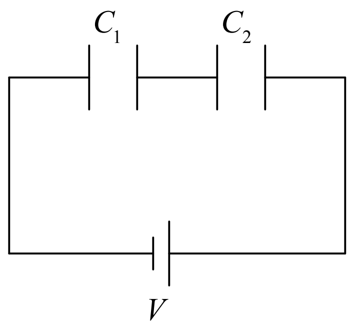
PHY481: Electromagnetism

Exam 2

Constant currents

Exam problem 2

Capacitors C_1, and C_2 are attached to a battery, potential V, as shown.



a) What are the $Q_1 \,\&\, Q_2,\ V_1 \,\&\, V_2$

Charges are the same. Potentials add to Battery potential.

$$Q_1 = Q_2 = Q; \quad \frac{Q}{C_1} + \frac{Q}{C_2} = V$$

$$Q = CV; \quad C = \frac{C_1 C_2}{C_1 + C_2}$$

b) Double spacing of C_2.

Capacitance is related to the geometry of capacitors. New charge and potential as in a)

$$E_i = \sigma_i / \varepsilon_0 = Q_i / A\varepsilon_0 = V_i / d_i$$

$$Q_i = \varepsilon_0 A_i V_i / d_i = C_i V_i; \quad C_i = \varepsilon_0 A_i / d_i$$

$$C_2' = \varepsilon_0 A_2 / d_2' = \varepsilon_0 A_2 / 2d_2 = C_2 / 2$$

$$Q' = C'V = \frac{C_1 C_2'}{C_1 + C_2'} V = \frac{C_1 C_2}{2C_1 + C_2} V; \quad C' = \frac{C_1 C_2}{2C_1 + C_2}; \quad V_1' = \frac{Q'}{C_1}; \quad V_2' = \frac{2Q'}{C_2}$$

c) What happens to the stored energy?

$$U = \frac{1}{2} C V^2; \quad U' = \frac{1}{2} C' V^2$$

$$\frac{\Delta U}{U} = \frac{C'}{C} - 1 = \frac{-C_2}{2C_2 + C_1} = -\frac{1}{2 + C_1/C_2}$$

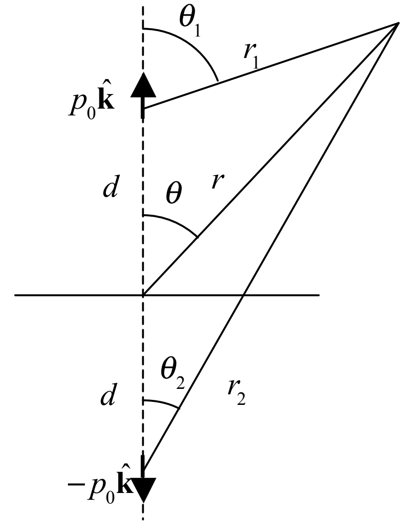
$$\frac{\Delta U}{U} = -\frac{1}{2 + C_1/C_2}$$

Other exam problems

- Problem 3: M.o.I, see S & P, Ch. 4, p. 103 - 104
- Problem 4. Fourier: see S & P, Ch. 5, Ex. 1, p. 138 - 141
- Problem 5.
  - 1 & 2 dipoles
  - Geometry

$$V_i(r_i,\theta_i) = \frac{p_i \cos\theta_i}{4\pi\varepsilon_0 r_i^2}$$

$$V(r,\theta) = \frac{p_0}{4\pi\varepsilon_0}\left[\frac{\cos\theta_1}{r_1^2} - \frac{\cos\theta_1}{r_2^2}\right]$$



$$r_i = (r^2 + d^2 \mp 2rd\cos\theta)^{1/2}; \quad \cos\theta_i = \frac{\left(r\cos\theta \mp d\right)}{(r^2 + d^2 \mp 2rd\cos\theta)^{1/2}}$$

  - Substitution

$$V(r,\theta) = \frac{p_0}{4\pi\varepsilon_0}\left[\frac{\cos\theta_1}{r_1^2} - \frac{\cos\theta_1}{r_2^2}\right] = \frac{p_0}{4\pi\varepsilon_0 r^2}\left[\frac{\left(\cos\theta - d/r\right)}{(1+\frac{d^2}{r^2} - 2\frac{d}{r}\cos\theta)^{3/2}} - \frac{\left(\cos\theta + d/r\right)}{(1+\frac{d^2}{r^2} + 2\frac{d}{r}\cos\theta)^{3/2}}\right]$$

