PHY492: Nuclear & Particle Physics

Lecture 3

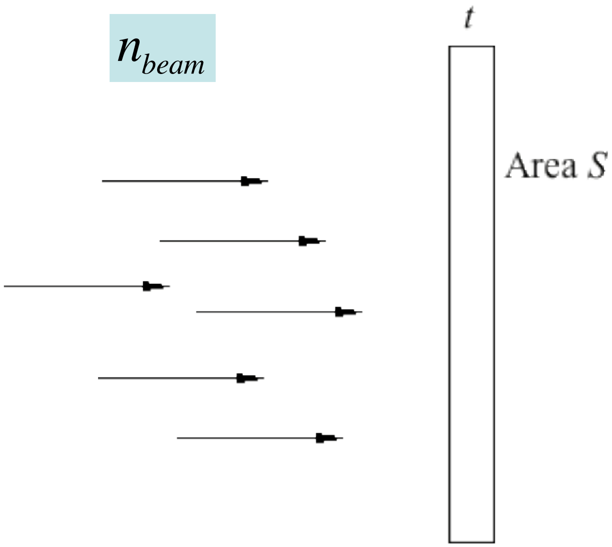
Homework 1

Nuclear Phenomenology

Measuring cross sections in thin targets

beam particles/s

n_{beam}



$$m_T = \rho t S$$ mass of target

$$n_{moles} = \frac{m_T}{A}$$ # of target moles
atomic mass A

$$n_{nuclei} = n_{moles} A_0$$ # target nuclei

$$\frac{n_{nuclei}}{S}$$ # target nuclei
per unit area

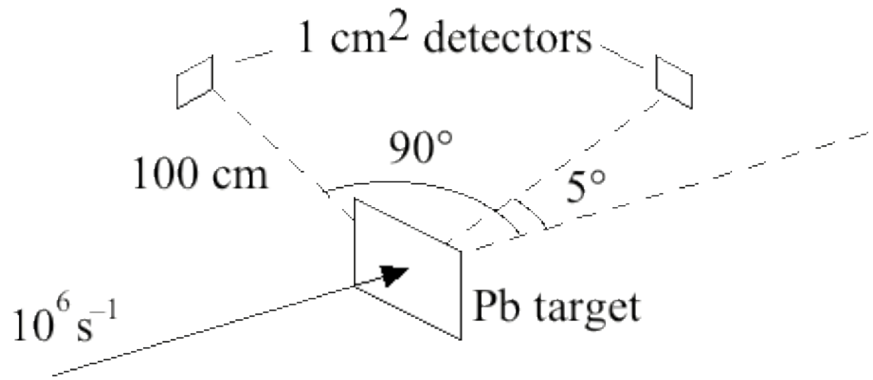
scattered particles
detected per second

$$n_{\text{detected}} = n_{beam} \frac{n_{nuclei}}{S} \frac{d\sigma}{d\Omega}(\theta,\phi) d\Omega_{\text{de tector}}$$

Problem 1.4

Rutherford scattering of 10 MeV α particles on Lead foil, 0.1 cm thick

What is the counting rate at 90° in the lab, 100 cm away, in a counter 1 cm² if there are 10^6 α particles/s hitting the target?



S = area of target is not given?

$$dn = N_0 \frac{N}{S}\frac{d\sigma}{d\Omega}(\theta,\phi)\,d\Omega$$
$$= N_0 A_0 \frac{\rho t}{A}\frac{d\sigma}{d\Omega}(\theta,\phi)\,d\Omega$$

$\rho = \text{mass/cm}^3$ $\quad$ $\rho t = \text{mass/cm}^2$

$\frac{\rho t}{A} = \text{moles/cm}^2$ $\quad$ $A_0 \frac{\rho t}{A} = \text{atoms/cm}^2$

$$\frac{d\sigma}{d\Omega}(\theta) = \left(\frac{ZZ'e^2}{4E}\right)^2 \sin^{-4}(\theta/2)$$

$$\frac{ZZ'e^2}{4E} = \frac{ZZ'e^2(\hbar c)}{4E(\hbar c)} = \frac{ZZ'}{4E}\alpha\hbar c$$
$$= \frac{ZZ'}{4E}\frac{197\ \text{MeV}\bullet\text{fm}}{137}$$

$dn = 0.5\ \text{s}^{-1}$

What about at 5° ?

