Prof. Jack Baldwin
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Office hrs (BPS 3270):
3-4 Mon
2:30-3:30 Tu
4-5 Fri

OBAFGKM Winners

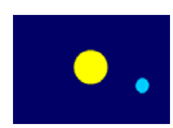
- 6 points will be added to clicker grade for contest winners; 3 points for all who submitted to the OBAFGKM contest.

- Old bread always forms green kelpy mold—J Atkinson
- Oh Beckham, a freaking great kick man—P Chen
- Only beer and food get Ken motivated—L Cooper
- Only bad acrobats forget gravity kills missers—S Greenberg
- On big and fat gumdrops kookaburra munches—A Jacobs
- Only bored astronomers find gratification knowing mnemonics—L Jones
- Our brilliant and friendly grandma knits mittens—E Keller
- Odorous breath and foul gas kill me—E Swanson
- Only Barbie's alibi fixed guilty Ken's mess—S Vanderploeg
- Obviously, boys are forgetful. Girls know more.—E Weadock
- Older boys and freshmen girls keep mingling—R Wilkerson

Homework 7 is now open on Angel. Due late at night Monday.

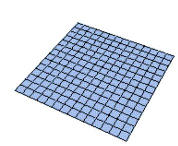
Motions under influence of gravity [13.3]

- Kepler
  - The planets move in ellipses with $P^2 = a^3$, etc.
  - Didn't know why.



- Newton
  - 3 laws of motion affect everything.
    - F = ma, etc.
  - Gravity = force = Gm_1m_2/r^2

- General Relativity
  - Gravity = distortion of space-time.



General relativity

- Worked out in 1907 - 1915
- Consistent with (incorporates) special relativity
- Describes motions of objects in presence of gravity
- Gravity = distortion into extra space-like dimensions.

[see Fig 13.11/13.12]

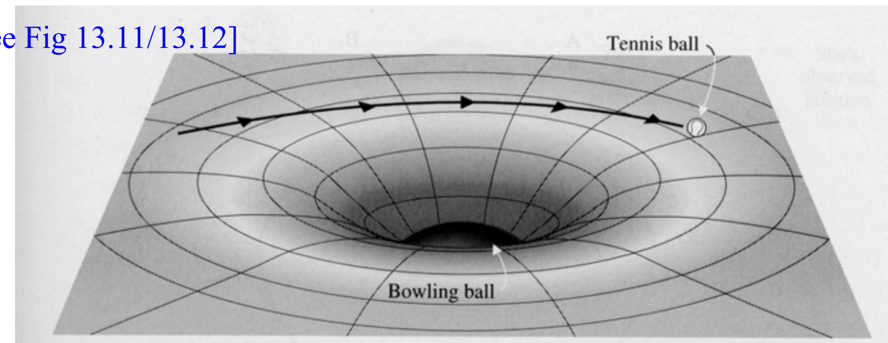


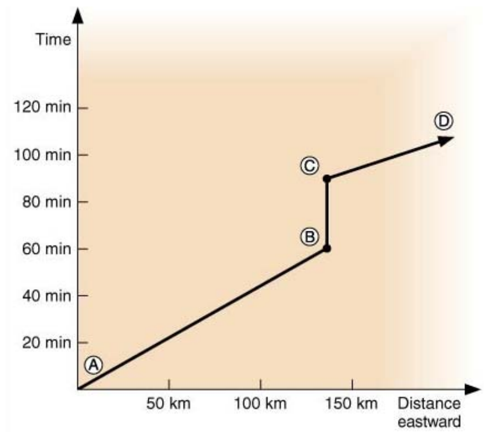
Figure 16.2 Rubber sheet analogy for curved space around the Sun.

How many dimensions do we live in?

- General Relativity
  - 3D space = “surface” in a 4D space
- Use easily visualized analogy
  - 2D surface in a 3D space
- Imagine a bug constrained to that 2D surface
  - Doesn’t know 3rd dimension exists.

Spacetime

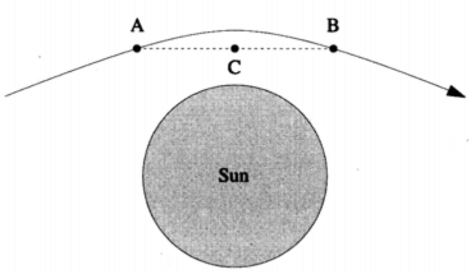
- Cross-talk between space & time.
  ➔ convenient to think of time as 4th dimension.
- But time is still different from space.
  - "space-like", "time-like" dimensions.
- Special Relativity:
  - 1 time-like, 3 space-like dimensions.
- General Relativity:
  - 1 time-like, 3 space-like dimensions = surface in 1+4D space-time.

