PHY481: Electromagnetism

Student Instructional Rating of Faculty

Homework 5

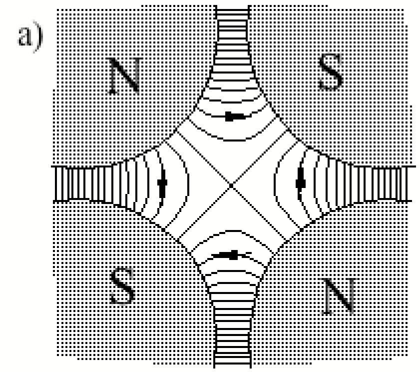
What do I need to study for the final exam?

HW 8

Problem 8.4

Positive particles headed out of slide, on the left or right, are bent *inward* (focused).

Positive particles headed out of slide, on the top or bottom, are bent *outward* (defocused).



$\mathbf{F} = q\mathbf{v}_0 \times \mathbf{B}; \quad \mathbf{B} = b\left(y\,\hat{\mathbf{i}} + x\,\hat{\mathbf{j}}\right)$ $\qquad \mathbf{v}_0 = v_0\hat{\mathbf{k}} = v_3\hat{\mathbf{k}}$

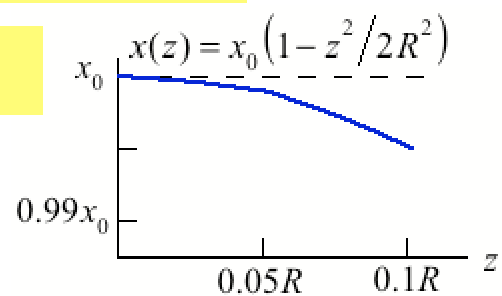
Newton's 3rd law: $m\ddot{\mathbf{x}} = \mathbf{F}; \quad m\ddot{x}_i = q\varepsilon_{i3k}v_3B_k$ $\qquad \ddot{x} = -\frac{q}{m}v_0B_y = -\frac{qv_0b}{m}x$

Solution: $x(t) = C\cos(\omega t + \phi); \quad \omega^2 = qv_0b/m$

Boundary conditions: $x(0) = x_0 = C; \quad \dot{x}(0) = 0 = -\omega C\sin\phi \quad \rightarrow \quad \phi = 0$

With $\omega t << 1$ $\qquad t \approx z/v_0$ $\qquad x(t) = C\cos(\omega t) \approx C\left(1 - \omega^2t^2/2\right)$

Trajectory: $x(z) \approx x_0\left(1 - z^2/2R^2\right)$ $\qquad R = \sqrt{mv_0/qb}$

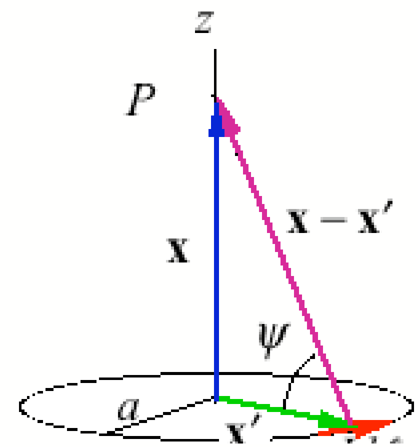


HW 8 (cont'd)

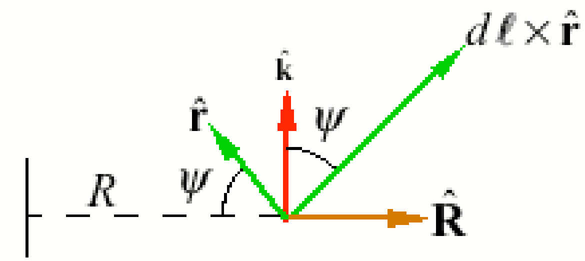
Problem 8.10

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{x}')\times\hat{\mathbf{r}}}{r^2}d^3x' = \frac{\mu_0 I}{4\pi}\int\frac{d\ell\times\hat{\mathbf{r}}}{r^2}$$

