PHY481: Electromagnetism

Constant currents and magnetic fields

Magnetic force and field

Magnetic force: $\mathbf{F} = q\mathbf{v}\times\mathbf{B}$

Perpendicular to direction of motion
Magnetic forces do no work $\Delta KE = 0$

Circular motion: $\mathbf{v}_0 \perp \mathbf{B}$

Centripetal acceleration caused by magnetic force. $\omega = v/R = qB/m$

$\mathbf{v}_0 \not\perp \mathbf{B}$

Helical motion. Helix has radius and pitch

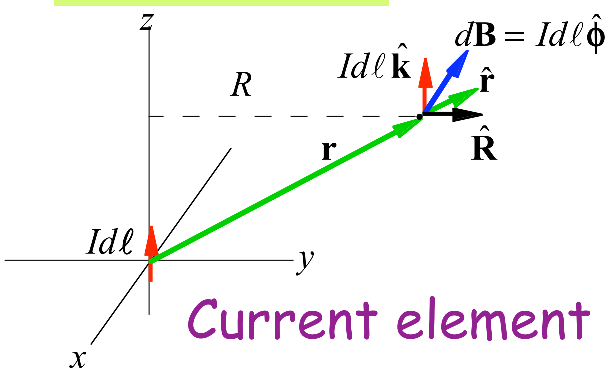
Crossed **E** and **B**: $\mathbf{E} \perp \mathbf{B}$

Exists an un-deflected velocity $\mathbf{v} = \mathbf{E}\times\mathbf{B}/B^2$

Currents make magnetic fields: Biot-Savart Law

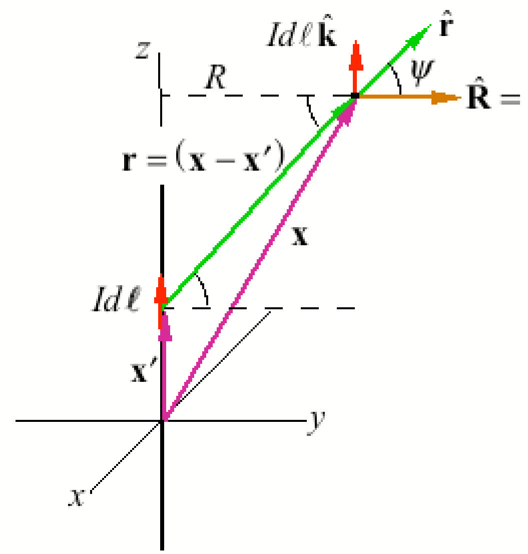
$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Id\boldsymbol{\ell}\times\hat{\mathbf{r}}}{r^2}$$

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int_{wire}\frac{Id\boldsymbol{\ell}\times(\mathbf{x}-}{|\mathbf{x}-\mathbf{x}'|^3}$$



Current element

Wire carryi
current



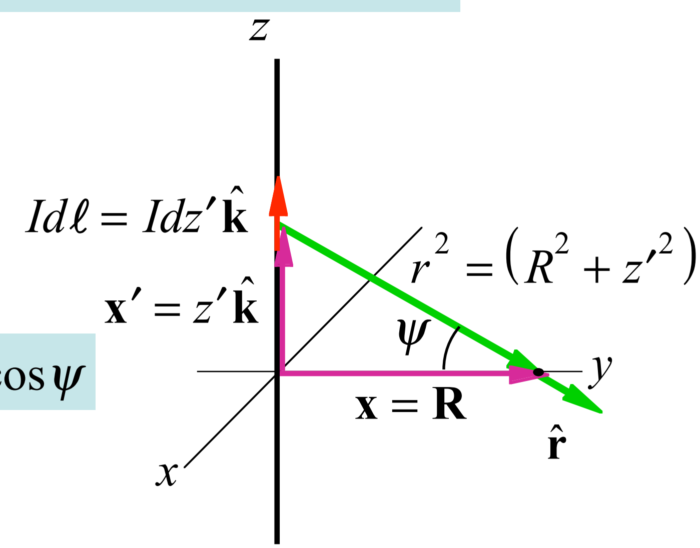
Magnetic field of long wire carrying current

(the hard way)

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int_{wire} \frac{I d\boldsymbol{\ell}\times(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}$$

$$\mathbf{x} = R\hat{\mathbf{R}} + z\hat{\mathbf{k}} \qquad \mathbf{x}' = z'\hat{\mathbf{k}} \qquad d\boldsymbol{\ell} = dz'\hat{\mathbf{k}}$$

