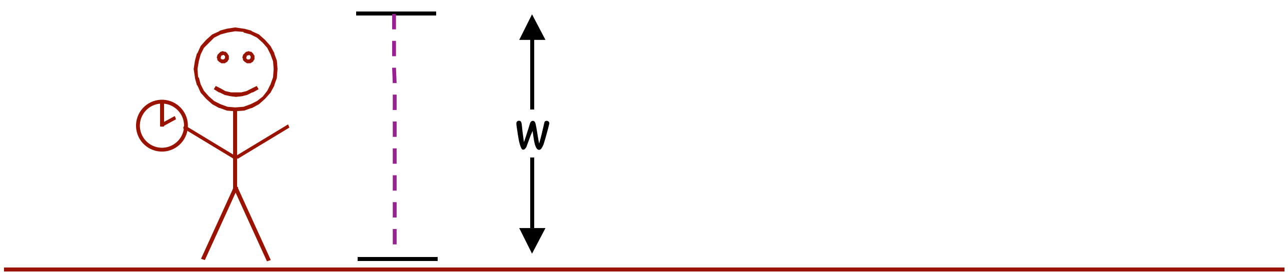
Gedanken Experiments

A light clock:

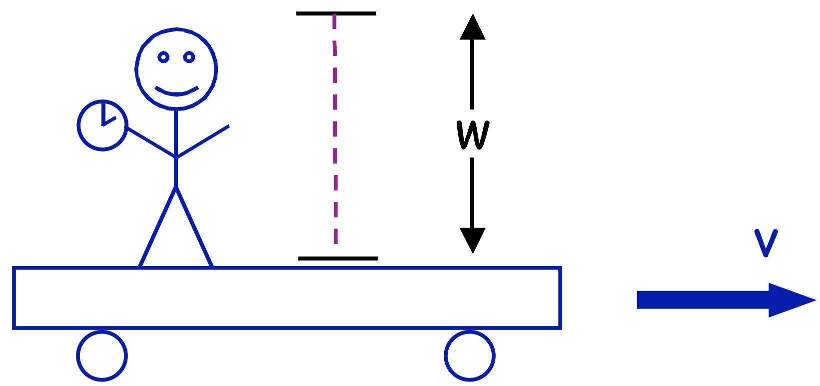


It ticks every $\Delta t = 2\,w/c$ seconds.
One can synchronize ordinary clocks with it.

Time Dilation



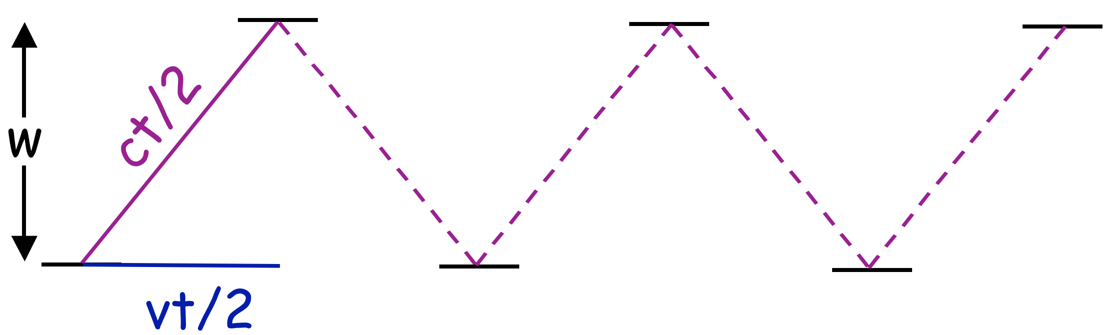
O_G: Observer on Ground



O_T: Observer on Truck

O_T's clock as seen from the ground:

$$c = 3 \times 10^8 \text{ m/s}$$



$$(ct/2)^2 - (vt/2)^2 = w^2$$

Time for one round trip of light, as seen from the ground:

$$\boxed{t = (2\,w/c) \Big/ \sqrt{1 - v^2/c^2}}$$

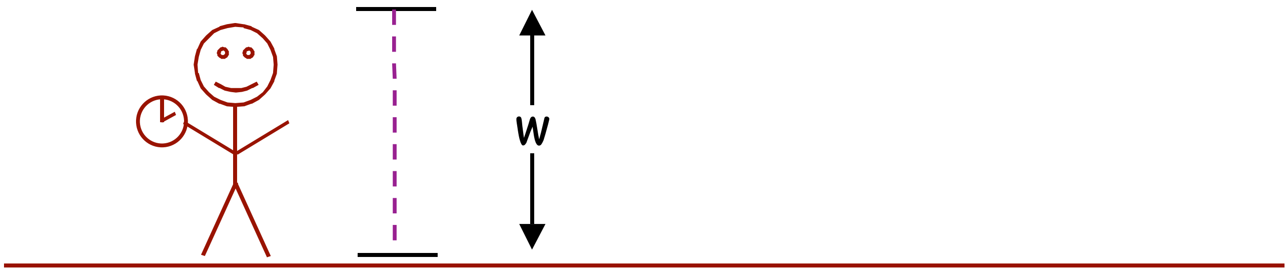
For v = 0.6c, $t = (2\,w/c) \times 1.25$

All of O_T's processes slow down compared to O_G as seen by O_G.

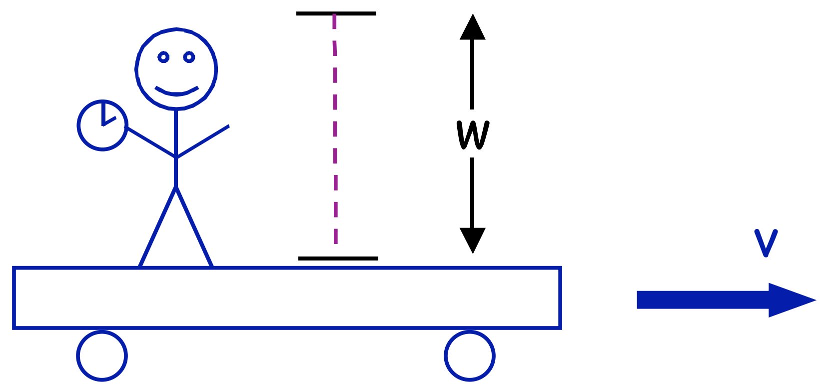
Similarly,

All of O_G's processes slow down compared to O_T as seen by O_T.

Length Contraction



O_G: Observer on Ground



O_T: Observer on Truck

Device on truck makes mark on track each time clock ticks.

As seen from ground:

Distance between marks

$$= (\text{time between ticks}) \times v$$

$$= \left[(2\,w/c)\Big/\sqrt{1 - v^2/c^2}\,\right] v$$

As seen from truck:

Distance between marks

= (time between ticks) x v

= (2 w/c) v

(To the person on the truck the time between ticks is (2 w/c).)

(Distance measured on truck)

$$= \sqrt{1 - v^2/c^2}$$

x (distance measured on ground)