$$d\ell = a\,d\phi\,\hat{\boldsymbol{\phi}}$$



$$= \frac{\mu_0 I a}{4\pi\left(a^2+z^2\right)}\int d\phi\,\hat{\boldsymbol{\phi}}\times\hat{\mathbf{r}}$$



$$= \frac{\mu_0 I a}{4\pi\left(a^2+z^2\right)}\int d\phi\left(\hat{\mathbf{k}}\cos\psi + \hat{\mathbf{R}}\sin\psi\right)$$

$$\int_0^{2\pi} d\phi\,\hat{\mathbf{R}} = 0$$

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0 I a^2}{2\left(a^2+z^2\right)^{3/2}}\hat{\mathbf{k}}$$

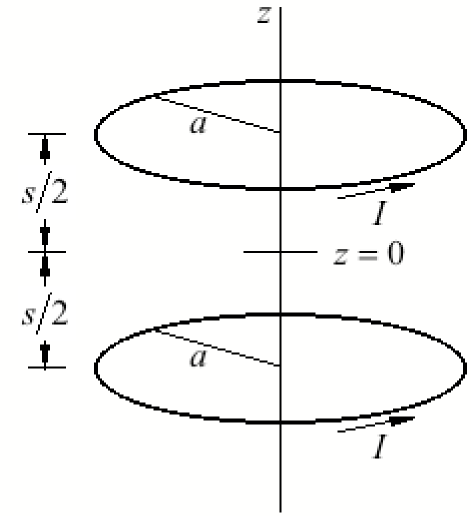
$$\hat{\mathbf{R}} = \hat{\mathbf{i}}\cos\phi + \hat{\mathbf{j}}\sin\phi$$

$$\cos\psi = a\Big/\left(a^2+z^2\right)^{1/2}$$

HW 8 (cont'd)

Problem 8.10 (cont'd)

$$B_z(z) = \frac{\mu_0 I a^2}{2\left[a^2 + (z+s/2)^2\right]^{3/2}} + \frac{\mu_0 I a^2}{2\left[a^2 + (z-s/2)^2\right]^{3/2}}$$



$$\frac{dB_z}{dz} = -\frac{3\mu_0 I a^2}{2}\left\{\frac{(z+s/2)}{\left[a^2 + (z+s/2)^2\right]^{5/2}} + \frac{(z-s/2)}{\left[a^2 + (z-s/2)^2\right]^{5/2}}\right\}$$

$$\frac{dB_z(0)}{dz} = 0$$

$$\frac{d^2 B_z}{dz^2} = -\frac{3\mu_0 I a^2}{2}\left\{\begin{array}{c}\dfrac{1}{\left[a^2 + (z+s/2)^2\right]^{5/2}} + \dfrac{1}{\left[a^2 + (z-s/2)^2\right]^{5/2}} \\ -\dfrac{5(z+s/2)^2}{\left[a^2 + (z+s/2)^2\right]^{7/2}} - \dfrac{5(z-s/2)^2}{\left[a^2 + (z-s/2)^2\right]^{7/2}}\end{array}\right\}$$

$$0 = 1 - \frac{5(s/2)^2}{\left[a^2 + (s/2)^2\right]}$$

$$\frac{d^2 B_z(0)}{dz^2} = -3\mu_0 I a^2 \left\{\frac{1}{\left[a^2 + (s/2)^2\right]^{5/2}} - \frac{5(s/2)^2}{\left[a^2 + (s/2)^2\right]^{7/2}}\right\} = 0$$

$$s = a$$

HW 8 (cont'd)

Problem 8.14

$$\mathbf{B} = axy\,\hat{\mathbf{i}} + by^2\,\hat{\mathbf{j}}$$

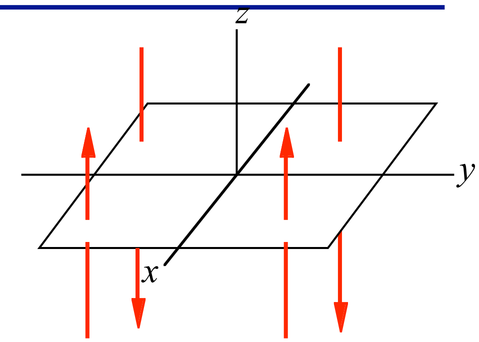
$$\nabla \cdot \mathbf{B} = 0 = ay + 2by$$