$dn = 34{,}000\ \text{s}^{-1}$

Problem 1.1: Rutherford scattering total cross section

Differential Cross section

$$\frac{d\sigma}{d\Omega}(\theta) = \left(\frac{ZZ'e^2}{4E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$$

Total Cross section

$$\sigma_{total} = \int d\Omega \frac{d\sigma}{d\Omega}(\theta)$$

$$d\Omega = \sin\theta d\theta d\phi$$

in D&F Eq. 1.32

$$\sin^2\frac{\theta_b}{2} = \frac{1}{1+\frac{4b^2E^2}{\left(ZZ'e^2\right)^2}}$$

Scattering at angles larger than θ_b

$$\sigma_{TOT}\left(\theta > \theta_b\right) = 8\pi\left(\frac{ZZ'e^2}{4E}\right)^2 \int_{\sin\frac{\theta_b}{2}}^{1} \frac{dx}{x^3}$$

$$= 4\pi\left(\frac{ZZ'e^2}{4E}\right)^2 \left(\frac{1}{\sin^2\frac{\theta_b}{2}} - 1\right) = 4\pi\left(\frac{ZZ'e^2}{4E}\right)^2 \frac{4b^2E^2}{\left(ZZ'e^2\right)^2}$$

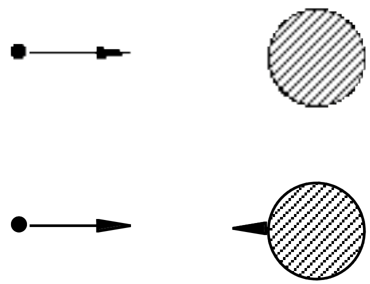
$$= 4\pi\left(\frac{ZZ'e^2}{4E}\right)^2 \left(\frac{2E}{ZZ'e^2}\right)^2 b^2 = \pi b^2$$