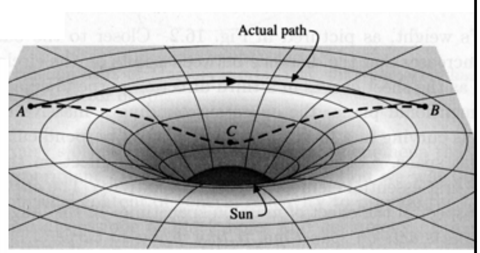


Gravity

- Everything finds shortest path in spacetime.
- Photons (light) find shortest path of all, because they move the fastest.

Photon follows solid line, not the dashed line through point C.





Dashed line is actually a longer path when curved space is taken into account.

The mathematical solution:

$$R_{\eta\eta} = -\frac{2a^2 \frac{\partial\psi}{\partial\theta}\cot\theta}{\delta\psi} + \frac{2ac\frac{\partial\psi}{\partial\eta}\cot\theta}{\delta\psi} + \frac{a\frac{\partial c}{\partial\eta}\cot\theta}{\delta} - \frac{\frac{\partial a}{\partial\eta}c\cot\theta}{2\delta} - \frac{a\frac{\partial a}{\partial\theta}\cot\theta}{2\delta} - \frac{2a^2\frac{\partial^2\psi}{\partial\theta^2}}{\delta\psi}$$

$$-\frac{2a^2(\frac{\partial\psi}{\partial\theta})^2}{\delta\psi^2} + \frac{4ac\frac{\partial\psi}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta\psi^2} - \frac{a^2\frac{\partial d}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta d\psi} + \frac{ac\frac{\partial d}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta d\psi} + \frac{2a\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta\psi} - \frac{\frac{\partial a}{\partial\eta}c\frac{\partial\psi}{\partial\theta}}{\delta\psi}$$

$$-\frac{3a\frac{\partial a}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta\psi} - \frac{2a^2c\frac{\partial c}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} + \frac{2a^2b\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{a^2\frac{\partial b}{\partial\eta}c\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{a\frac{\partial a}{\partial\eta}bc\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} + \frac{a^3\frac{\partial b}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta^2\psi}$$

$$+\frac{a^2\frac{\partial a}{\partial\theta}b\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{2ab\frac{\partial^2\psi}{\partial\eta^2}}{\delta\psi} - \frac{2\frac{\partial^2\psi}{\partial\eta^2}}{\psi} + \frac{4ac\frac{\partial^2\psi}{\partial\eta\partial\theta}}{\delta\psi} - \frac{2ab(\frac{\partial\psi}{\partial\eta})^2}{\delta\psi^2} + \frac{6(\frac{\partial\psi}{\partial\eta})^2}{\psi^2}$$

$$+\frac{ac\frac{\partial d}{\partial\theta}\frac{\partial\psi}{\partial\eta}}{\delta d\psi} - \frac{ab\frac{\partial d}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta d\psi} - \frac{2c\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta\psi} + \frac{\frac{\partial a}{\partial\theta}c\frac{\partial\psi}{\partial\eta}}{\delta\psi} - \frac{2a\frac{\partial b}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta\psi} + \frac{\frac{\partial a}{\partial\eta}b\frac{\partial\psi}{\partial\eta}}{\delta\psi}$$

$$+\frac{2a^2b\frac{\partial c}{\partial\theta}\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} - \frac{2abc\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} - \frac{a^2\frac{\partial b}{\partial\theta}c\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} - \frac{a\frac{\partial a}{\partial\theta}bc\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} + \frac{a^2b\frac{\partial b}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} + \frac{a\frac{\partial a}{\partial\eta}b^2\frac{\partial\psi}{\partial\eta}}{\delta^2\psi}$$

$$+\frac{a\frac{\partial c}{\partial\eta}\frac{\partial d}{\partial\theta}}{2\delta d} - \frac{\frac{\partial a}{\partial\eta}c\frac{\partial d}{\partial\theta}}{4\delta d} - \frac{a\frac{\partial a}{\partial\theta}\frac{\partial d}{\partial\theta}}{4\delta d} - \frac{\frac{\partial^2 d}{\partial\eta^2}}{2d} + \frac{(\frac{\partial d}{\partial\eta})^2}{4d^2} - \frac{c\frac{\partial c}{\partial\eta}\frac{\partial d}{\partial\eta}}{2\delta d}$$

$$+\frac{\frac{\partial a}{\partial\theta}c\frac{\partial d}{\partial\eta}}{4\delta d} + \frac{\frac{\partial a}{\partial\eta}b\frac{\partial d}{\partial\eta}}{4\delta d} + \frac{a\frac{\partial^2 c}{\partial\eta\partial\theta}}{\delta} - \frac{a\frac{\partial^2 b}{\partial\eta^2}}{2\delta} - \frac{a\frac{\partial^2 a}{\partial\theta^2}}{2\delta} + \frac{ac\frac{\partial c}{\partial\eta}\frac{\partial c}{\partial\theta}}{\delta^2}$$

