PHY481: Electrostatics

Semester plans

Introductory E&M review (1)

Syllabus

Syllabus for PHY481, Fall 2007

Lecturer: Prof. Carl Bromberg; E-mail: bromberg@pa.msu.edu; Office: Rm. 3225 BPS; Phone:5-9200 Ext. 2122; Office hrs : Mon. & Wed. 2 – 3 & 4 - 5, or by appointment.

Grader: Kiseok Chang; E-mail: changkis@msu.edu; Office:1300 BPS, Phone: 59200 x 2080. See Kiseok for grading issues. If not satisfied, see Prof. Bromberg in office hours.

IF you have not completed a 2-Semester introductory course in Calculus-based Physics (preferably an honors course), see me during office hours.

Lectures:

- Mon. & Wed., **Note: 12:30 - 1:40 pm**, in room 1420 BPS (see Course Schedule).
- Textbooks:
- 1) "Electromagnetism", Pollack and Stump, ISBN 0-8053-8567-3, Addison-Wesley
  2) An Introductory text: Fishbane et al., "Physics for Scientists and Engineers", or 2nd Ed, Halliday et al., "Physics", 5th Ed., or similar texts.

Course Topics:

- Course covers the topics shown in the **Course Schedule** (on the next page).
- Lectures may not cover all topics presented in the **Reading Assignments**. All topics, in the assigned reading or presented in lecture may appear on an exam.
- Lectures on Mon. and Wed. will be posted on the Course Web site by Fri of that week.

Homework (HW) and Exams:

- There will be 8 homework assignments with due dates as indicated in the **Course Schedule**. Homework handed in late will not be graded, but will be logged. All missing homework assignments must be submitted by the last class on 5 - Dec., or you risk an incomplete (I) grade for the course.
- Two, **60 minute exams** will be given **FRIDAY** on the dates indicated in the Course Schedule, and a **2 hr Final Exam** on **13-Dec.** covering the work of the entire semester. Only simple calculators allowed, no formula or graphing capability
- Documented medical (or other) excuses for **one** exam will be considered on a case by case basis. Resolution may involve an oral exam.

Grades

- HW (1 point/problem ~50 total points), 2 exams (100 points each), Final Exam (200 points). Straight scale, with >360 points receiving a 4.0 with cuts 30 points lower for each 0.5 in grade.
- Frequently check the **WEB** site, **http://www.pa.msu.edu/courses/PHY481**, for announcements, lecture slides, HW and exam solutions, scores and grades.

Miscellaneous

- No HEAD-phones, IPODs, CD-players, CELL-phones, or HATs in class.

PHY481 Schedule
Fall 2007

W	D	Date	L	Subjects	P&S	HW	HW Due
1	M	27-Aug	1	Intro E&M - review(1)		HW1:Show derivations of E & V	
	W	29-Aug	2	Intro E&M - review(2)		for the 8 charge distributions in	
2	M	3-Sep		**Labor Day - no classes**		the Table on the class web site	
	W	5-Sep	3	Vector tools	2		1
3	M	10-Sep	4	Integral Theorems			
	W	12-Sep	5	Curvilinear coordinates			
4	M	17-Sep	6	Coulomb's law & electric fields (E)	3		
	W	19-Sep	7	HW-2 solutions, etc.		Ch.2 2-3,6,8-10,12-14,19,22,24,27	2
5	M	24-Sep	8	Curl and divergence of E			
	W	26-Sep	9	Gauss's Law			
6	M	1-Oct	10	HW-3a solutions, etc.		Ch.3 3,5-8,10,15-16,19,24,40,42	3a
	W	3-Oct	11	Electric potential & energy			
7	M	8-Oct	12	Dipole and multipoles			
	W	10-Oct	13	HW-3b solutions, etc.		Ch.3 22-23,25-26,30,32,33	3b
	F	**12-Oct**		**60-min. Exam (Lectures 1-13)**			
8	M	15-Oct	14	Exam 1 solutions, etc.	4		
	W	17-Oct	15	Parallel plate capacitor			
9	M	22-Oct	16	Potentials by method of images			
	W	24-Oct	17	Potentials for spheres and cylinders			
10	M	29-Oct	18	HW-4 solutions, etc.	5	Ch. 4 3-4,6-7,10a,14-16,18,20,22	4
	W	31-Oct	19	Solving Laplace's equation			
11	M	5-Nov	20	Potentials with polar angle dependence			
	W	7-Nov	21	HW-5 sol.utions, etc.		Ch. 5 3,7,11,13,15-16,28,32	5
12	**F**	**9-Nov**		**60-min. Exam (Lectures 1-21)**			
	M	12-Nov	22	Exam 2 solutions, etc.	7		
	W	14-Nov	23	Electric Currents			
13	M	19-Nov	24	Resistance			
	W	21-Nov	25	HW-7 solutions, etc.	8	Ch. 7 2-3,5,7,9,11,15,18	7
14	M	26-Nov	26	Biot-Savart Law			
	W	28-Nov	27	Ampere's Law			
15	M	3-Dec	28	Vector potential & magnetic dipole			
	W	5-Dec	29	HW-8 solutions, etc.		Ch. 8 4,10,14,18,22,34,37	8
	Th	**13-Dec**		**Final Exam 12:45 - 2:45**			