Area of disk radius *b*

Problem 1.3: N.R. two body scattering

$$\varsigma = \frac{m_{\text{beam}}}{m_{\text{target}}}$$

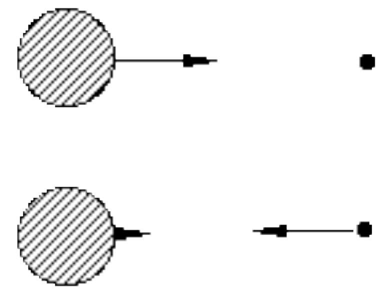
$\varsigma = 0.05$

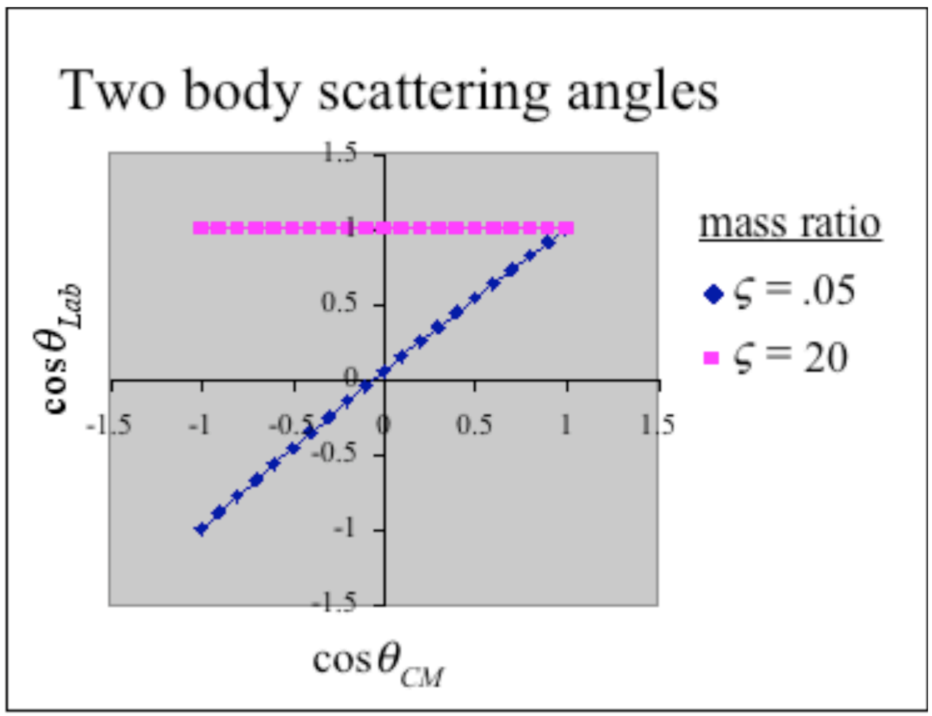


Target stationary
(laboratory frame)

Net zero momentum
(center of mass frame)

$\varsigma = 20$





Problem 1.5: Isotropic Center of Mass scattering

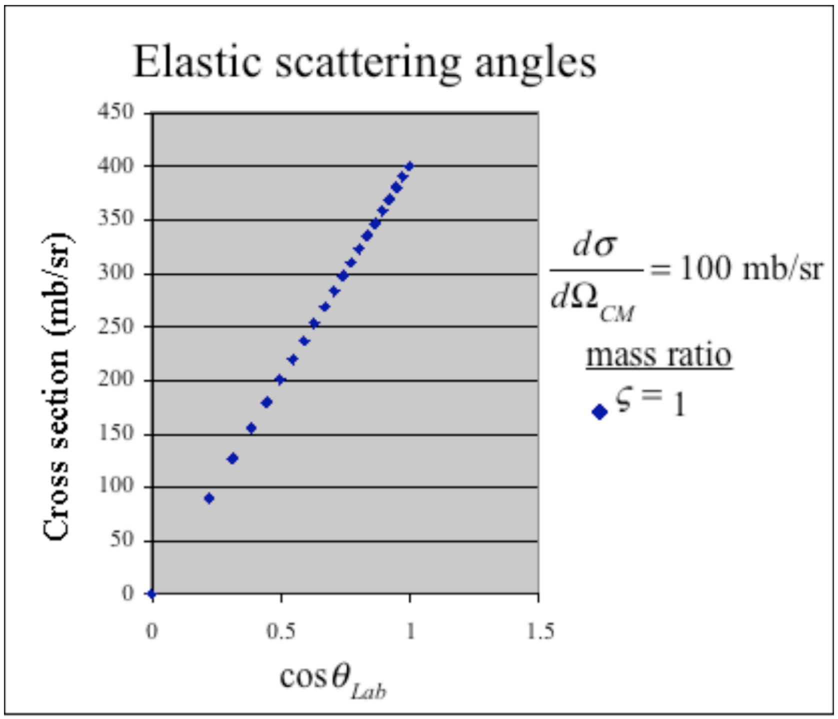
Angle transformation

$$\cos\theta_{\text{Lab}} = \frac{\cos\theta_{\text{CM}} + \varsigma}{\left(1 + 2\varsigma\cos\theta_{\text{CM}} + \varsigma^2\right)^{1/2}}$$