But there's more…

(hairy math II)

$$-\frac{a\frac{\partial a}{\partial\theta}c\frac{\partial c}{\partial\theta}}{2\delta^2} - \frac{a\frac{\partial a}{\partial\eta}b\frac{\partial c}{\partial\theta}}{2\delta^2} - \frac{a\frac{\partial b}{\partial\eta}c\frac{\partial c}{\partial\eta}}{2\delta^2} - \frac{a^2\frac{\partial b}{\partial\theta}\frac{\partial c}{\partial\eta}}{2\delta^2} + \frac{a\frac{\partial a}{\partial\eta}\frac{\partial b}{\partial\theta}c}{4\delta^2} - \frac{a\frac{\partial a}{\partial\theta}\frac{\partial b}{\partial\eta}c}{4\delta^2}$$

$$+\frac{a^2\frac{\partial a}{\partial\theta}\frac{\partial b}{\partial\theta}}{4\delta^2} + \frac{a^2(\frac{\partial b}{\partial\eta})^2}{4\delta^2} + \frac{a\frac{\partial a}{\partial\eta}b\frac{\partial b}{\partial\eta}}{4\delta^2} + \frac{a(\frac{\partial a}{\partial\theta})^2 b}{4\delta^2}$$

$$R_{\eta\theta} = -\frac{2ac\frac{\partial\psi}{\partial\theta}\cot\theta}{\delta\psi} + \frac{2ab\frac{\partial\psi}{\partial\eta}\cot\theta}{\delta\psi} - \frac{2\frac{\partial\psi}{\partial\eta}\cot\theta}{\psi} - \frac{\frac{\partial d}{\partial\eta}\cot\theta}{2d} - \frac{\frac{\partial a}{\partial\theta}c\cot\theta}{2\delta} + \frac{a\frac{\partial b}{\partial\eta}\cot\theta}{2\delta}$$

$$-\frac{2ac\frac{\partial^2\psi}{\partial\theta^2}}{\delta\psi} - \frac{2ac(\frac{\partial\psi}{\partial\theta})^2}{\delta\psi^2} + \frac{4ab\frac{\partial\psi}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta\psi^2} + \frac{2\frac{\partial\psi}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\psi^2} - \frac{ac\frac{\partial d}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta d\psi} + \frac{ab\frac{\partial d}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta d\psi}$$

$$-\frac{\frac{\partial d}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{d\psi} + \frac{2a\frac{\partial c}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta\psi} - \frac{3\frac{\partial a}{\partial\theta}c\frac{\partial\psi}{\partial\theta}}{\delta\psi} + \frac{2a\frac{\partial b}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta\psi} + \frac{\frac{\partial a}{\partial\eta}b\frac{\partial\psi}{\partial\theta}}{\delta\psi} - \frac{2a^2b\frac{\partial c}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta^2\psi}$$

$$+\frac{2abc\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} + \frac{a^2\frac{\partial b}{\partial\theta}c\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} + \frac{a\frac{\partial a}{\partial\theta}bc\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{a^2b\frac{\partial b}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{a\frac{\partial a}{\partial\eta}b^2\frac{\partial\psi}{\partial\theta}}{\delta^2\psi} - \frac{2bc\frac{\partial^2\psi}{\partial\eta^2}}{\delta\psi}$$

$$+\frac{4ab\frac{\partial^2\psi}{\partial\eta\partial\theta}}{\delta\psi} - \frac{6\frac{\partial^2\psi}{\partial\eta\partial\theta}}{\psi} - \frac{2bc(\frac{\partial\psi}{\partial\eta})^2}{\delta\psi^2} + \frac{ab\frac{\partial d}{\partial\theta}\frac{\partial\psi}{\partial\eta}}{\delta d\psi} - \frac{\frac{\partial d}{\partial\theta}\frac{\partial\psi}{\partial\eta}}{d\psi} - \frac{bc\frac{\partial d}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta d\psi}$$