Magnetic field does not depend on z.
Any z will do. Pick z = 0 to simplify



$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0 I}{4\pi}\int_{-\infty}^{\infty} \frac{dz'\,\hat{\mathbf{k}}\times\hat{\mathbf{r}}}{(R^2+z'^2)}$$

$$\hat{\mathbf{r}} = \hat{\mathbf{R}}\cos\psi - \hat{\mathbf{k}}\sin\psi$$

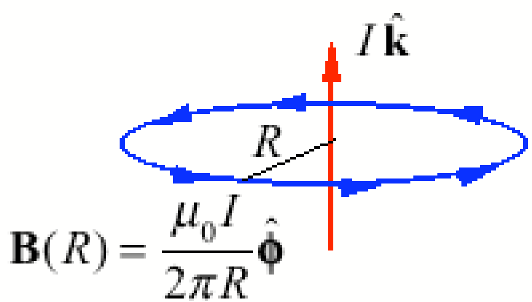
$$\hat{\mathbf{k}}\times\hat{\mathbf{r}} = \hat{\mathbf{k}}\times\hat{\mathbf{R}}\cos\psi = \hat{\boldsymbol{\phi}}\cos\psi$$

$$\cos\psi = R/(R^2+z'^2)^{1/2}$$

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0 IR}{4\pi}\int_{-\infty}^{\infty} \frac{dz'}{(R^2+z'^2)^{3/2}}\hat{\boldsymbol{\phi}}$$

$$\int_{-\infty}^{\infty}\frac{dz'}{(R^2+z'^2)^{3/2}} = \left[\frac{z'}{R^2\sqrt{R^2+z'^2}}\right]_{-\infty}^{\infty} = \frac{2}{R^2}$$

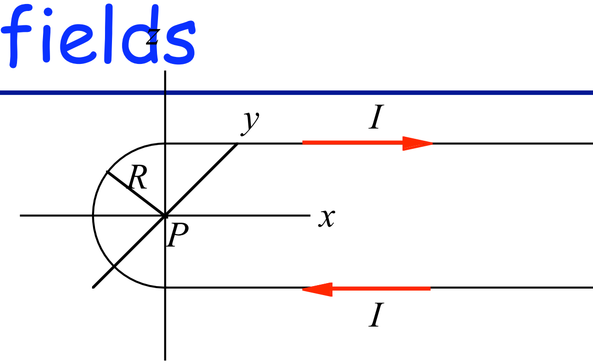
$$\mathbf{B}(R) = \frac{\mu_0 I}{2\pi R}\hat{\boldsymbol{\phi}}$$



Magnetic field

More magnetic fields

Field at center of a narrow loop
= 2 straight wires + 1/2 ring

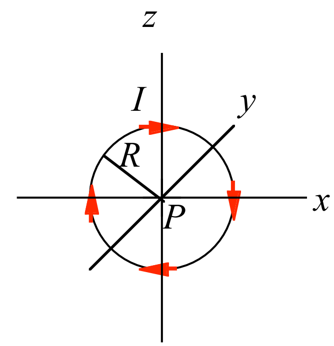


Field of long straight wire

$$\mathbf{B} = \frac{\mu_0 I}{2\pi R}\hat{\phi}$$

end of long wire

$$\mathbf{B} = \frac{\mu_0 I}{4\pi R}\hat{\phi}$$



Field at center of a current ring

$$\mathbf{B} = \int d\mathbf{B} = \frac{\mu_0}{4\pi}\int \frac{I d\ell \times (-R\hat{\mathbf{r}})}{R^3}$$

$$= \hat{\mathbf{j}}\frac{\mu_0 I}{4\pi R^2}\int_0^{2\pi} R d\phi = \hat{\mathbf{j}}\frac{\mu_0 I}{2R}$$

$-R\hat{\mathbf{r}}$ because $\mathbf{r}$ points from d$\mathbf{l}$ to P

Field at center of 1/2 a current ring

$$\mathbf{B} = \frac{\mu_0 I}{4R}$$

Field at center of narrow loop

$$\mathbf{B} = \hat{\mathbf{j}}\frac{\mu_0 I}{2R}\left(\frac{1}{2} + \frac{1}{\pi}\right) \qquad \mathbf{B} = \hat{\mathbf{j}}\frac{\mu_0 I}{4\pi R}(\pi + 2)$$