Plan of attack

- Why take PHY481, a difficult undergraduate physics course?
- Compassionate review
  - 2 lectures reviewing Intro E&M concepts, including brief descriptions and solving "familiar" problems.
  - Homework assignment: to derive E & V for typical charge distributions.
- Followed by the traditional course content
  - Develop advanced mathematics and techniques
  - Full description of each topic in Electrostatics, using advanced mathematics, and solving problems with a large range of difficulty
  - Exams: ~30% at an Intro E&M level, ~70% with focus on advanced techniques.

Properties of classical electric charge

- Electric Charge
  - Property of matter associated with the electromagnetic force
  - Quantized in integer units (or 1/3) of electron charge *e*

$$e = 1.6 \times 10^{-19}\ \mathrm{C}$$

  - Two signs + (proton) and – (electron), also neutral (neutron)
- All matter begins as a collection of neutral atoms
  - Electrons can move from one object to another (nuclei difficult)
  - To make an object positive, remove electrons
  - To make an object negative, add electrons
  - An object with charge +q, implies a net charge –q elsewhere.
- Charge densities ρ, σ, λ with simple space dependences

Volume	Surface	Line
$dq = \rho dV$	$dq = \sigma dS$	$dq = \lambda d\ell$

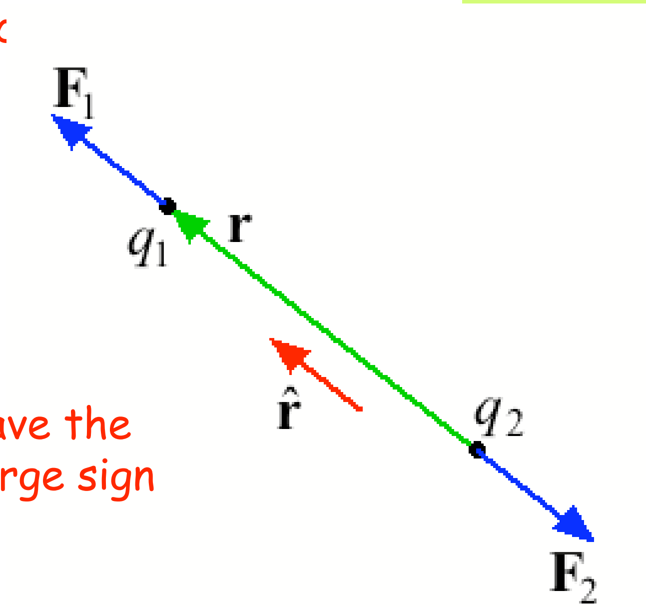
Force between charges

- Force between two charges, Coulomb's Law:

Force on 1 is
in direc

$$\frac{1}{\ \varepsilon_0}\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

Will need new
notation later!



q_1 & q_2 have the
same charge sign

$$\frac{1}{4\pi\varepsilon_0} = 9.0\times10^9\ \frac{\text{Nm}^2}{\text{C}^2}$$

$$\hat{\mathbf{r}} = \frac{\mathbf{r}}{r}\ \text{points from } q_2 \text{ to } q_1$$