$$a = -2b; \quad \mathbf{B} = b\left(-2xy\,\hat{\mathbf{i}} + y^2\,\hat{\mathbf{j}}\right)$$

$$\nabla \times \mathbf{B} = 2bx\,\hat{\mathbf{k}} = \mu_0 \mathbf{J}$$

$$\mathbf{J} = 2bx / \mu_0\,\hat{\mathbf{k}}$$

y independent, changes sign ±x
zero in yz-plane



Problem 8.18

$$\int \mathbf{B} \cdot d\ell = \mu_0 I_{encl} = \mu_0 \int \mathbf{J} \cdot d\mathbf{A}$$

$$J_1 = I / \pi a^2$$

$$J_3 = -I / \pi\left(c^2 - b^2\right)$$

$$2\pi r B_1 = \mu_0 \frac{I}{\pi a^2} \pi r^2 \qquad B_1 = \frac{\mu_0 I r}{2\pi a^2}$$

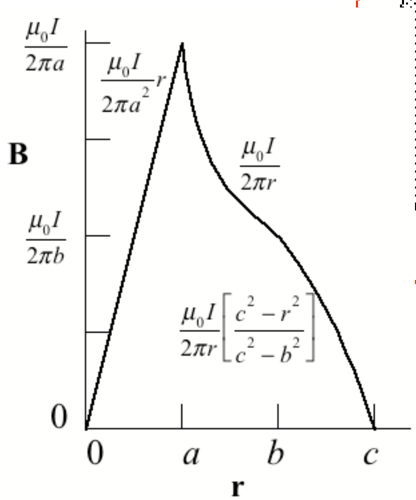
$$2\pi r B_2 = \mu_0 I \qquad B_2 = \frac{\mu_0 I}{2\pi r}$$

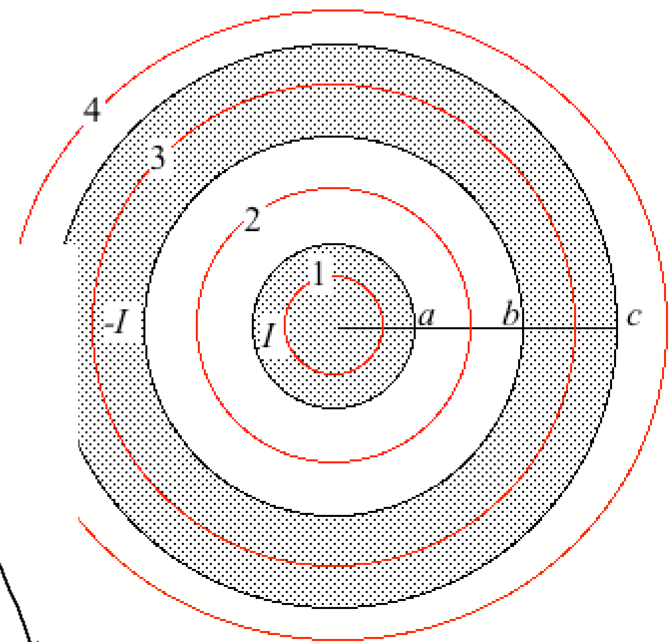
$$2\pi r B_3 = \mu_0 \left(I - \frac{I\left(r^2 - b^2\right)}{\left(c^2 - b^2\right)} \right)$$

$$B_3 = \frac{\mu_0 I}{2\pi r}\left[\frac{c^2 - r^2}{c^2 - b^2}\right]$$

$$2\pi r B_4 = 0$$

$$B_4 = 0$$





HW 8 (cont'd)