$$+\frac{2b\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta\psi} - \frac{3\frac{\partial b}{\partial\eta}c\frac{\partial\psi}{\partial\eta}}{\delta\psi} + \frac{a\frac{\partial b}{\partial\theta}\frac{\partial\psi}{\partial\eta}}{\delta\psi} + \frac{2\frac{\partial a}{\partial\theta}b\frac{\partial\psi}{\partial\eta}}{\delta\psi} + \frac{2abc\frac{\partial c}{\partial\theta}\frac{\partial\psi}{\partial\eta}}{\delta^2\psi} - \frac{2ab^2\frac{\partial c}{\partial\eta}\frac{\partial\psi}{\partial\eta}}{\delta^2\psi}$$

and still more…

(hairy math III)

$$+\frac{ab\frac{\partial b}{\partial\eta}c\frac{\partial\psi}{\partial\eta}}{\delta^2\psi}+\frac{\frac{\partial a}{\partial\eta}b^2c\frac{\partial\psi}{\partial\eta}}{\delta^2\psi}-\frac{a^2b\frac{\partial b}{\partial\theta}\frac{\partial\psi}{\partial\eta}}{\delta^2\psi}-\frac{a\frac{\partial a}{\partial\theta}b^2\frac{\partial\psi}{\partial\eta}}{\delta^2\psi}+\frac{\frac{\partial d}{\partial\eta}\frac{\partial d}{\partial\theta}}{4d^2}-\frac{\frac{\partial a}{\partial\theta}c\frac{\partial d}{\partial\theta}}{4\delta d}$$

$$+\frac{a\frac{\partial b}{\partial\eta}\frac{\partial d}{\partial\theta}}{4\delta d}-\frac{\frac{\partial^2 d}{\partial\eta\partial\theta}}{2d}-\frac{\frac{\partial b}{\partial\eta}c\frac{\partial d}{\partial\eta}}{4\delta d}+\frac{\frac{\partial a}{\partial\theta}b\frac{\partial d}{\partial\eta}}{4\delta d}-\frac{\frac{\partial c}{\partial\eta}\frac{\partial c}{\partial\theta}}{\delta}+\frac{\frac{\partial a}{\partial\theta}\frac{\partial c}{\partial\theta}}{2\delta}$$

$$+\frac{c\frac{\partial^2 c}{\partial\eta\partial\theta}}{\delta}+\frac{\frac{\partial b}{\partial\eta}\frac{\partial c}{\partial\eta}}{2\delta}-\frac{\frac{\partial^2 b}{\partial\eta^2}c}{2\delta}-\frac{\frac{\partial^2 a}{\partial\theta^2}c}{2\delta}-\frac{\frac{\partial a}{\partial\eta}\frac{\partial b}{\partial\theta}}{4\delta}+\frac{\frac{\partial a}{\partial\theta}\frac{\partial b}{\partial\eta}}{4\delta}$$

$$+\frac{ab\frac{\partial c}{\partial\eta}\frac{\partial c}{\partial\theta}}{\delta^2}-\frac{\frac{\partial a}{\partial\eta}bc\frac{\partial c}{\partial\theta}}{2\delta^2}-\frac{a\frac{\partial a}{\partial\theta}b\frac{\partial c}{\partial\theta}}{2\delta^2}-\frac{a\frac{\partial b}{\partial\theta}c\frac{\partial c}{\partial\eta}}{2\delta^2}-\frac{ab\frac{\partial b}{\partial\eta}\frac{\partial c}{\partial\eta}}{2\delta^2}+\frac{a\frac{\partial a}{\partial\theta}\frac{\partial b}{\partial\theta}c}{4\delta^2}$$

$$+\frac{a\left(\frac{\partial b}{\partial\eta}\right)^2c}{4\delta^2}+\frac{\frac{\partial a}{\partial\eta}b\frac{\partial b}{\partial\eta}c}{4\delta^2}+\frac{\left(\frac{\partial a}{\partial\theta}\right)^2bc}{4\delta^2}+\frac{a\frac{\partial a}{\partial\eta}b\frac{\partial b}{\partial\theta}}{4\delta^2}-\frac{a\frac{\partial a}{\partial\theta}b\frac{\partial b}{\partial\eta}}{4\delta^2}$$

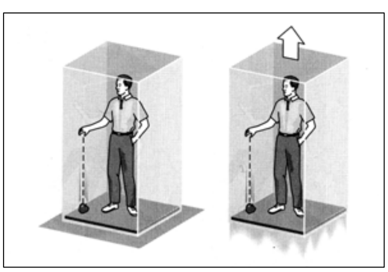
$$R_{\theta\theta}=-\frac{2ab\frac{\partial\psi}{\partial\theta}\cot\theta}{\delta\psi}+\frac{2bc\frac{\partial\psi}{\partial\eta}\cot\theta}{\delta\psi}-\frac{\frac{\partial d}{\partial\theta}\cot\theta}{d}-\frac{c\frac{\partial c}{\partial\theta}\cot\theta}{\delta}+\frac{\frac{\partial b}{\partial\eta}c\cot\theta}{2\delta}+\frac{a\frac{\partial b}{\partial\theta}\cot\theta}{2\delta}$$