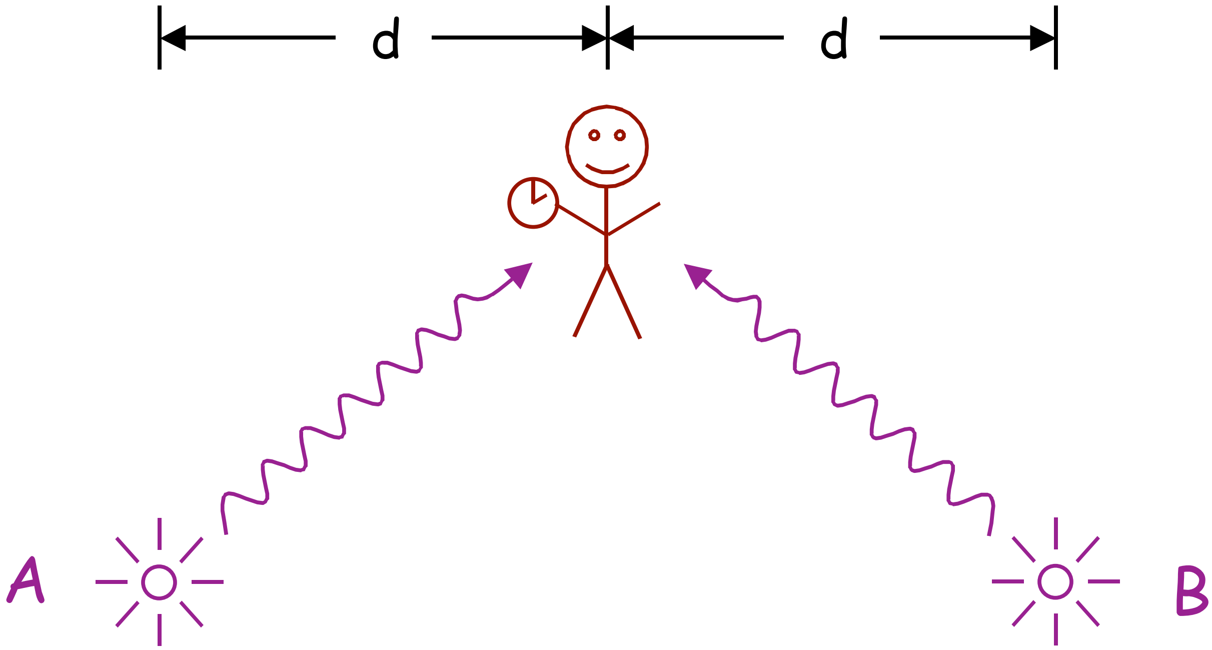
As seen from a moving frame, rest distances contract.

(L-F contraction)

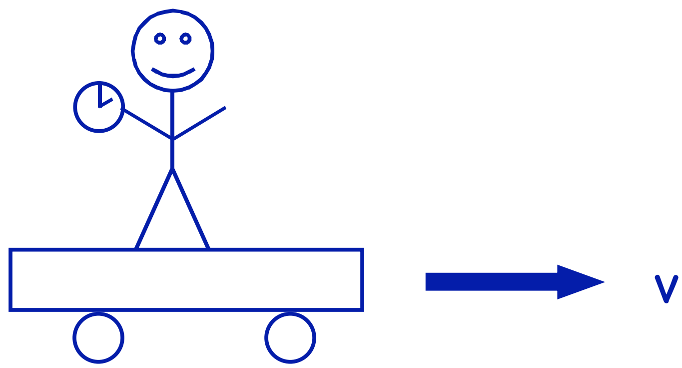
Simultaneity

Events occur at a well defined position and a time (x,y,z,t).

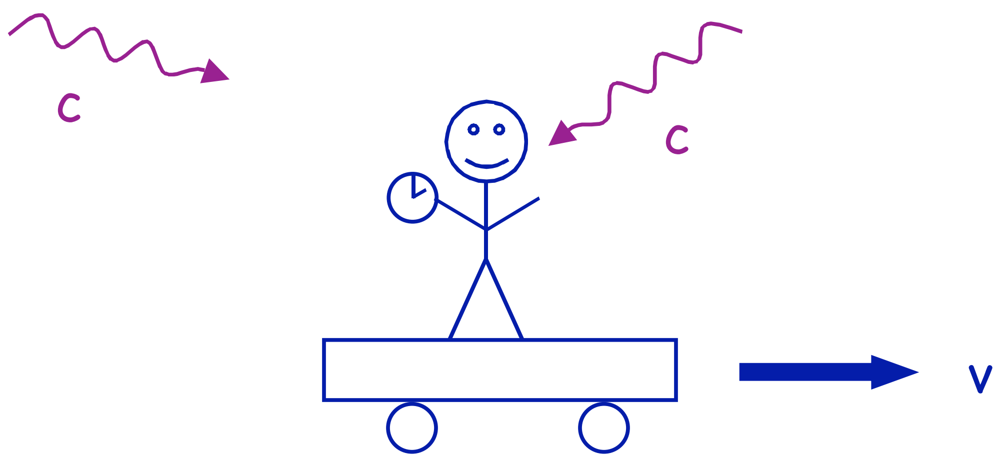
But events that are simultaneous (same t) in one inertial frame are not necessarily simultaneous in another frame.