Problem 8.22

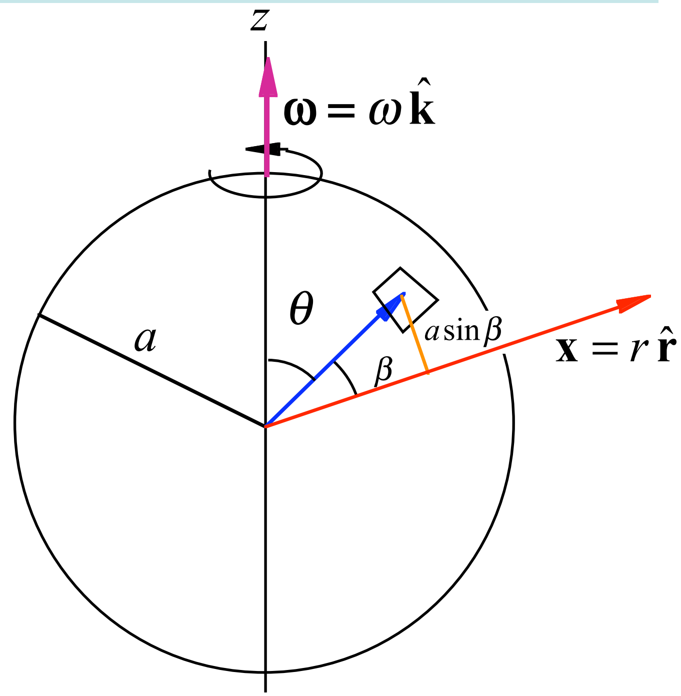
$\mathbf{A}_{in}$ & $\mathbf{A}_{out}$ given in text

$$\hat{\mathbf{k}}\times\hat{\mathbf{r}} = \sin\theta\,\hat{\boldsymbol{\phi}} \qquad \hat{\mathbf{k}}\times r\hat{\mathbf{r}} = r\sin\theta\,\hat{\boldsymbol{\phi}}$$

$$\nabla\times\left(r\sin\theta\,\hat{\boldsymbol{\phi}}\right) = \frac{\hat{\mathbf{r}}}{r\sin\theta}\frac{\partial}{\partial\theta}\left(r\sin^2\theta\right) - \frac{\hat{\boldsymbol{\theta}}}{r}\frac{\partial}{\partial r}\left(r^2\sin\theta\right)$$
$$= 2\left(\hat{\mathbf{r}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta\right) = 2\hat{\mathbf{k}}$$

$$\mathbf{B}_{in} = \nabla\times\mathbf{A}_{in} = \frac{\mu_0\sigma a\omega}{3}\nabla\times\left(\hat{\mathbf{k}}\times r\hat{\mathbf{r}}\right)$$

$$\mathbf{B}_{in} = \frac{2\mu_0\sigma a\omega}{3}\hat{\mathbf{k}}$$



$$\mathbf{B}_{out} = \nabla\times\mathbf{A}_{out} = \frac{\mu_0\sigma a^4\omega}{3}\nabla\times\left(\hat{\mathbf{k}}\times\frac{\hat{\mathbf{r}}}{r^2}\right)$$
$$= \frac{\mu_0\sigma a^4\omega}{3}\nabla\times\left(\hat{\mathbf{k}}\times\frac{\hat{\mathbf{r}}}{r^2}\right) = \frac{\mu_0\sigma a^4\omega}{3}\nabla\times\left(\frac{\sin\theta}{r^2}\hat{\boldsymbol{\phi}}\right)$$

$$\mathbf{B}_{out} = \frac{\mu_0\sigma a^4\omega}{3r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right)$$

$$\mathbf{m} = m\hat{\mathbf{k}}$$

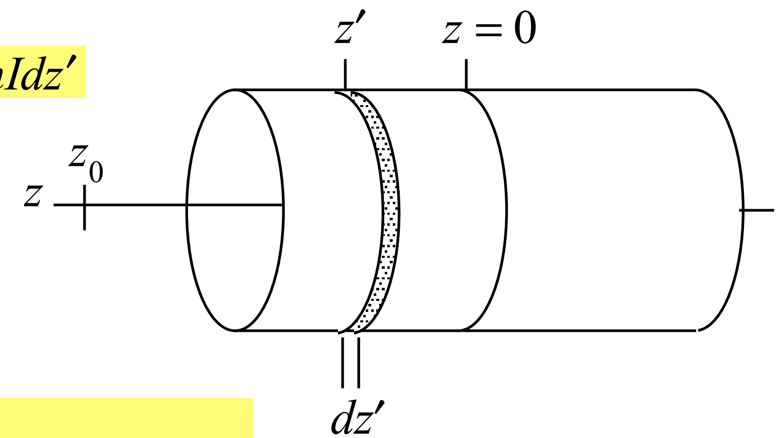
$$\mathbf{B}_{dipole} = \frac{\mu_0}{4\pi r^3}\left(3\hat{\mathbf{r}}\left(\mathbf{m}\cdot\hat{\mathbf{r}}\right) - \mathbf{m}\right) = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right)$$