$$-\frac{2ab\frac{\partial^2\psi}{\partial\theta^2}}{\delta\psi}-\frac{2\frac{\partial^2\psi}{\partial\theta^2}}{\psi}-\frac{2ab\left(\frac{\partial\psi}{\partial\theta}\right)^2}{\delta\psi^2}+\frac{6\left(\frac{\partial\psi}{\partial\theta}\right)^2}{\psi^2}+\frac{4bc\frac{\partial\psi}{\partial\eta}\frac{\partial\psi}{\partial\theta}}{\delta\psi^2}-\frac{ab\frac{\partial d}{\partial\theta}\frac{\partial\psi}{\partial\theta}}{\delta d\psi}$$

Oops! This only is for a 2-dimensional axisymmetric case. More complicated situations are much worse!

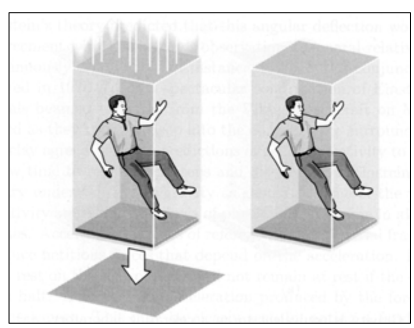
The Principle of Equivalence

- A thought experiment: falling elevators.



Gravity

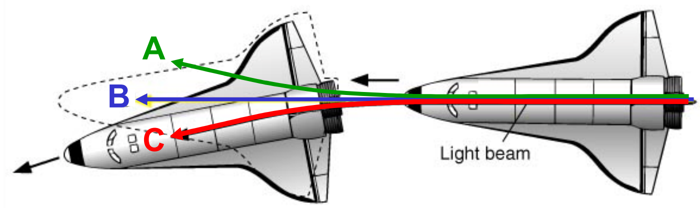
Upwards acceleration, no gravity.



Falling due to gravity

No gravity

- Can't tell difference between gravity & acceleration
- …or between freefall & no gravity.
- So *any* experiment should give same answer in either case.



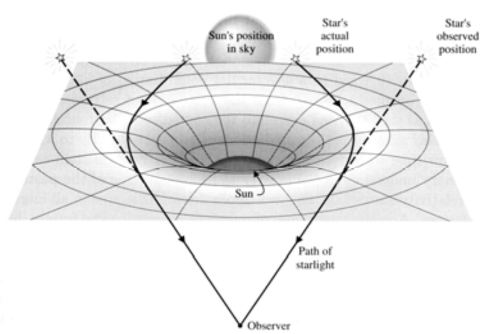
The Equivalence Principle at Work

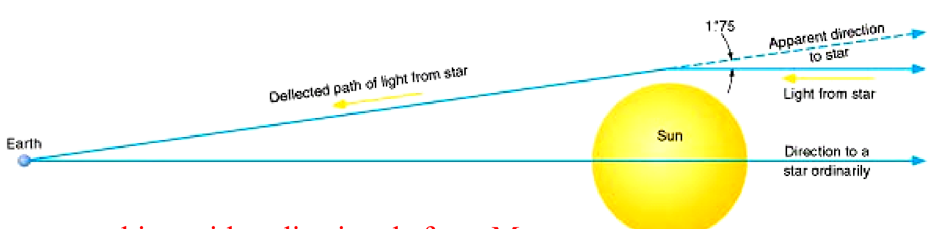
Clicker Question. *What path does the light beam take inside the Space Shuttle?*

A. Path A
B. Path B
C. Path C
D. None of the above

Tests (Proofs) of General Relativity

- Bending of starlight in Sun's gravitational field
  - Seen during 1919 eclipse





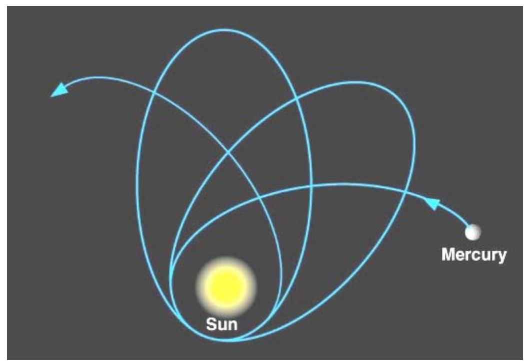
+ same thing with radio signals from Mars.