The light from the two flashes reach O_G at the same time. He sees them as simultaneous.

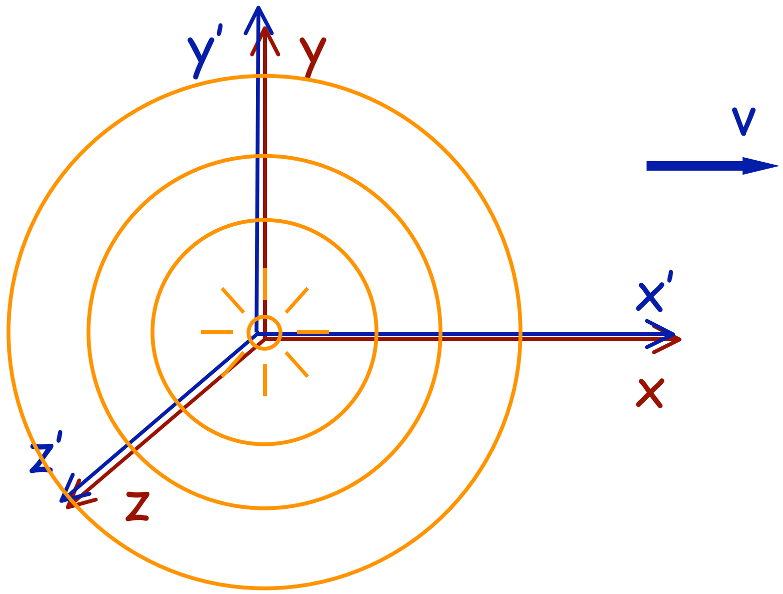


O_T passes O_G just as the lights flash.



But light from B reaches O_T first. Since both light beams started the same distance from her, and both travel at speed c, she concludes that B must have flashed before A.

Lorentz Transformations



- Flashbulb at origin just as both axes are coincident.
- Wavefronts in both systems must be spherical:

$$x^2 + y^2 + z^2 = c^2t^2 \qquad \text{and}$$

$$x'^2 + y'^2 + z'^2 = c^2t'^2$$

- Inconsistent with a Galilean transformation
- Also cannot assume $t=t'$.

Assuming:

- Principle of relativity
- linear transformation $(x,y,z,t) \rightarrow (x',y',z',t')$

Lorentz Transformations (section 2.4)