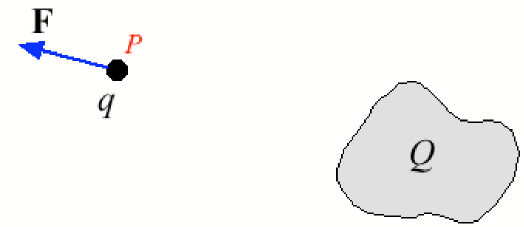
q_1 and q_2 carry charge sign

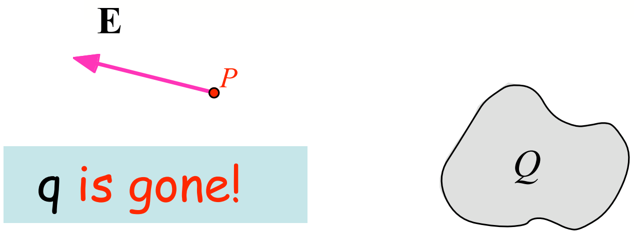
- Superposition principle
  - Force on charge q from charges $q_1, q_2, \ldots\ q_k$ is the vector sum of forces between q and each of the charges.
  - No interference between action of the charges

The electric field

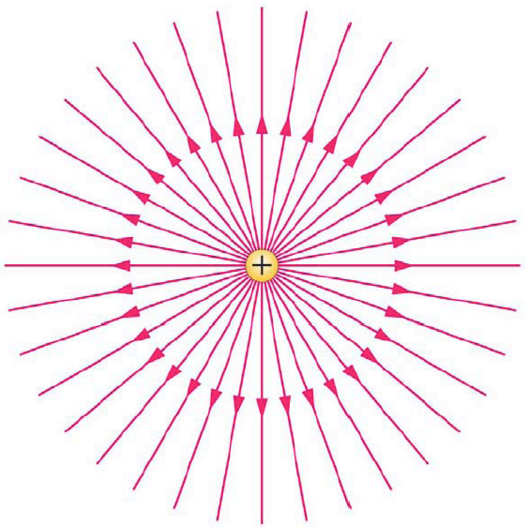
- Charge creates an electric field
  - A very small positive charge *q* placed at a point P experiences a force **F** from a collection of charge Q (seems positive).
  - The electric field **E** at the point P is defined as

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$





- Electric field lines
  - begin on + charge and end on – charge
  - direction of **E** is along field lines
  - **E** field lines do not cross
  - density of lines is proportional to field magnitude



positive point charge

Electric fields from charge distributions

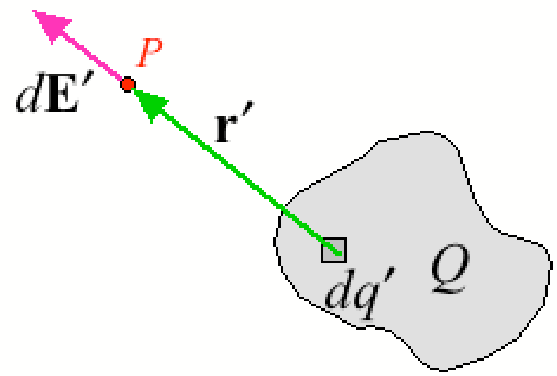
- Integration over charge distributions

Will need new notation later!

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\int \frac{dq'}{r'^2}\hat{\mathbf{r}}'$$

$$dq' = \rho dV'$$

$$Q = \int \rho dV'$$



- E-fields of simple charge distributions & density
  - Sphere, cylinder, box - volume charge density: ρ
  - Sphere, cylinder, box, sheet - surface charge density: σ
  - Thin line, ring - linear charge density: λ
  - Sheets and lines may be of infinite extent