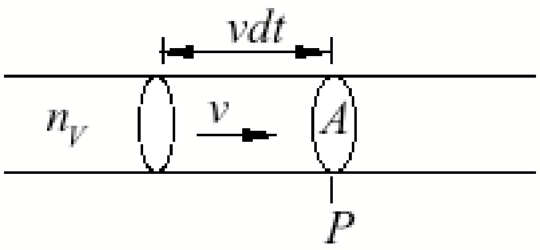
  - For $r \gg d$, ignore terms r^2/d^2 and higher

$$V(r,\theta) \approx \frac{p_0}{4\pi\varepsilon_0 r^2}\left[\left(\cos\theta - d/r\right)\left(1 + 3\frac{d}{r}\cos\theta + \ldots\right) - \left(\cos\theta + d/r\right)\left(1 - 3\frac{d}{r}\cos\theta + \ldots\right)\right]$$

$$V(r,\theta) \approx \frac{p_0}{\pi\varepsilon_0 r^2}\frac{d}{r}\frac{\left(3\cos^2\theta - 1\right)}{2} = Cr^{-3}P_2(\cos\theta), \quad C = \frac{p_0 d}{\pi\varepsilon_0}$$

Ch. 7 Electric Currents: charge carriers

$n_V =$ no. of charge carriers per unit volume



Charge passing P per unit time

$$dQ = qn_V A v dt$$

$$I = \frac{dQ}{dt} = qn_V A v$$

$n_L =$ no. of charge carriers per unit length

$$I = qn_L v \qquad v = v_{drift}$$

Examples

Cu, atomic mass 63, mass density 9 g/cm^3.
Assume 1 charge carrier (e.g, an electron) per Cu atom.
How many charge carriers in 1 cm^3 of Cu ?

How big is a Coulomb?

$$q = 1.6 \times 10^{-19}\ \text{C}$$

$$n_V = \left(6 \times 10^{23} / 63\ \text{g}\right)\left(9\ \text{g/cm}^3\right) = 9 \times 10^{22} / \text{cm}^3$$

right answer!
(but rather lucky)

Wire 1 mm^2 area. How long a wire has 1 Coulomb of charge carriers?

$$Q = \left(1.6 \times 10^{-19} \text{C}\right)\left(9 \times 10^{22}\ \text{cm}^{-3}\right)\left(1 \times 10^{-2}\ \text{cm}^2\right) L \qquad \Rightarrow \quad L = 0.07\ \text{mm}$$

The wire carries a current of 1 A (1 C/s). What is the speed of the carriers?

$$v_{drift} = 0.07\ \text{mm/s} \qquad v_{drift} = 1\ \text{mm/s}\ \ (15\ \text{A})$$

Lights turn on immediately, why?

Origin of Ohm's law

Stationary charges ---> electrostatics
Constant current ---> magnetostatics

What is the origin of Ohm's law, $V = IR$?

Key is thermal velocity v_{th} of electrons is >> than 1 mm/s.

$\begin{pmatrix}\text{Classical calculations are} \\ \text{within } \pm 1 \text{ order of mag.}\end{pmatrix}$

Time between collisions τ involves mean the free path $\langle \lambda \rangle$

$\begin{pmatrix}\text{QM description} \\ \text{required here}\end{pmatrix}$

$$\tau = \langle \lambda \rangle / v_{th}$$

Electron drift velocity is then

$$v_{drift} = a\tau = \frac{qE}{m}\frac{\langle \lambda \rangle}{v_{th}} = \frac{qV}{mL}\frac{\langle \lambda \rangle}{v_{th}}$$

Current from previous slide

$$I = qn_L v_{drift} = \frac{q^2 n_L}{mL}\frac{\langle \lambda \rangle}{v_{th}} V \qquad I = \frac{V}{R}$$

Resistance per unit length

$$\frac{R}{L} = \frac{mv_{th}}{q^2 n_L \langle \lambda \rangle}$$

Resistivity ρ_R

$$\rho_R = \frac{RA}{L}$$

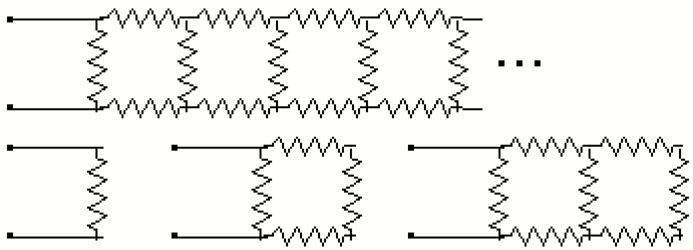
Conductivity σ_R (beware, σ also surface Q density)

$$\sigma_R = \frac{L}{RA} = \frac{n_L}{A}\frac{q^2 \langle \lambda \rangle}{mv_{th}}$$

$$\sigma_R = \frac{n_V q^2 \langle \lambda \rangle}{mv_{th}}$$

Resistors

Each resistor has the value R. What is the total resistance, R_T, of this infinite set of resistors?