Tests (Proofs) of General Relativity

- Precession of Mercury
  - 1/90 degree per century in excess of amount expected from Newton's laws.
    - Easy to observe because of long time span of observations.
  - GR predicts this.
  - Need extra planet to explain it with Newton's laws.

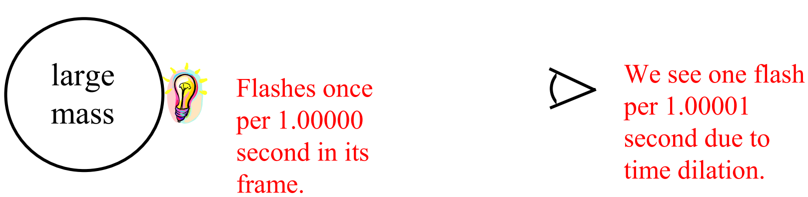
Orbits in strongly curved spacetime

GR vs Newtonian orbits

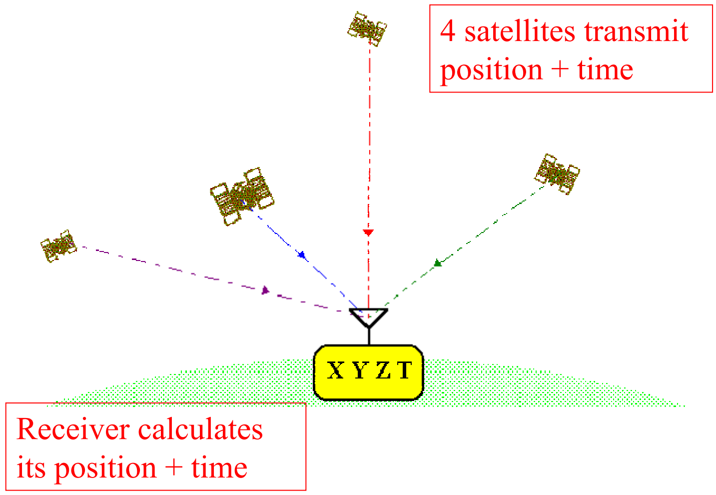


Tests (Proofs) of General Relativity

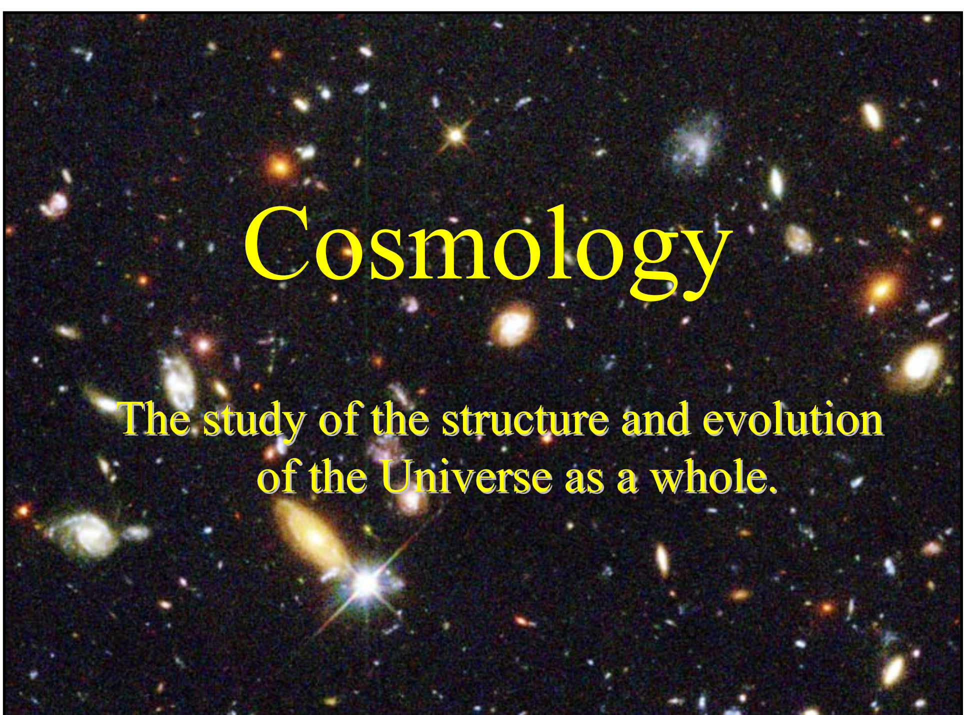
- Time also runs slower in stronger gravitational field
  - General relativistic time dilation .
- ➔ gravitational redshift: light waves emitted at different frequency than we receive them.
    - Observed from surface of white dwarfs.