$$x' = \gamma\,(x - v\,t)$$

$$y' = y$$

$$z' = z$$

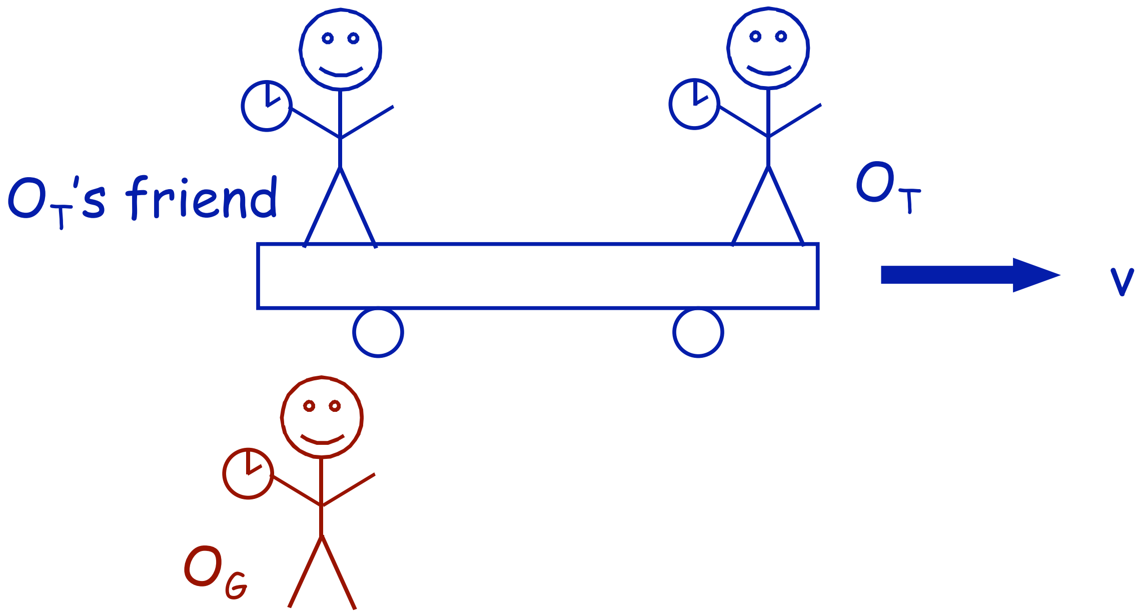
$$t' = \gamma\,(t - v\,x / c^2)$$

With $\boxed{\gamma = 1 / \sqrt{1 - v^2/c^2}\,.}$

(Often also define $\boxed{\beta = v / c\,.}$)

Time Dilation (again)

Proper time: time T_0 measured between two events at the same position in an inertial frame.



O_G's clock: $T_0 = t_2 - t_1,$ $\quad (x_2 - x_1 = 0)$

O_T's clock: $T' = t'_2 - t'_1$

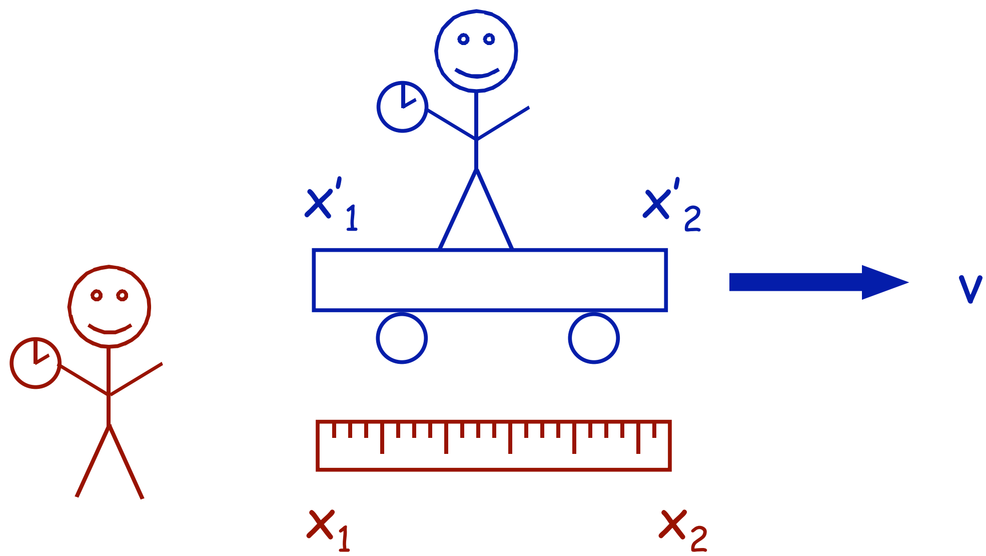
$$t'_2 - t'_1 = \gamma\left(t_2 - t_1 - v/c^2\,(x_2 - x_1)\right)$$

$$T' = \gamma T_0 > T_0$$

Clocks, as seen by observers moving at a relative velocity, run slow.

Length Contraction (again)

Proper length: distance L_0 between points that are at rest in an inertial frame.



O_T on truck measures its length to be $L_0 = x'_2 - x'_1$. This is its proper length.

O_G on ground measures its length to be $L = x_2 - x_1$, using a meter stick at rest ($t_2 = t_1$).

Then

$$L_0 = x'_2 - x'_1 = \gamma(x_2 - x_1 - v\,(t_2 - t_1))$$
$$= \gamma L$$

O_G measures $L = L_0/\gamma < L_0$.

Truck appears contracted to O_G.

An application

Muon decays with the formula:

$$N = N_0\, e^{-t/\tau}$$

N_0 = number of muons at time t=0.
N = number of muons at time t seconds later.
τ= 2.19 x 10^{-6} seconds is mean lifetime of muon.