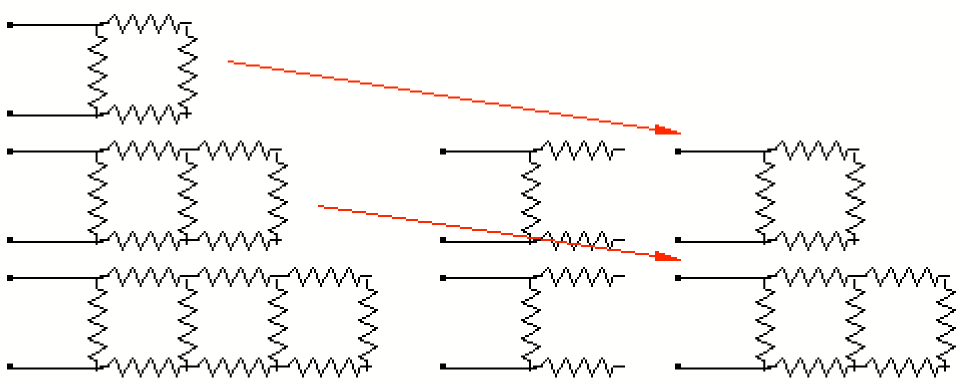


One resistor: $R_T = R$

Four resistors: $R_T = R+$ (3R in parallel)

N resistors: $R_T =$ not obvious

The trick!



Current and current densities

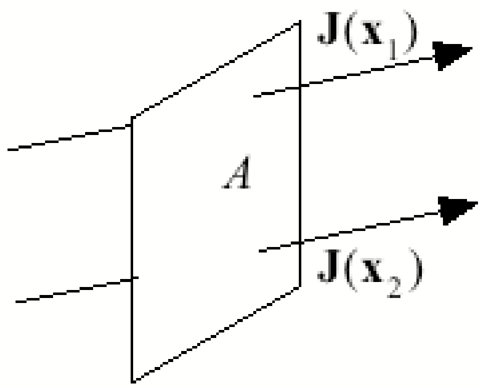
No charge can accumulate in a uniform wire carrying current. Charge going in = charge going out.

$\rho_c(\mathbf{x}) = \text{charge density}$ $\qquad \dfrac{\partial \rho_c}{\partial t} = 0$

Beware: ρ is also used for resistivity of a material.

Volume current density $\mathbf{J}(\mathbf{x}) = n_V q \mathbf{v}(\mathbf{x})$ (3 D)

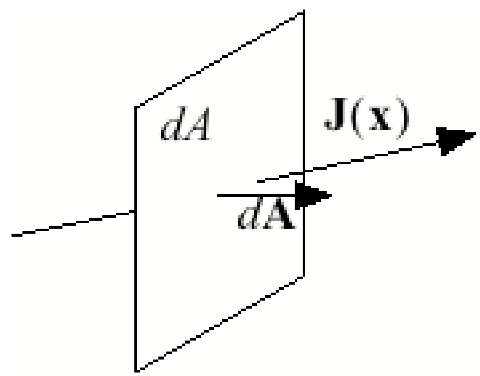
Current density **J** is position dependent



Current is charge through A per unit time

$I = dQ/dt$ (1 D)

Local current density **J** for a differential area element

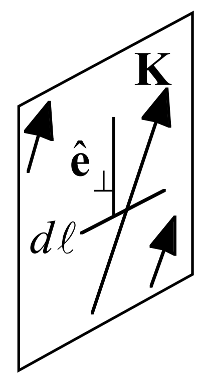


Current through *dA*

$dI = \mathbf{J}(\mathbf{x}) \cdot d\mathbf{A}$ (1 D)

$\mathbf{K}(\mathbf{x}) = n_S q \mathbf{v}(\mathbf{x})$ (2 D)

Surface current **K**



Current crossing $d\ell$

$dI = \mathbf{K} \cdot \hat{\mathbf{e}}_\perp d\ell$ (1 D)

Continuity equation -- conservation of charge

Continuity equation: $$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_c}{\partial t}$$

Remember that ρ_c and **J** are functions of 3D position

$$\rho_c(\mathbf{x}) \;\; \& \;\; \mathbf{J}(\mathbf{x})$$

Integrate over volume V, and use Gauss's theorem

$$\int_V \nabla \cdot \mathbf{J}\, d^3x = -\int_V \frac{\partial \rho_c}{\partial t}\, d^3x$$

Rate of change of charge inside of V

Continuity equation, integral form.

Flux of **J** through surface S

$$\int_S \mathbf{J} \cdot d\mathbf{A} = -\frac{d}{dt}\int_V \rho_c d^3x$$

Rate of change of charge inside of V

Boundary condition on **J** at a surface

Discontinuity in normal component of **J** at a surface

$$J_{2n} - J_{1n} = -\frac{\partial \sigma_c}{\partial t}$$

Rate of change of surface charge density

$\frac{\partial \rho_c}{\partial t} = 0$ and $\frac{\partial \sigma_c}{\partial t} = 0$ with constant currents

Reminiscent of $$E_{2n} - E_{1n} = \frac{\sigma_c}{\varepsilon_0}$$