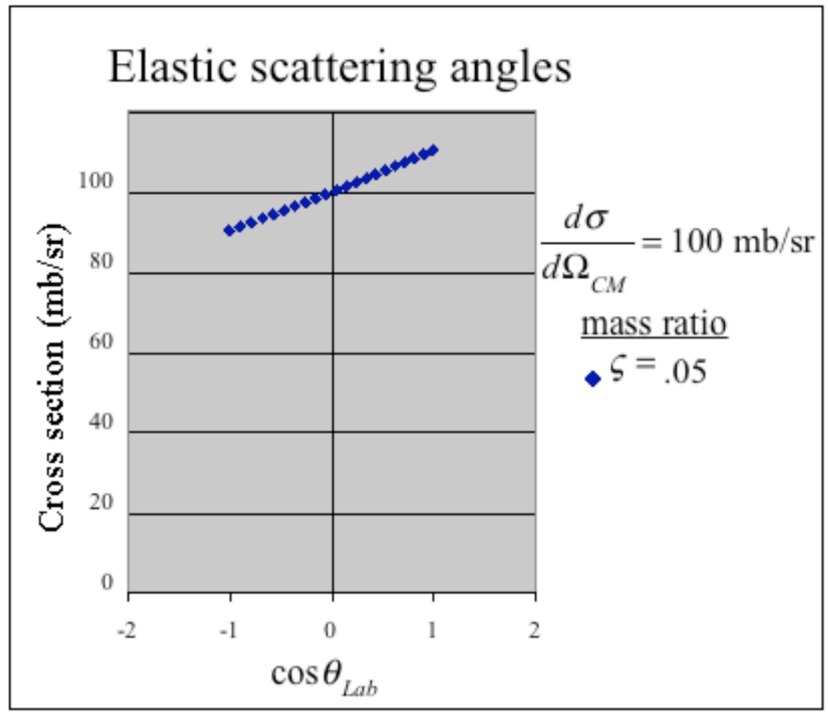
Cross section transformation

$$\frac{d\sigma}{d\Omega_{\text{Lab}}} = \frac{d\sigma}{d\Omega_{\text{CM}}} \frac{\left(1 + 2\varsigma\cos\theta_{\text{CM}} + \varsigma^2\right)^{3/2}}{\left|1 + \varsigma\cos\theta_{\text{CM}}\right|}$$

No backward scattering



Lab scattering nearly isotropic



Problem 1.7: Relativistic electron

Electron mass $m = 0.511\ \mathrm{MeV/c^2}$

Given electron momentum $pc = mc^2$

Total energy $E = \sqrt{p^2c^2 + m^2c^4} = \sqrt{2}mc^2$

Velocity $\beta = \dfrac{pc}{E} = \dfrac{1}{\sqrt{2}}$

Gamma $\gamma = \left(1 - \beta^2\right)^{-\frac{1}{2}} = \sqrt{2}$

Kinetic energy $T = E - mc^2 = \left(\sqrt{2} - 1\right)mc^2$

Problem 1.9: Ultra-relativistic limit $\tilde{\beta} \to 1$

Ultra-relativistic limit

$$\tan\theta_{Lab} = \frac{\tilde{\beta}\sin\theta_{CM}}{\gamma_{CM}\left(\tilde{\beta}\cos\theta_{CM} + \beta_{CM}\right)} \to \frac{\sin\theta_{CM}}{\gamma_{CM}\left(\cos\theta_{CM} + 1\right)}$$

Maximum at $\theta_{CM} = \frac{\pi}{2}$ $\quad \tan\theta_{Lab} = \frac{1}{\gamma_{CM}}$

$$\begin{aligned} \theta_{Lab} &= \tan^{-1}\left(\frac{1}{\gamma_{CM}}\right) \\ &= \tan^{-1}(.1) = 5.7^\circ; \quad \gamma_{CM} = 10 \\ &= \tan^{-1}(.01) = 0.57^\circ; \quad \gamma_{CM} = 100 \end{aligned}$$