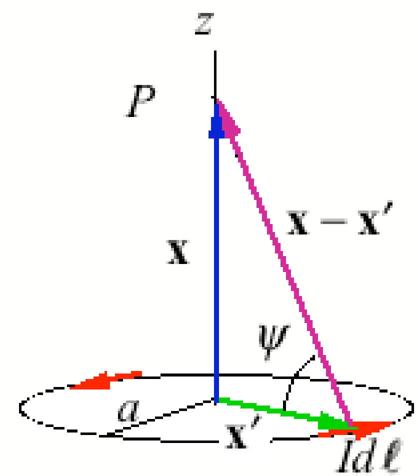
More magnetic fields

Field on axis of current loop

$$\mathbf{x} = z\hat{\mathbf{k}}; \quad \mathbf{x}' = R\hat{\mathbf{R}}; \quad Id\ell = ad\phi\hat{\boldsymbol{\phi}}$$

$$\mathbf{r} = \mathbf{x} - \mathbf{x}' = z\hat{\mathbf{k}} - a\hat{\mathbf{R}}$$

$$r\hat{\mathbf{r}} = \sqrt{a^2 + z^2}\,(\hat{\mathbf{k}}\sin\psi - \hat{\mathbf{R}}\cos\psi)$$



$$\hat{\boldsymbol{\phi}} \times \hat{\mathbf{r}} = \hat{\boldsymbol{\phi}} \times \hat{\mathbf{k}}\sin\psi - \hat{\boldsymbol{\phi}} \times \hat{\mathbf{R}}\cos\psi$$
$$= +\hat{\mathbf{R}}\sin\psi + \hat{\mathbf{k}}\cos\psi$$
$$\cos\psi = a\Big/\sqrt{a^2 + z^2}$$

Component in **R** direction will cancel in the integral over phi

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int \frac{Id\ell \times (\mathbf{x} - \mathbf{x}')\, d^3x'}{|\mathbf{x} - \mathbf{x}'|^3}$$

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0 I a^2}{4\pi}\int \frac{d\phi\,\hat{\mathbf{k}}}{\left(a^2 + z^2\right)^{3/2}}$$

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0 I a^2}{2\left(a^2 + z^2\right)^{3/2}}\hat{\mathbf{k}}$$

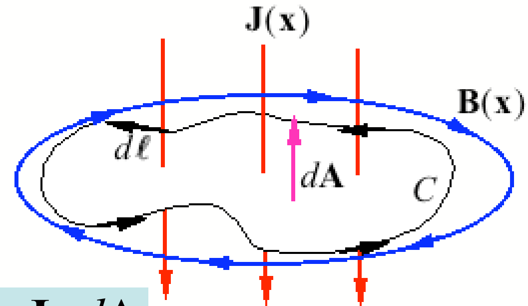
Ampere's Law

$\nabla \cdot \mathbf{B} = 0$ Magnetic fields are "solenoidal" (always true)

$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ Ampere's Law (constant currents only.
Maxwell's equations have another term

Integrate over a surface element:

$$\oint_S (\nabla \times \mathbf{B}) \cdot \mathbf{n}\, dA = \mu_0 \oint_S \mathbf{J} \cdot d\mathbf{A}$$



Apply Stokes's theorem:

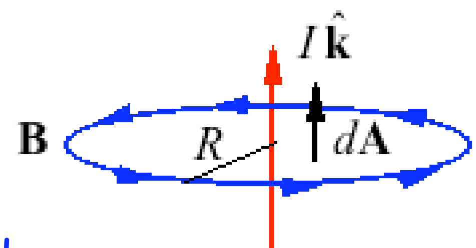
$$\oint_C \mathbf{B} \cdot d\ell = \mu_0 \oint_S \mathbf{J} \cdot d\mathbf{A}$$

$$dI = \mathbf{J} \cdot d\mathbf{A}$$

$$\oint_C \mathbf{B} \cdot d\ell = \mu_0 I_{encl}$$

Ampere's Law and symmetries

Straight wire (the easy way)