Suppose 1000 muons start at top of mountain d=2000 m high and travel at speed v=0.98c towards the ground. What is the expected number that reach earth?

Time to reach earth:
$t = d/v = 2000\text{m}/(0.98 \times 3 \times 10^8 \text{ m/s})$
$= 6.8 \times 10^{-6}$ s

Expect $N = 1000\, e^{-6.8/2.19} = 45$ muons.

But experimentally we see 540 muons!
What did we do wrong?

Time dilation: The moving muon's internal clock runs slow. It has only gone through

$$t' = 6.8 \times 10^{-6} \sqrt{1 - 0.98^2}\ \text{s}$$
$$= 1.35 \times 10^{-6}\ \text{s}$$

So $N = 1000\ e^{-1.35/2.19} = 540$ muons survive.

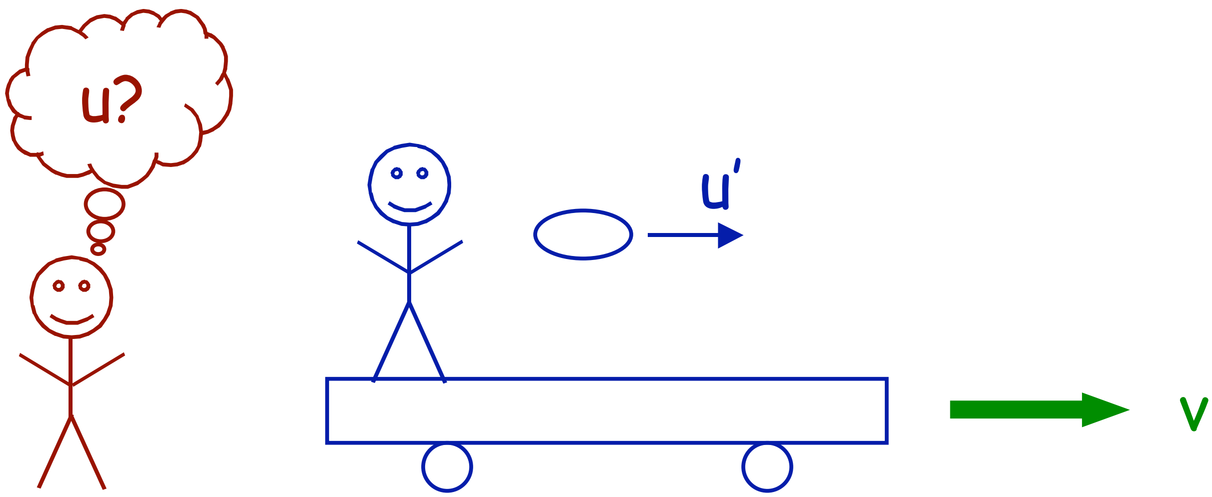
Alternate explanation: From muon's viewpoint, the mountain is contracted. Get same result.

Addition of velocities

Galilean formula ($u=u'+v$) is wrong.

Consider object, velocity u' as seen in frame of O_T who is on a truck moving with velocity v w.r.t the ground.

What is velocity u of the object as measured by O_G on the ground?



Recall $u = \Delta x/\Delta t$, $u' = \Delta x'/\Delta t'$.
Inverse Lorentz transformation formulae:

$$\Delta x = \gamma(\Delta x' + v \, \Delta t')$$

$$\Delta t = \gamma(\Delta t' + v \, \Delta x' / c^2)$$

$$u = \frac{\Delta x}{\Delta t} = \frac{\gamma(\Delta x' + v\,\Delta t')}{\gamma(\Delta t' + v\,\Delta x' / c^2)}$$

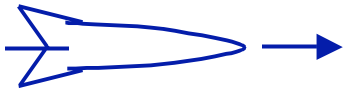
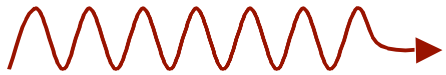
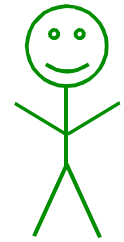
$$\boxed{u = \frac{u' + v}{1 + v\,u'/c^2}}$$