Problem 1.11: Scattering rates into a detector

Alpha T=8 MeV, flux = 10^4 s^{-1}, 0.1 mm Gold foil, $\rho = 19.3$ g/cm^3

$$\Delta\theta = .05 \text{ rad}, \ \theta = 90^\circ = \pi\frac{90}{180} \text{ rad} = 1.57 \text{ rad} \ \ (\text{note: } d\theta = \Delta\theta \text{ if } \frac{\Delta\theta}{\theta} << 1 \text{ is satisfied!})$$

$$\Delta\theta = .05 \text{ rad}, \ \theta = 5^\circ = \pi\frac{5}{180} \text{ rad} = 0.087 \text{ rad} \ \ (d\theta \neq \Delta\theta \text{ because } \frac{\Delta\theta}{\theta} << 1 \text{ not satisfied!})$$

$$\text{at } 90^\circ \quad \frac{d\sigma}{d\Omega}(\theta) = \left(\frac{ZZ'e^2}{4E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}} = 2.02 \text{ b/sr}; \quad \text{rate=3.5/s}$$

$$\text{at } 5^\circ \quad \frac{d\sigma}{d\Omega}(\theta) = 21{,}000/\text{s} \quad \text{is greater than beam rate!}$$

but $d\theta$ is not correct!

Nuclear Numerology

Nuclei are specified by three numbers *A*, *Z*, and *N*, and are labeled by the element *X*, in the periodic table with that *Z* :

$Z = \#$ of protons (atomic number)

$N = \#$ of neutrons (determines the isotope)

$A = Z + N$ (atomic weight, specifies the isotope)

$X =$ Mnemonic for element in periodic table

$$^{A}_{Z}X \text{ or } {}^{A}X^{Z} \text{ or } {}^{A}X_{Z}$$

For example, Calcium 40 :

^{40}Ca — *Z* is not specified except by looking in a Periodic Table of Elements

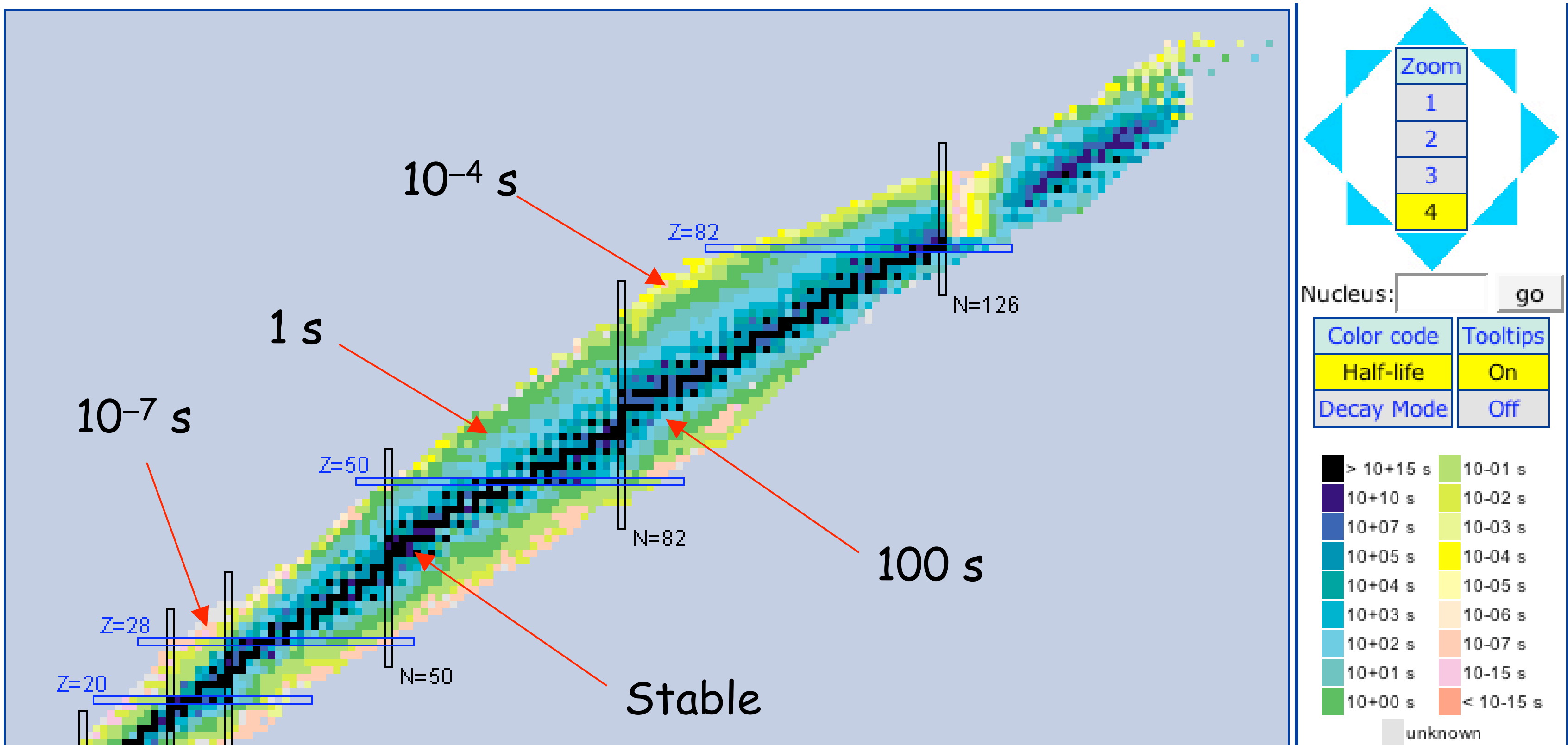
$^{40}_{20}\text{Ca}$ — *Z* = 20, and therefore N = A-Z = 20

