PHY492: Nuclear & Particle Physics

Lecture 4

Nature of the nuclear force

Reminder: Investigate

www.nndc.bnl.gov

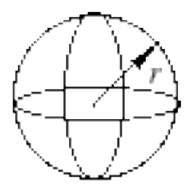
Physics of nuclei

Topics to be covered

- size and shape
- mass and binding energy
- charge distribution
- angular momentum (spin)
- symmetries (parity)
- magnetic moments
- radioactivity
- energy levels
- reactions

Nuclear size

Binding energy per nucleon is < 1% of the nucleon mass. The protons and neutrons in the nucleus retain their particle properties. Assume nuclei are spherical and have a constant density (not compressed).



$$\left.\begin{matrix} V \sim r^3 \\ V \sim A \end{matrix}\right\} r \sim A^{\frac{1}{3}}$$

Nuclear radius

$$r = \left(1.2 \times 10^{-15}\,\mathrm{m}\right) A^{\frac{1}{3}}$$

Nuclear mass and binding energy

The mass of a bound system is always less than the mass of its component parts. For example, the mass of the hydrogen atom is 13.5 eV/c^2 less than proton mass plus electron mass. When the hydrogen atom is formed, 13.5 eV is released in photons.

$$m_H c^2 - \left(m_p + m_e\right)c^2 = -13.5 \text{ eV}$$ electron binding energy in hydrogen

What is the sign of the binding energy?

D&F: the binding energy of a bound system is negative. So be it.

Nuclear binding energy

$$B.E. = M\left(A,Z\right)c^2 - \left(Zm_p + Nm_n\right)c^2 \quad \text{where } N = A - Z$$

$$\frac{B.E.}{A}$$ binding energy per nucleon,

~ the energy to remove one nucleon

Sometimes "mass excess" Δ is given in the tables.

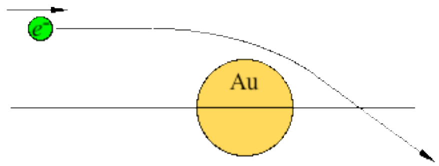
$$\Delta = M\left(A,Z\right)c^2 - A\cdot u$$

where u = 931.5 MeV, the average nucleon "mass" in ^{12}C.

$$B.E. = \Delta + A\cdot u - \left(Zm_p + Nm_n\right)c^2$$

Nuclear charge distribution via electron scattering

Electron scattering



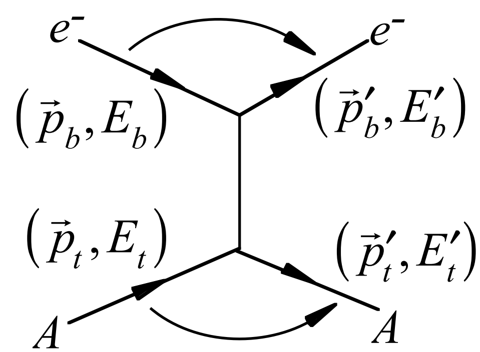
Relativistic invariants: t,s

Momentum transfer *squared* : t
(difference of final & initial momentum)^2

Center of Mass Energy *squared* : s
(effective m^2c^4 of beam + target)

$$t = \left(E_b' - E_b\right)^2 - \left(\vec{p}_b' - \vec{p}_b\right)^2$$

$$s = \left(E_b + E_t\right)^2 - \left(\vec{p}_b + \vec{p}_t\right)^2$$



Feynman diagram

$$t = \left(E_t' - E_t\right)^2 - \left(\vec{p}_t' - \vec{p}_t\right)^2$$

For target in the Lab frame

$$t = \left(E_t' - m_t c^2\right)^2 - \vec{p}_t'^2 c^2$$

$$= \left(E_t'^2 - \vec{p}_t'^2 c^2\right) - 2E' m_t c^2 + m_t^2 c^4$$

$$= -2m_t c^2\left(E' - m_t c^2\right) = -2m_t c^2 T_t' = -p_t'^2 c^2$$

N.R.

In lab, target starts at rest, picks up the transferred momentum

– momentum^2 of target

Electron scattering to probe nucleus

Rutherford scattering (θ)

$$\frac{d\sigma}{d\Omega}(\theta) = \left(\frac{ZZ'e^2}{4E}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$$

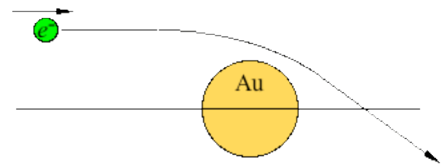
Momentum transfer q

$$q^2c^2 = -t = 2p^2c^2(1-\cos\theta)$$
$$q^2 = 2p^2(1-\cos\theta)$$

Trigonometry

$$\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$$
$$dq^2 = -2p^2 d(\cos\theta)$$
$$d\Omega = -2\pi d(\cos\theta)$$

Electron scattering



Momentum transfer[2]