Coordinate systems

- Unit vectors, differential line and volume

  - Cartesian

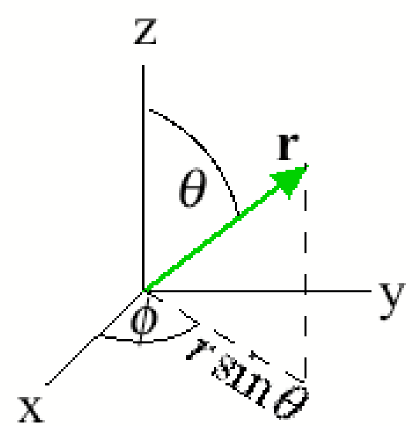
    $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$

    $$dx$$
    $$dV = dx\,dy\,dz$$

  - Cylindrical

    $\hat{\mathbf{r}}, \hat{\boldsymbol{\phi}}, \hat{\mathbf{k}}$

    $$dr \text{ (radial) or } rd\phi \text{ (ring)}$$
    $$dV = rdrd\phi dz$$



  - Spherical

    $\hat{\mathbf{r}}, \hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\phi}}$

    $$dr \text{ (radial)}, \quad rd\theta \text{ (polar)}, \quad r\sin\theta d\phi \text{ (ring)}$$
    $$dV = r^2 dr \sin\theta d\theta d\phi$$

  - Symmetry used to avoid angular complications
  - Radial and angular unit vectors needed later.

- Warm up! From the above, determine the volume and surface area of a cylinder & sphere of radius *R*.

Electric field from point charges

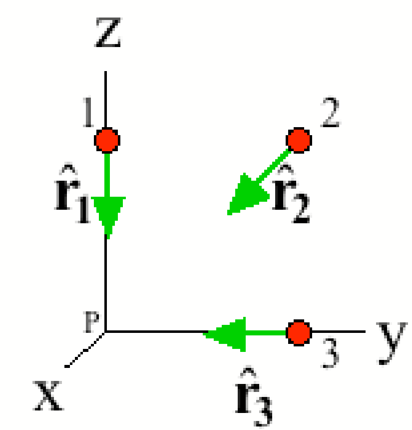
- At point *P*, the electric field $\mathbf{E}_p$ *generated* by *n* point charges is the vector sum of $\mathbf{E}_i$ from each charge:

$$\mathbf{E}_p = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i$$

  - Find electric field at the origin due to the three charges $q_{1\text{-}3}$ on corners of a square with side *a*.

$$\mathbf{E}_p = \frac{1}{4\pi\varepsilon_0}\left[-\frac{q_1}{a^2}\hat{\mathbf{k}} - \frac{q_2}{2a^2}\left(\frac{\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{2}}\right) - \frac{q_3}{a^2}\hat{\mathbf{j}}\right]$$

$$= \frac{-1}{4\pi\varepsilon_0 a^2}\left[\left(q_1 + \tfrac{\sqrt{2}}{4}q_2\right)\hat{\mathbf{k}} + \left(q_3 + \tfrac{\sqrt{2}}{4}q_2\right)\hat{\mathbf{j}}\right]$$

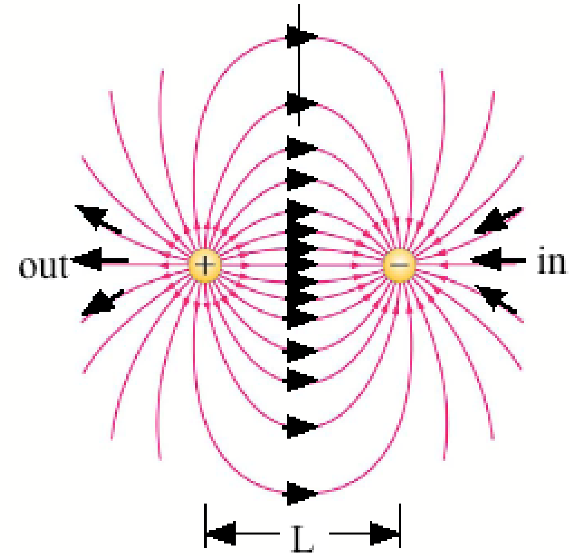


Dipole field on the bisector

- Field line's direction is out of +q and into −q
  - Definition of dipole moment vector with charges on the x-axis