$$\oint_C \mathbf{B} \cdot d\ell = \mu_0 I_{encl}$$

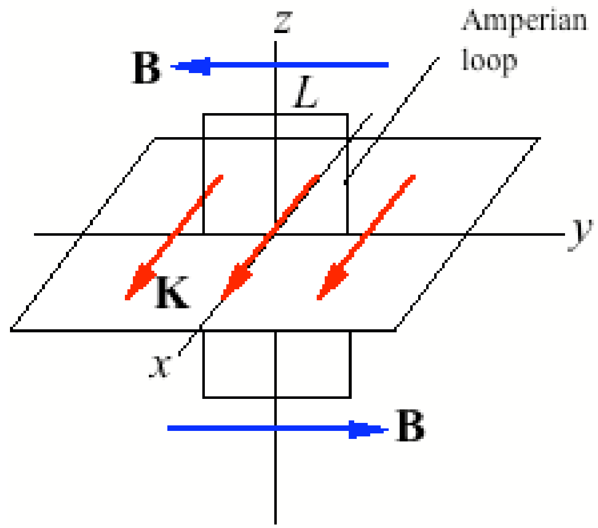
$$B_\phi 2\pi R = \mu_0 I$$

$$\mathbf{B}(R) = \frac{\mu_0 I}{2\pi R}\hat{\boldsymbol{\phi}}$$

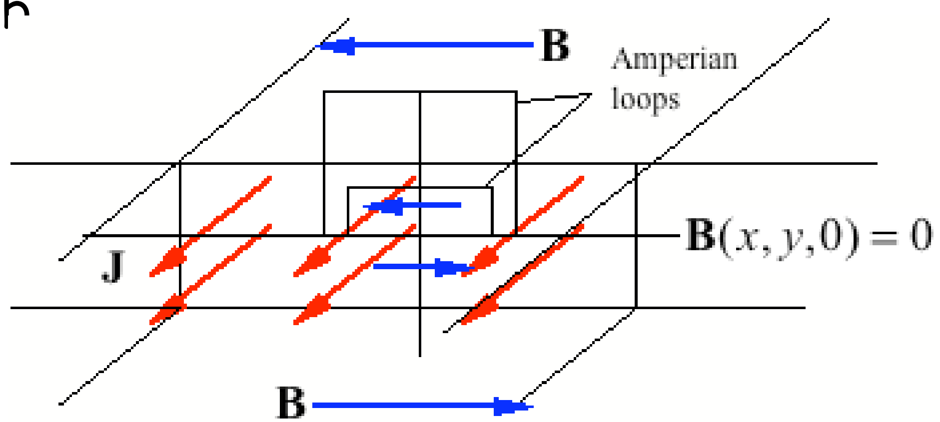
Right hand ru

More Ampere's law applications

Infinite current sheet



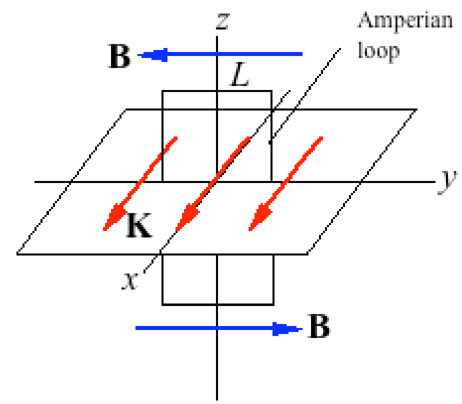
Infinite current slab



Ampere's law applications

Each line of current creates a B field in −y direction above the plane and +y direction below.

Infinite current sheet



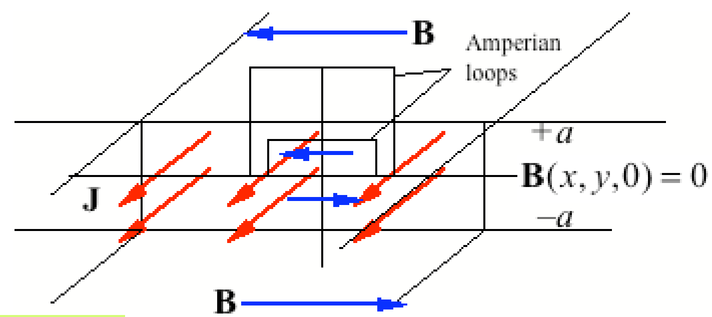
$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$ $\oint \mathbf{B} \cdot d\ell = \mu_0 I_{encl}$ $I_{encl} = \mathbf{K} \cdot \hat{\mathbf{i}} L = KL$

$$2BL = \mu_0 KL; \quad B = \frac{\mu_0 K}{2}$$

$$\mathbf{B} = -\frac{\mu_0 K}{2}\hat{\mathbf{j}} \quad z > 0; \quad \mathbf{B} = +\frac{\mu_0 K}{2}\hat{\mathbf{j}} \quad z > 0$$

B field on the center line must be zero

Infinite current slab



$$\oint \mathbf{B} \cdot d\ell = \mu_0 I_{encl}$$

Inside

$I_{encl} = \int \mathbf{J} \cdot d\mathbf{A} = JLz$

$\oint \mathbf{B} \cdot d\ell = (-B\hat{\mathbf{j}})(-L\hat{\mathbf{j}}) = BL$

$\mathbf{B}_{top} = -\mu_0 Jz\,\hat{\mathbf{j}}$ $\mathbf{B}_{bottom} = \mu_0 Jz\,\hat{\mathbf{j}}$

Outside

$I_{encl} = \int \mathbf{J} \cdot d\mathbf{A} = JLa$

$\oint \mathbf{B} \cdot d\ell = BL$

$\mathbf{B}_{top} = -\mu_0 Ja\,\hat{\mathbf{j}}$ $\mathbf{B}_{bottom} = \mu_0 Ja\,\hat{\mathbf{j}}$