Assume only the direction of beam changes so that E,|p| = E′,|p′|

$$t = (E'_b - E_b)^2 - (\vec{p}'_b - \vec{p}_b)^2 c^2$$
$$= -2p_b p'_b c^2 + 2p_b p'_b c^2 \cos\theta$$
$$= -2p^2c^2(1-\cos\theta)$$

v = velocity of electron $\approx c$

$Z = 1$

Rutherford electron scattering (q)

$$\frac{d\sigma}{dq^2}(\theta) = \frac{4\pi(Z'e^2)^2}{v^2}\frac{1}{q^4}$$

Point-like scattering

Rutherford scattering

$$\frac{d\sigma}{dq^2} = \frac{4\pi\left(Z'e^2\right)^2}{v^2 q^4}$$

Mott included effects of electron spin (relativistic quantum mechanics)

$$\left[\frac{d\sigma}{dq^2}(\theta)\right]_{\text{Mott}} = 4\cos^2\frac{\theta}{2}\left[\frac{d\sigma}{dq^2}(\theta)\right]_{\text{Rutherford}}$$

What happens if the charge distribution is not point-like?

In other words, the electron approaches the nuclear surface?

$$\frac{d\sigma}{dq^2} = \left|F(q)\right|^2 \left[\frac{d\sigma}{dq^2}\right]_{\text{Mott}}$$

Nuclear form factor

Form factors -nuclear charge distribution

Nuclear charge distribution

$$\rho(r) = Ze\, f(r)$$

Nuclear form factor

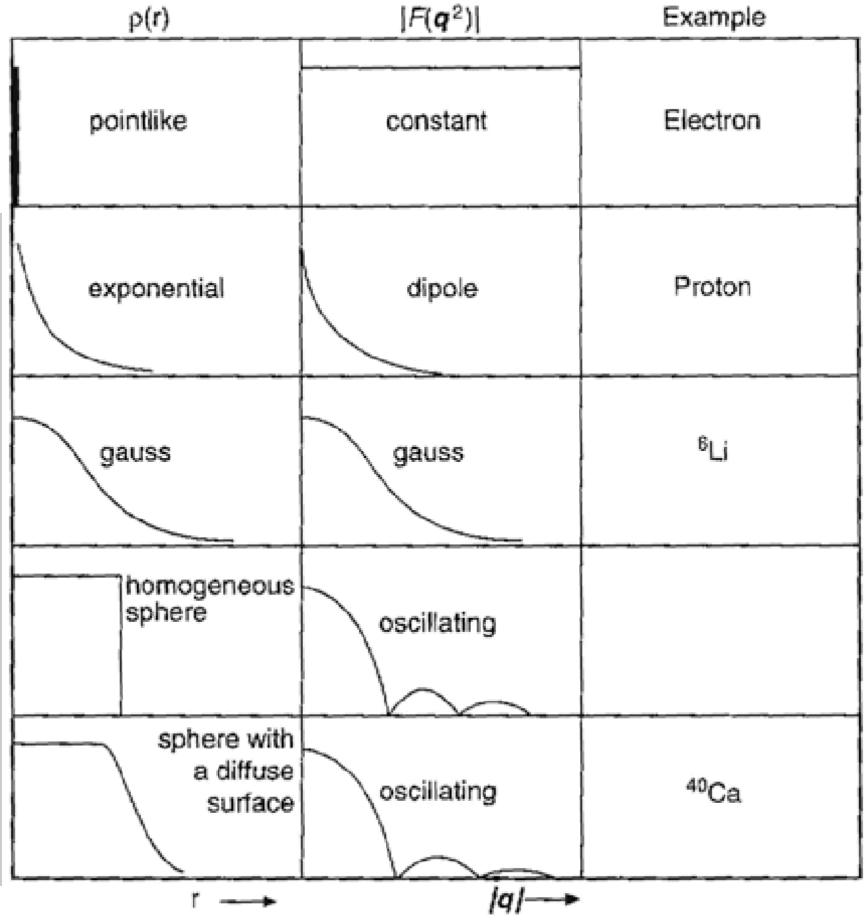
$$F(q) = \int d^3r\, f(r)\, e^{iq \bullet r/\hbar}$$

Modified differential cross section

$$\frac{d\sigma}{dq^2} = \left|F(q)\right|^2 \left[\frac{d\sigma}{dq^2}\right]_{\text{Mott}}$$

$f(r)$	$\left\|F(q^2)\right\|^2$
$(1/4\pi)\delta(r)$	1
$(a^3/8\pi)\exp(-ar)$	$\left(1 + \frac{q^2}{2a^2\hbar^2}\right)^{-2}$
$(a^2/2\pi)^{3/2}\exp\left(-\frac{r^2}{2a^2}\right)$	$\exp\left(\frac{-q^2}{2a^2\hbar^2}\right)$
$\frac{3}{4\pi R^3}(r < R)$	$\frac{3\hbar^3}{(qR)^3}\left[\sin\frac{qR}{\hbar} - \frac{qR}{\hbar}\cos\frac{qR}{\hbar}\right]$