$$\mathbf{p} = q\mathbf{L} = -qL\hat{\mathbf{i}}$$

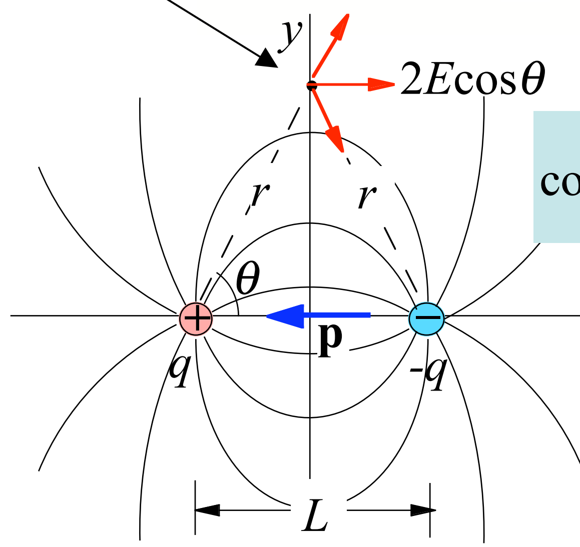
  - On the bisector, the vertical components cancel, horizontal components add.



$$\mathbf{E} = \frac{2}{4\pi\varepsilon_0}\frac{q}{r^2}\cos\theta\,\hat{\mathbf{i}} = \frac{1}{4\pi\varepsilon_0}\frac{qL}{r^3}\hat{\mathbf{i}}$$

$$= \frac{-\mathbf{p}}{4\pi\varepsilon_0 r^3}$$

Note minus sign



$$\cos\theta = \frac{L/2}{r}$$

Far from the dipole $r \approx y$

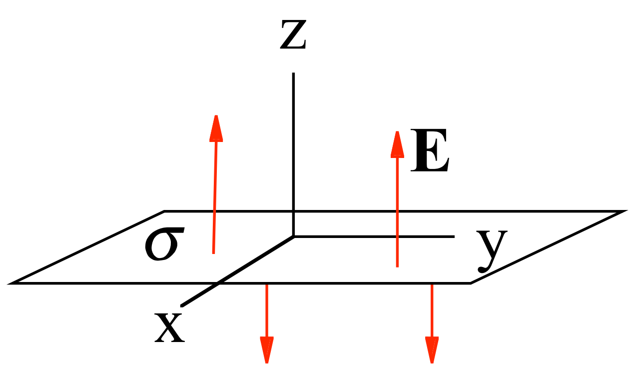
$$\mathbf{E} = \frac{-\mathbf{p}}{4\pi\varepsilon_0 y^3}$$

Uniformly charged infinite plane

- For an infinite horizontal plane the only reasonable direction for the electric field **E** is vertical.
- Electric field can be determined by integrating over the charge distribution (try it yourself). It is not too surprising that the field is the same at all distances above the plane.

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{k}} \text{ (above)}$$

$$\mathbf{E} = -\frac{\sigma}{2\varepsilon_0}\hat{\mathbf{k}} \text{ (below)}$$

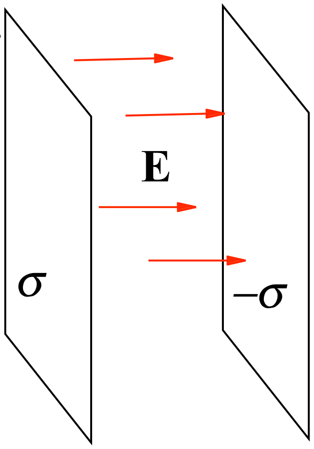


- The change in the electric field going from below to above

$$\Delta E = \frac{\sigma}{\varepsilon_0}$$

Parallel charge sheets

- Two infinite sheets of charge are separated by a constant distance d. One sheet has a charge density +σ and the other a charge density −σ.
  - Outside, the electric fields point in opposite directions
  - Between the sheets the electric fields point in the same direction.



$$\mathbf{E}_{outside} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{i}} + \frac{\sigma}{2\varepsilon_0}(-\hat{\mathbf{i}}) = 0$$