Neutron deficit	Stable isotopes	Neutron excess
^{34}Ca	^{A}Ca, $\text{A} = 40, 42, 43, 44, 46, 48$	^{57}Ca

Alice's Restaurant
http://www.nndc.bnl.gov/

For binding energies
http://t2.lanl.gov/data/map.html

Half Lives (www.nndc.bnl.gov)



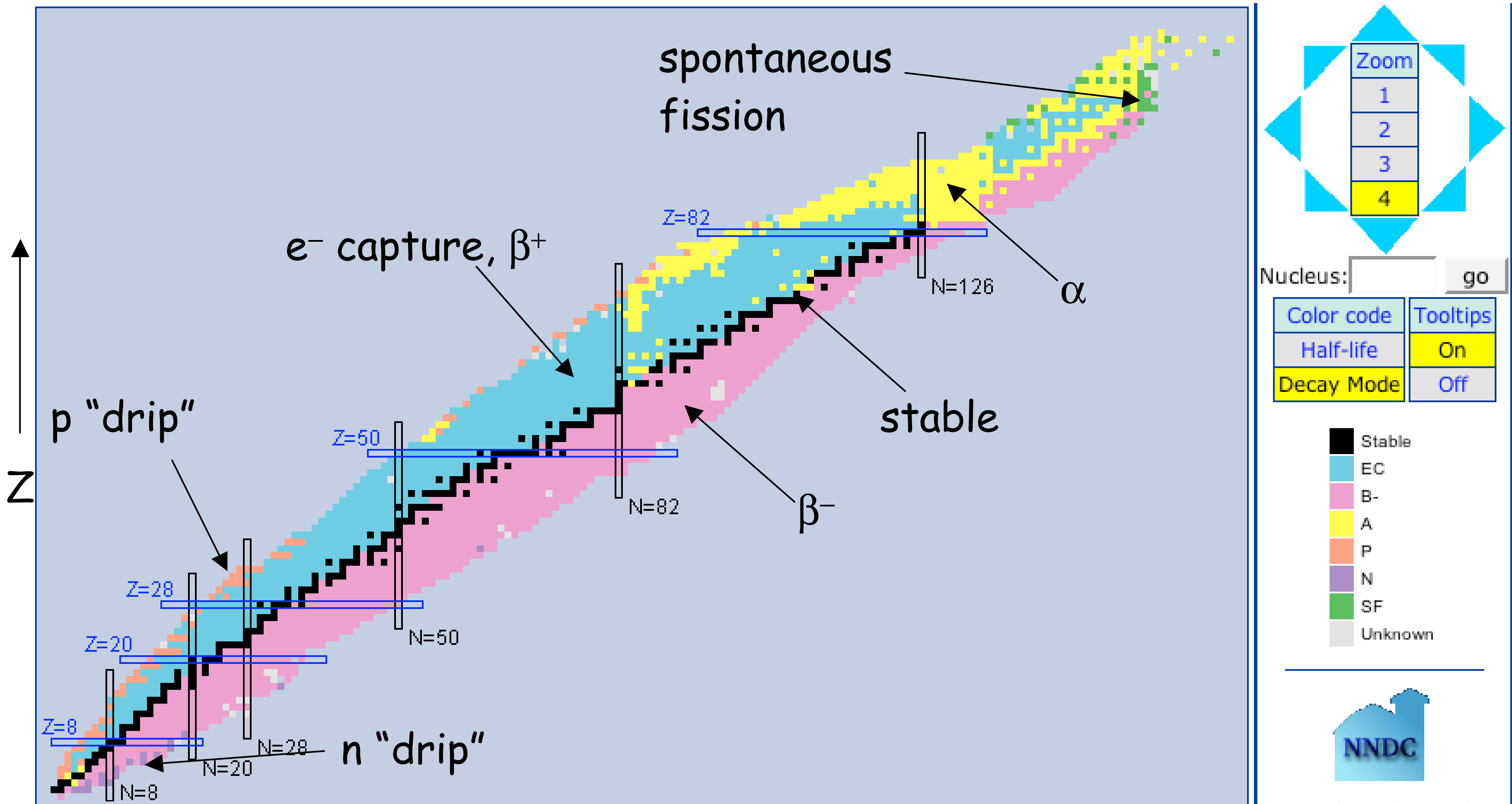
NNDC ENSDF NSR
Nuclear Wallet Cards

Ground and isomeric state information for $^{46}_{20}$Ca

E(level) (MeV)	Jπ	Δ(MeV)	$T_{1/2}$	Abundance	Decay Modes
0.0	0+	-43.1349	> 0.28E+16 y	0.004% *3*	$2\beta^-$

A list of levels and a level scheme are available

Decay modes (www.nndc.bnl.gov)



NNDC ENSDF NSR
Nuclear Wallet Cards

Ground and isomeric state information for $^{163}_{64}$Gd

E(level) (MeV)	Jπ	Δ(MeV)	$T_{1/2}$	Decay Modes
0.0	(5/2-,7/2+)	-61.4880 Syst	68 s 3	β^- : 100.00 %

A list of levels, a level scheme and decay radiation information are available

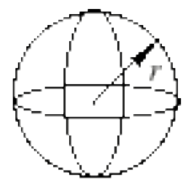
Physics of nuclei

Topics to be covered

- size and shape →
- mass and binding energy
- charge distribution
- angular momentum (spin)
- symmetries (parity)
- magnetic moments
- radioactivity
- energy levels
- reactions

Nuclear size

Binding energy per nucleon is < 1% of the nucleon mass. The protons and neutrons in the nucleus retain their particle properties. Assume nuclei are spherical and have a constant density (not compressed).



$$\left.\begin{array}{l} V \sim r^3 \\ V \sim A \end{array}\right\} r \sim A^{\frac{1}{3}}$$

Nuclear radius

$$r = \left(1.2 \times 10^{-15}\,\mathrm{m}\right) A^{\frac{1}{3}}$$