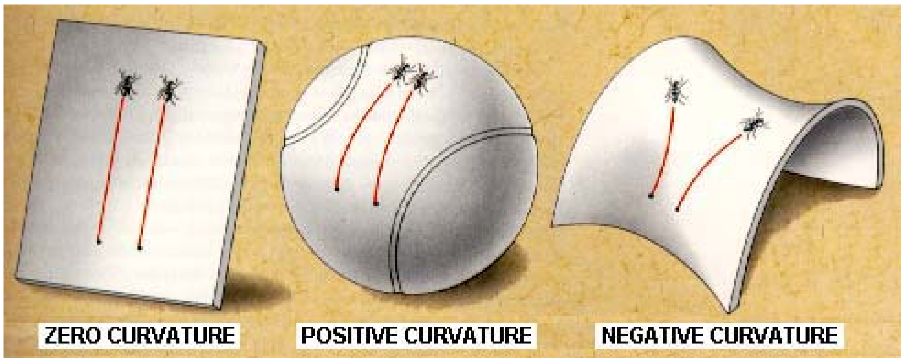
GPS Navigation



- General Relativity ➔ time runs faster in the satellites.
- If no correction applied, 6 mile position error after only 1 day!

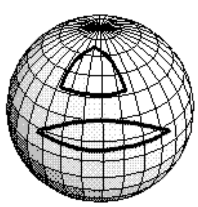


The Shape of the Universe

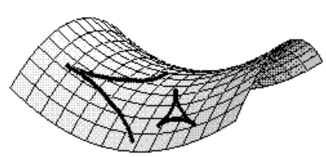


Some possible geometries

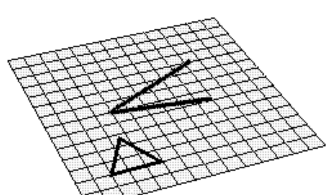
Geometrical Tests



Positive
Curvature

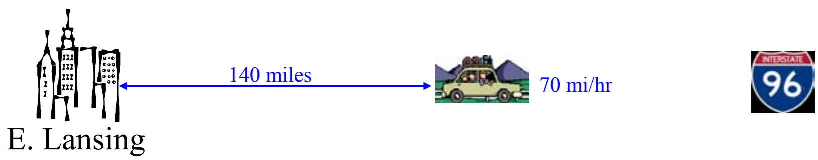


Negative
Curvature



Flat
(zero curvature)

The Expanding Universe



How long has it been since this car left East Lansing?

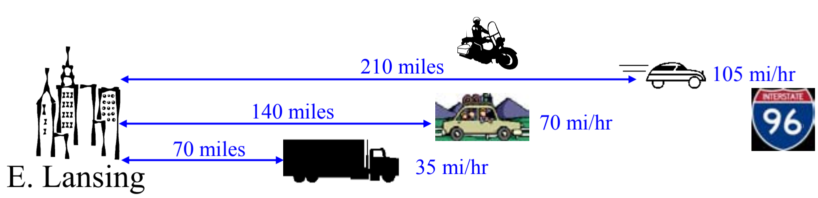
A. There is not enough information to know.

B. 1/2 hour

C. 1 hour

D. 2 hours

More traffic

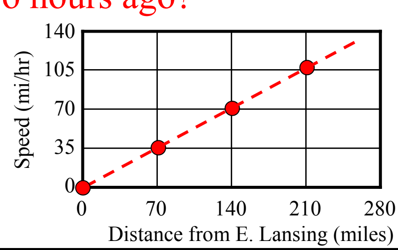


Where were all of these cars two hours ago?