Outside plates field is zero

$$\mathbf{E}_{inside} = \frac{\sigma}{\varepsilon_0}\hat{\mathbf{i}}$$

Field between the plates

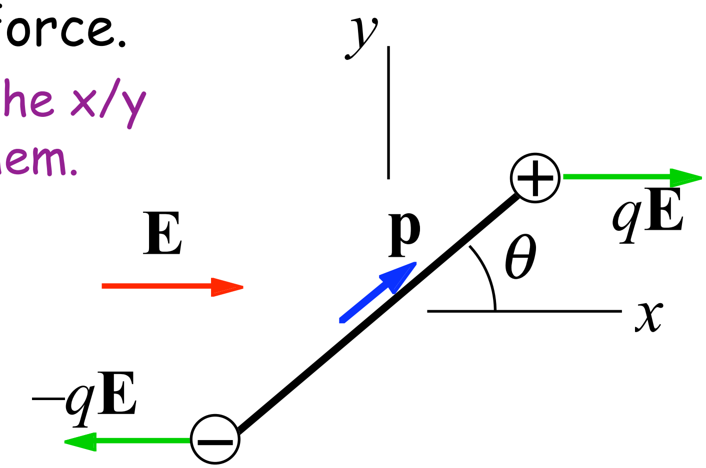
- Uniform electric field E, applies a constant force on a small particle with charge q and mass m.

$$\mathbf{F} = q\mathbf{E} \;\text{ and } \mathbf{a} = \frac{\mathbf{F}}{m} = \frac{q}{m}\mathbf{E}$$

Torque on a small electric dipole

- An electric dipole **p** in a uniform electric field **E** experiences a net torque $\mathbf{N}$ and no net force.
  - Choose coordinates where **p** and **E** lie in the x/y plane. **p** and **E** have an angle θ between them.

$$\mathbf{N} = \mathbf{p} \times \mathbf{E} = pE\sin\theta(-\hat{\mathbf{k}})$$



- In addition to a torque, an electric field **E** with a divergence (spatial changes) will generate, a net force **F** on an electric dipole, **p** :

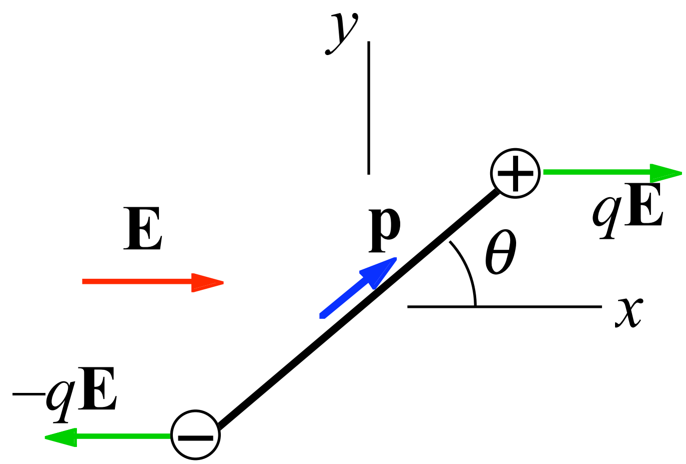
Cartesian coordinates

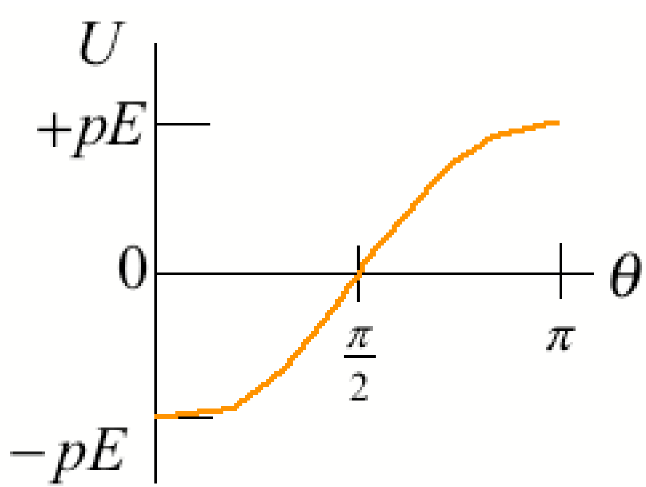
$$\mathbf{F} = p_i \frac{\partial E_j}{\partial x_i} \hat{\mathbf{e}}_j$$