For u' and v much less than c:

$$u \approx u' + v$$

Velocities in y and z directions are also modified (due to $t' \neq t$, see section 2.6)

Examples:

Rocket
$v=0.5\,c$

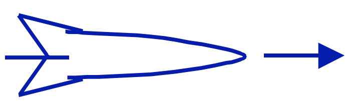
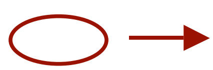
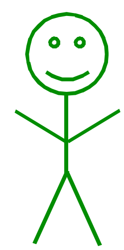
Light pulse
$u'=c$

Observer

Observer sees light move at

$$u = \frac{0.5c + c}{1+(0.5c)(c)/c^2} = c$$

Light moves at $c=3\times10^8$ m/s <u>in all frames</u>.

Rocket
$v=0.8\,c$

Projectile
$u'=0.5c$

Observer

Observer sees projectile move at

$$u = \frac{0.5c + 0.8c}{1+(0.5)(0.8)} = 0.93c$$

Massive objects always move at speeds < c.

The Twin Paradox

Suppose there are two twins, Henry and Albert. Henry takes a rocket ship, going near the speed of light, to a nearby star, and then returns. Albert stays at home on earth.

Albert says that Henry's clocks are running slow, so that when Henry returns he will still be young, whereas Albert is an old man.

But Henry could just as well say that Albert is the one moving rapidly, so Albert should be younger after Henry returns!

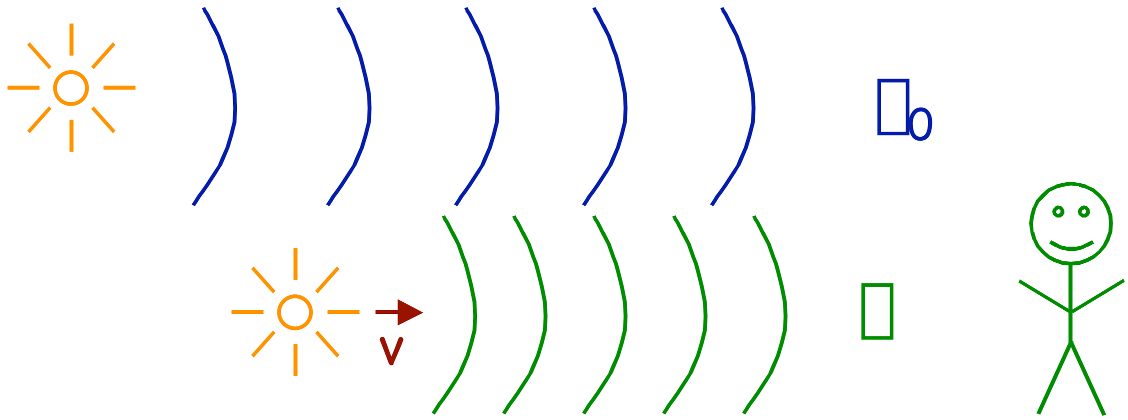
Who is right?

The first scenario is the correct one.

The situation is <u>not</u> symmetric, because the rocket has to decelerate, turn around and accelerate again to return to earth. Thus, Henry is not in an inertial frame throughout the trip. He does return younger than Albert.

Relativistic Doppler Effect

Light source and observer approach each other with relative velocity, v.
Light is emitted at frequency ν_0.



Observer sees light at a higher frequency:

$$\nu = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}}\,\nu_0 \quad \text{with } \beta = v/c$$