$$m = \frac{4\pi a^4\sigma\omega}{3}$$

HW 8 (cont'd)

Problem 8.34

Using result of Problem 8.10 and $dI = nI dz'$



$$dB_z(0,0,z_0) = \frac{\mu_0 a^2 nI dz'}{2\left[a^2 + (z_0 - z')^2\right]^{3/2}}$$

$$B_z = \frac{\mu_0 a^2 nI}{2} \int_{-\ell/2}^{\ell/2} \frac{dz'}{\left[a^2 + (z_0 - z')^2\right]^{3/2}} \quad x = z_0 - z',\ dz' = -dx$$

$$= \frac{\mu_0 a^2 nI}{2} \int_{z_0-\ell/2}^{z_0+\ell/2} \frac{dx}{\left[a^2 + x^2\right]^{3/2}} = \frac{\mu_0 a^2 nI}{2}\left[\frac{x}{a^2\sqrt{a^2+x^2}}\right]_{z_0-\ell/2}^{z_0+\ell/2}$$

$$\mathbf{B} = \frac{\mu_0 a^2 nI\,\hat{\mathbf{k}}}{2}\left\{\frac{\ell/2 - z_0}{\left[a^2 + (\ell/2 - z_0)^2\right]^{1/2}} + \frac{\ell/2 + z_0}{\left[a^2 + (\ell/2 + z_0)^2\right]^{1/2}}\right\}$$

HW 8 (cont'd)

Problem 8.37

$$\mathbf{A}(\mathbf{x}) = -\frac{A_0 a}{r} e^{-r^2/a^2} \hat{\boldsymbol{\phi}}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = -\frac{2A_0}{a} e^{-r^2/a^2} \hat{\mathbf{k}}$$

$$\mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B} = -\frac{4A_0}{\mu_0 a^3} r e^{-r^2/a^2} \hat{\boldsymbol{\phi}}$$

$r < a$

To plot

$$\mathbf{B} = -\frac{2A_0}{a} e^{-x^2} \hat{\mathbf{k}}, \qquad x = r/a$$

$$\mathbf{J} = -\frac{4A_0}{\mu_0 a^2} x e^{-x^2} \hat{\boldsymbol{\phi}}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{2A_0}{a^2} r e^{-r^2/a^2} \hat{\boldsymbol{\phi}}$$

$$\mathbf{J} = \frac{1}{\mu_0} \nabla \times \mathbf{B} = \frac{4A_0}{\mu_0 a^4} \left(a^2 - r^2\right) e^{-r^2/a^2} \hat{\mathbf{k}}$$

$r > a$

To plot

$$\mathbf{B} = \frac{2A_0}{a} x e^{-x^2} \hat{\boldsymbol{\phi}}, \qquad x = r/a$$

$$\mathbf{J} = \frac{4A_0}{\mu_0 a^2} \left(1 - x^2\right) e^{-x^2} \hat{\mathbf{k}}$$

Final Exam

(Exam is on 12/13, 12:45-2:45, Rm 1279 Anthony)

What do I need to study for the final exam?

Homework, Exams, and Text-example type problems

- At this level in the study of E&M
  - Coordinate systems other than Cartesian cannot be avoided
  - Differential operators: gradient, divergence, curl, and Laplacian are part and parcel of E&M at this level
  - Solving Laplace's equation is a useful skill
  - Expansions with electric and magnetic dipole (& occasionally higher) terms appear often in physical problems
- Course failures are rare (unless there is evidence of giving up).
- Read the syllabus - missed HOMEWORK handed in before Exam.