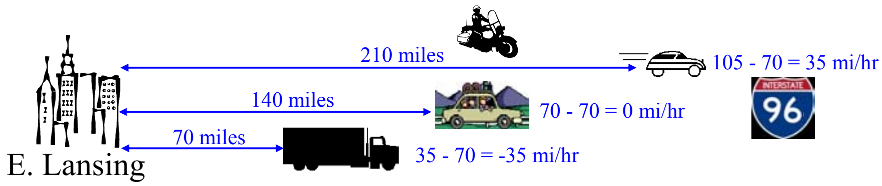
A. Scattered all over Michigan

B. Somewhere west of Chicago

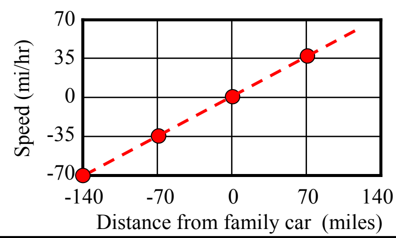
C. All together in E. Lansing

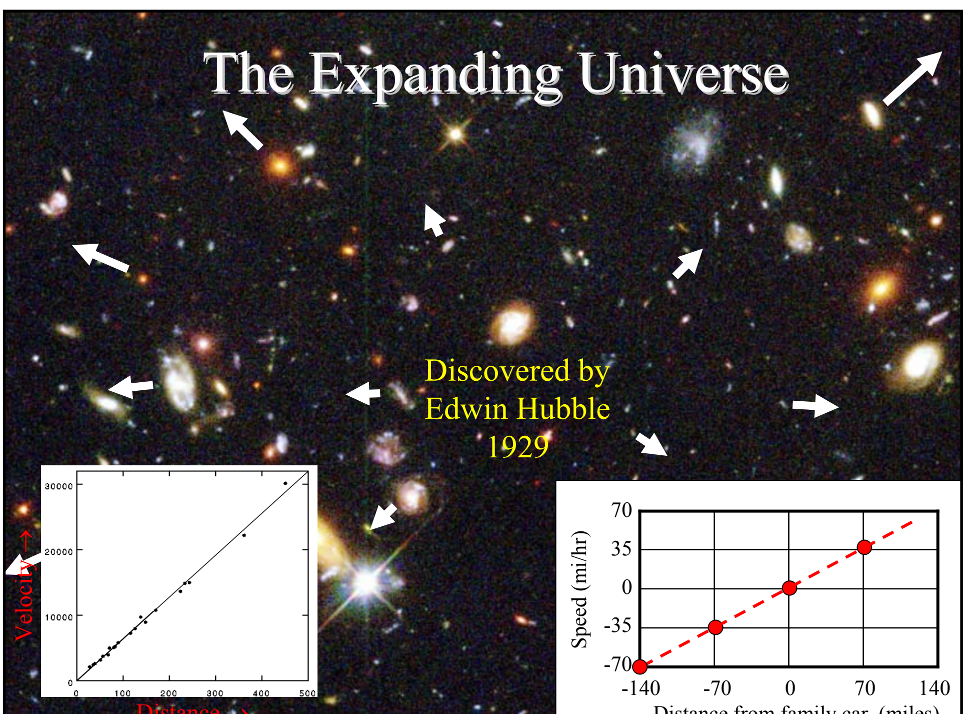


A New Reference Frame



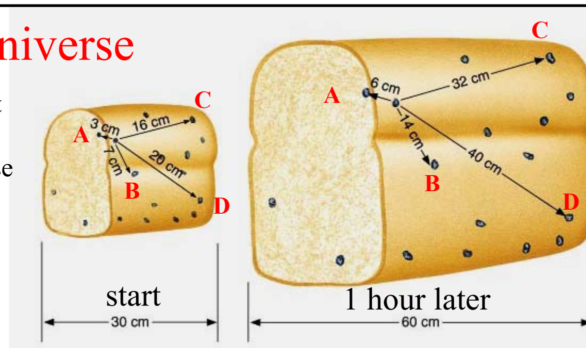
What does the plot look like now?



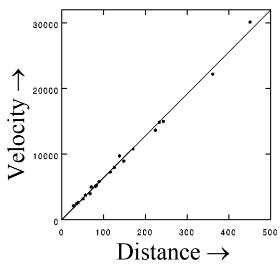


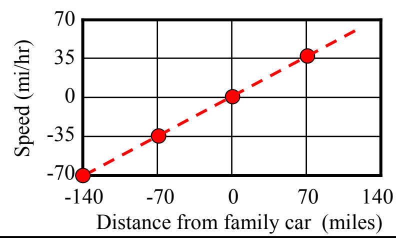
The Expanding Universe

- We are unlikely to be at exact center.
- ➔ Scale of the whole universe is expanding.
- Galaxies all recede from each other
  - Except for small random motions.

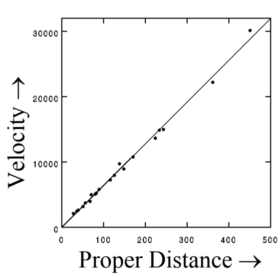


Looks same from any raisin

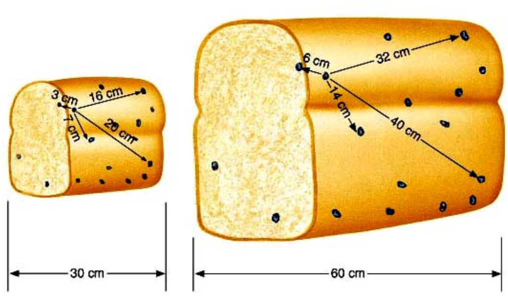




The Expanding Universe



- Cosmological principle
  - Universe looks the same from any point.
- Expanding Universe
  - Hubble's Law
- Scale Factor = R(time) = R(t)
  - Proper Distance = R(t) x (co-moving distance).



R(t) increased by factor 2

The Evolving Universe