The general expression needs operators to be covered later

Energy of dipole in electric field

- Potential energy *U* of the electric dipole **p** in uniform electric field **E**:

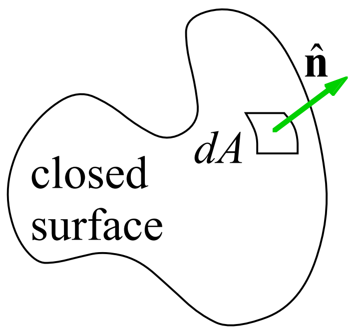




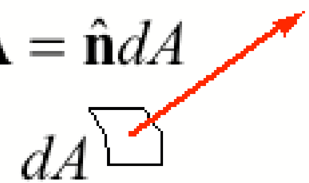
$-pE\cos\theta$

Gauss's Law

- Electric field passing through a closed (mathematical) surface
  - A surface enclosing NO net charge has a zero net field leaving or entering the surface.
  - A surface enclosing a positive (negative) charge has a net field leaving (entering) the surface proportional to the enclosed charge.



$$d\mathbf{A} = \hat{\mathbf{n}}\, dA$$



$$\int_S \mathbf{E} \cdot d\mathbf{A} = \frac{q_{encl}}{\varepsilon_0}$$

General expression for Gauss's Law

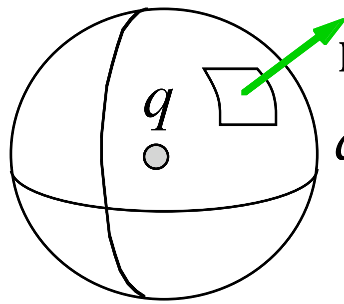
  - For symmetric charge distributions, pick an enclosing surface where **E** and *d***A** are everywhere parallel to each other.

Coulomb's Law <---> Gauss's Law

- For symmetric charge distributions, pick enclosing surfaces, so that **E** and *d***A** are are parallel to each other.
  - For a point charge at the origin, use a spherical surface, radius *R*, centered on the charge (makes direction of normal = radial)

Electric field at surface

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 R^2}\hat{\mathbf{r}}$$



$$\hat{\mathbf{n}} = \hat{\mathbf{r}}$$

$$dA = R^2 \sin\theta d\theta d\phi$$

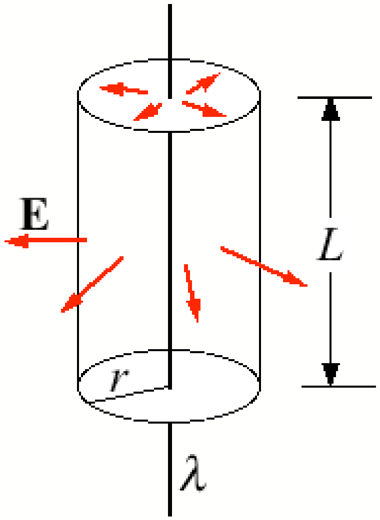
Evaluate Gauss's Integral

$$\int_S \mathbf{E}\cdot d\mathbf{A} = \frac{q}{4\pi\varepsilon_0 R^2}\int_0^{\pi} R^2 \sin\theta d\theta \int_0^{2\pi} d\phi = \frac{q}{\varepsilon_0}$$

- This is a "proof" that Gauss's law follows directly from the Coulomb Force Law for point charges, and their derived electric fields.

Field of a line of charge - use Gauss's Law

- Consider an infinitely long line of charge with linear charge density λ, and a cylindrical gaussian surface.
  - The electric field is parallel to the surface at the top and bottom of the cylinder, **E**•d**A** is zero.
  - The electric field is perpendicular to the surface and therefore parallel to the surface normal.



$$q_{encl} = \lambda L$$

$$\int_S \mathbf{E}\cdot d\mathbf{A} = Er\int_0^{2\pi} d\phi \int_0^{L} dz$$

$$E2\pi rL = \frac{q_{encl}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}; \qquad \mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}$$