PHY251 Fall 2007 Week 12: Lab practical exam
THE SPRING: Hooke's Law

OBJECTIVES:

- To investigate how a spring behaves when it is stretched under the influence of an external force. To verify that this behavior is accurately described by Hooke's Law.
- To measure the spring constant $k$.

THEORY

An ideal spring is remarkable in the sense that it is a system where the generated force is \textbf{linearly dependent} on how far it is stretched. Hooke's law describes this behavior, and you would like to verify this in lab today. Hooke's Law states that to extend a string by an amount $\Delta x$ from its previous position, one needs a \textbf{force} $F$ which is determined by $F = k \Delta x$. Here $k$ is the \textbf{spring constant} which is a quality particular to each spring. Therefore in order to verify Hooke's Law, you must verify that the force $F$ and the distance the spring is stretched are proportional to each other (that just means linearly dependent on each other). The constant of proportionality is $k$.

In our case the external force is determined by attaching a mass $m$ to the end of the spring. The mass will of course be acted upon by gravity, so the force exerted downward on the spring will be $F_g = mg$. See Figure 1. Consider the forces exerted on the attached mass. The force of gravity ($mg$) is pointing downward. The force exerted by the spring ($k \Delta x$) is pulling upwards. When the mass is attached to the spring, the spring will stretch until it reaches the point where the two forces are equal but pointing in \textbf{opposite directions}:

$$F_s - F_g = 0$$

or

$$k \Delta x = F_g = mg$$

This point where the forces balance each other is known as the \textbf{equilibrium point}. The spring + mass system can stay at the equilibrium point indefinitely as long as no additional external forces act on it. The relationship (1) allows us to determine the spring constant $k$ when $m$, $g$, and $\Delta x$ are known or can be measured. This is how you will be determining $k$ today.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{spring_equilibrium.png}
\caption{The spring in equilibrium.}
\end{figure}
PROCEDURE
Determine the initial mass, \( m_0 \), by weighing the support table. Next, attach the support table for the masses to the spring. Measure the position of the end of the spring after the table has been attached. This position is the initial position \( x_0 \).

Start the measurement with an attached mass of 120 grams. Then increase the mass in steps of 20 grams, making a total of 5 measurements. Measure the corresponding position of the spring for each mass. This results in a series of measurements \( m_i \) and \( x_i \). To calculate the forces due to gravity and the spring calculate \( \Delta x_i = x_i - x_0 \) and \( \Delta m_i = m_i - m_0 \). Also calculate the error on the distance the spring extends, \( \delta(\Delta x_i) \). The corresponding forces for gravity and the spring are \( F^{F_g}_i = \Delta m g \) and \( F^{F_s}_i = k \Delta x_i \). Right now you do not know \( k \), so you will only have your spreadsheet calculate \( F_g \) for you. But remember, at equilibrium positions such as we are measuring, \( F_g \) equals \( F_s \)!

Graph \( F_g \) vs. \( \Delta x \). Add error bars for \( \Delta x \) to your graph. Now have the computer fit your plot with a best fit line. The slope and its uncertainty determine the spring constant \( k \) in Hooke's Law.

Hand in:
- Your spreadsheet.
- The formula view of the spreadsheet.
- The graph with linear fit and error bars for \( \Delta x \).
- Answers to the questions.

Introduction and conclusion are not needed for this lab.

Questions:
1. What is the uncertainty on \( \Delta x \)?
2. What value did you measure for the spring constant \( k \)? What is the uncertainty on your measurement?
3. The spring constant is a property of each spring. It depends on the material and the length of the spring. Measure the length of your spring and calculate the spring constant from its length and the conversion equation for your spring found in the lab. What is this calculated spring constant and its uncertainty?
4. Is the spring constant you found in this experiment consistent with the value you calculated in the previous question?
5. How would \( F_g \) and \( \Delta x \) change if the experiment was done on the moon where the gravitational acceleration is six times smaller than on earth?
6. For the single spring shown in Figure 2, \( \Delta x \) is 4.0 cm when a weight of 20 grams is attached.
   What is \( \Delta x \) for a weight of 20 grams if the single spring is replaced by two identical springs in parallel?
   What is \( \Delta x \) for a weight of 20 grams if the single spring is replaced by two identical springs in series?

Figure 2: Replacing a single spring with two springs in series or in parallel.