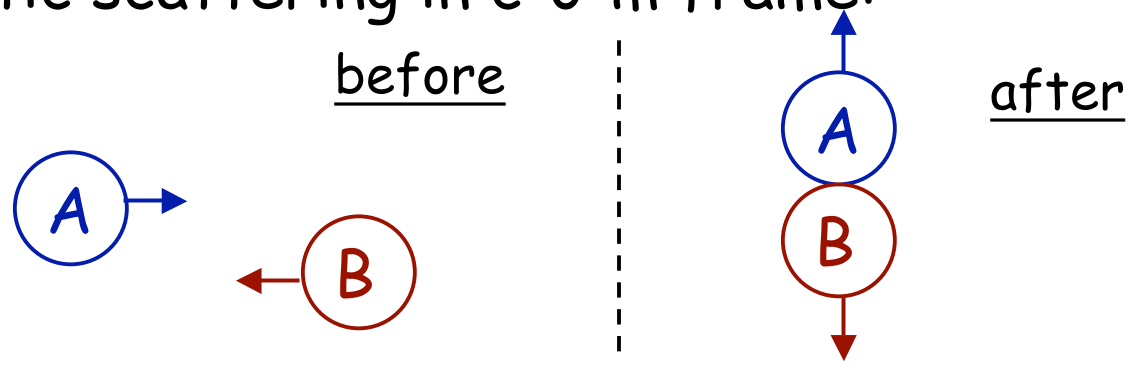
- If source is receding, the formula still holds but now β is <u>negative.</u>

We know that the universe is expanding, because light from distance galaxies is red-shifted, indicating motion away from us.

Relativistic Momentum

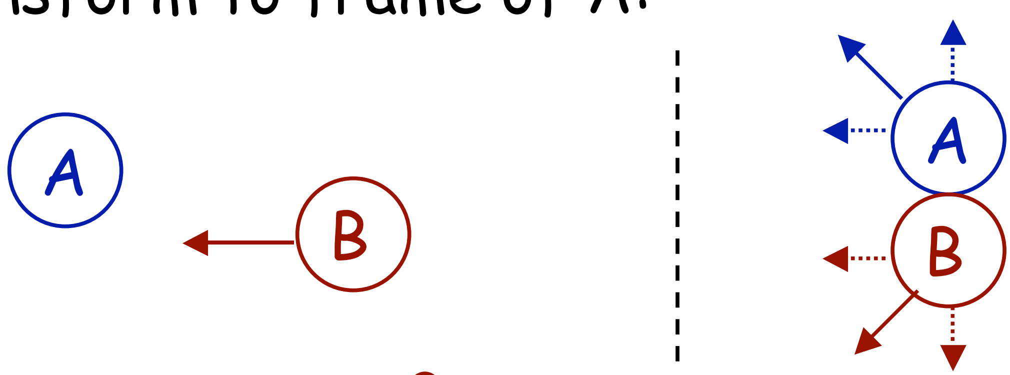
Requirement: momentum is conserved in all inertial frames.
Assume: $\vec{p} = m\,\vec{v}$.

Elastic scattering in c-o-m frame:



$$p_x: \quad mu + m(-u) = 0$$

Transform to frame of *A*:



$$p_x: \quad 0 + m\left(\frac{-2u}{1+u^2/c^2}\right) \neq m(-2u)$$

It doesn't work!

Relativistic momentum:

$$\vec{p} = \gamma m \vec{v} = \frac{m\vec{v}}{\sqrt{1-v^2/c^2}}$$

Relativistic Kinetic Energy:

$$K = (\gamma - 1)\, mc^2$$

$$= \left(\frac{1}{\sqrt{1-v^2/c^2}} - 1 \right) mc^2$$

For small velocities, $v/c \ll 1$:

$$K = \left(1 + 1/2\, (v/c)^2 + \ldots \qquad - 1 \right) mc^2$$

$$\approx 1/2\ m\, v^2$$

For large velocities $v \rightarrow c$:

$$K \longrightarrow$$

Massive objects always travel at speeds less than c.

Relativistic Energy

According to Einstein, even a mass at rest has energy:

$$E_0 = m\,c^2 \qquad \text{(rest energy)}$$

Thus, the total energy of a moving object is

$$\begin{aligned} E &= K + E_0 \\ &= (\gamma - 1)\, mc^2 + mc^2 \\ &= \gamma mc^2 \end{aligned}$$

It is straightforward to show:

$$E^2 - p^2c^2 = m^2c^4$$

For a massless particle (e.g. a photon):

$$E = |\vec{p}|\, c$$

In general

$$v = \frac{|\vec{p}|\, c^2}{E} = \frac{\gamma m v\, c^2}{\gamma m c^2}$$

For a massless particle this gives

$$v = c$$

Massless particles travel at the speed of light c.