Fourier transform of distribution



Obtain Rutherford scattering via quantum mechanics

Assume:
- particles are plane waves (known as the Born Approximation)
- target recoil energy is neglected
- particles have spin 0 (electrons are fermions - spin 1/2)
- point like scattering (charge distribution included later)

Fermi's Golden Rule

$$P = \frac{2\pi}{\hbar}\left|\langle\psi'|H|\psi\rangle\right|^2 \rho(E')$$

Probability per unit time to make a transition from initial state ψ to any one of many final states ψ', with a density of final states $\rho(E')$ ← tricky

Free particle wave functions: $\psi = \frac{1}{\sqrt{V}} e^{i\vec{p}\bullet\vec{r}/\hbar}, \psi' = \frac{1}{\sqrt{V}} e^{i\vec{p}'\bullet\vec{r}/\hbar}$

Matrix element

$$\langle\psi'|H|\psi\rangle = \int d^3r\left[\psi'^* H(r)\psi\right] = \frac{e}{V}\int d^3r\, V(r)\, e^{i\vec{q}\bullet\vec{r}/\hbar}$$

$$\psi'^* = \frac{1}{\sqrt{V}} e^{-i\vec{p}'\bullet\vec{r}/\hbar}$$

momentum transfer

$$\vec{q} = \vec{p}' - \vec{p}$$

Coulomb potential

$$V(r) = \frac{Z'e}{r} \longrightarrow \langle\psi'|H|\psi\rangle = \frac{Z'e^2}{V}\int \frac{e^{iqr\cos\theta/\hbar}}{r} r^2 dr\, d(\cos\theta)\, d\phi$$

Evaluation of the matrix element

$$\langle \psi' | H | \psi \rangle = \frac{Z'e^2}{V} \int \frac{e^{iqr\cos\theta/\hbar}}{r} r^2 dr d(\cos\theta) d\phi$$

$$= \frac{2\pi Z'e^2}{V} \int_0^\infty r dr \int_{-1}^{1} e^{iqr\cos\theta/\hbar} d(\cos\theta)$$

$$= \frac{2\pi Z'e^2}{V} \int_0^\infty r dr \left[(2\hbar / qr) \left(e^{iqr/\hbar} - e^{-iqr/\hbar} \right) / 2i \right]$$

$$= \frac{4\pi Z'e^2\hbar}{qV} \int_0^\infty dr \sin(qr/\hbar) = -\frac{4\pi Z'e^2\hbar^2}{q^2 V} \left[\cos(qr/\hbar) \right]_0^\infty$$

$$= \frac{4\pi Z'e^2\hbar^2}{q^2 V}$$

Scattering probability per unit time

Other factors in golden rule

Density of final states $\rho(E)$

$$dp_x dx = 2\pi\hbar \text{ (each state)}$$

$$dn(p') = \frac{V d^3 p'}{(2\pi\hbar)^3} = \frac{V p'^2 dp' d\Omega}{(2\pi\hbar)^3}$$

$$dE' = v' dp'$$

$$\rho(E') = \frac{dn(p')}{dE'} = \frac{V}{v'} \frac{p'^2 d\Omega}{(2\pi\hbar)^3}$$

Probability ---> cross sections

$$P = \frac{2\pi}{\hbar} \left| H_{fi} \right|^2 \rho(E')$$

(scatters/second/beam particle/target particle)

$$\frac{V}{v'} = \frac{\psi \text{ normalization}}{\text{beam velocity}}; \quad v = v'$$

$$\frac{V}{v} P = \frac{V}{v} \frac{2\pi}{\hbar} \left| H_{fi} \right|^2 \rho(E')$$

$$d\sigma = \frac{V}{v} P$$

$$\frac{\text{(scatters/second)}}{\text{(beam/second)(targets/unit-area)}}$$

Fermi's "Golden Rule" ---> Rutherford scattering

$$d\sigma = \frac{V}{v}\frac{2\pi}{\hbar}\left|H_{fi}\right|^2 \rho(E')$$

$$d\sigma = \frac{2\pi V}{v\hbar}\left[\frac{4\pi Z' e^2 \hbar^2}{q^2 V}\right]^2 \left[\frac{V}{v}\frac{p'^2}{(2\pi\hbar)^3}\right] d\Omega$$

$$= \frac{2\pi\left[4\pi Z' e^2\right]^2 p'^2}{(2\pi)^3 v^2 q^4}\left(\frac{\pi dq^2}{p'^2}\right)$$

$$\frac{d\sigma}{dq^2} = \frac{4\pi\left(Z' e^2\right)^2}{v^2 q^4}$$

Rutherford scattering

Trigonometry

$$q^2 = 2p^2(1-\cos\theta)$$

$$dq^2 = -2p^2 d\cos\theta$$

$$d\Omega = -2\pi d\cos\theta$$