PHY492: Nuclear & Particle Physics

Lecture 5

Angular momentum

Nucleon magnetic moments

Nuclear models

Orbital angular momentum

Orbital angular momentum, L

$$\text{Classical: } \vec{L} = \vec{r} \times \vec{p}; \quad \text{Quantum: } \vec{p} \to -i\hbar\nabla, \; L_z = -i\hbar\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x} \right)$$

eigenfunctions & eigenvalues:

$$\vec{L}^2\psi_{\ell m} = \ell(\ell+1)\hbar^2\psi_{\ell m}$$
$$L_z\psi_{\ell m} = m\hbar\psi_{\ell m} \qquad -\ell \le m \le \ell$$

$$L = \sqrt{\ell(\ell+1)}\hbar, \quad \ell \text{ an integer}$$

L_z takes $(2\ell+1)$ (odd number of) possible values

$$\sqrt{\ell(\ell+1)}\hbar > \ell\hbar, \text{ therefore } L > L_z = m\hbar$$

Coordinate representation if angular distributions are desired

Spherical Harmonics: $\psi_{\ell m} = Y_{\ell m}(\theta,\phi) = f_{lm}(\theta)e^{im\phi}$

$$L_z = -ih\frac{\partial}{\partial\phi}$$

Matrix representation if only eigenvalues are of interest

eigenstates & eigenvalues:

$$\vec{L}^2|\ell,m\rangle = \ell(\ell+1)\hbar^2|\ell,m\rangle$$
$$L_z|\ell,m\rangle = m\hbar|\ell,m\rangle$$

Spin angular momentum

Many “elementary” particles, e.g., electron and neutrino, carry one half unit ($\hbar$) of angular momentum known as “spin”. With no spatial dimension, nothing is “spinning” or orbiting. Protons and neutrons are spin 1/2, therefore, nuclei can have s = 1/2 integer.

Matrix representation

$$\vec{S}^2\left|s,m_s\right\rangle = s(s+1)\hbar^2\left|s,m_s\right\rangle$$

$$S_z\left|s,m_s\right\rangle = m_s\hbar\left|s,m_s\right\rangle$$

Fermi-Dirac statistics

$$s=\tfrac{1}{2},\ m_s=\pm\tfrac{1}{2}$$

$$s=\tfrac{1}{2},1,\tfrac{3}{2}\ldots;\ -s\le m_s\le s$$

Angular momentum coupling

Nuclear energy levels have an L •S component:

$$\vec{J}=\vec{L}+\vec{S}$$

$$\vec{J}^2=\vec{L}^2+\vec{S}^2+2\vec{L}\cdot\vec{S}$$

$$\vec{L}\cdot\vec{S}=\tfrac{1}{2}\left(\vec{J}^2-\vec{L}^2-\vec{S}^2\right)$$

Eigenvalues

$$\left|\ell-s\right|\le j\le\left|\ell+s\right|$$

$$J^2\left|\ell,s,j,m_j\right\rangle = j(j+1)\hbar^2\left|\ell,s,j,m_j\right\rangle$$

$$L^2\left|\ell,s,j,m_j\right\rangle = \ell(\ell+1)\hbar^2\left|\ell,s,j,m_j\right\rangle$$

$$S^2\left|\ell,s,j,m_j\right\rangle = s(s+1)\hbar^2\left|\ell,s,j,m_j\right\rangle$$

$$J_z\left|\ell,s,j,m_j\right\rangle = m_j\hbar\left|\ell,s,j,m_j\right\rangle$$

Expectation value of L•S

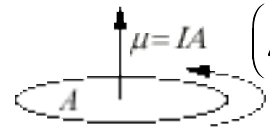
$$\left\langle \ell,s,j,m_j\right|\vec{L}\cdot\vec{S}\left|\ell,s,j,m_j\right\rangle$$

$$=\left\langle \ell,s,j,m_j\right|\tfrac{1}{2}\left(\vec{J}^2-\vec{L}^2-\vec{S}^2\right)\left|\ell,s,j,m_j\right\rangle$$

$$=\frac{\hbar^2}{2}\left\{j(j+1)-\ell(\ell+1)-s(s+1)\right\}$$

Nucleon spin and magnetic moments

Classical energy of magnetic dipole in a magnetic field: $U = -\vec{\mu}\cdot\vec{B}$



$\left(\vec{\mu} = \frac{I}{c}\vec{A}\right)_{\text{cgs}}$

$$\mu = IA = \frac{qv}{2\pi r}\left(\pi r^2\right) = \frac{q}{2m}mvr = \frac{q}{2m}L$$

$$I = \frac{qv}{2\pi r}; \quad A = \pi r^2$$

Orbital angular momentum: $L = n\hbar, \quad n = 0,1,2\ldots$

Quantum mechanics has only two values of for the projection of the magnetic dipole on the B axis, ±1 . $U = \mp\mu B$

Bohr magneton: $\mu_B = \dfrac{e\hbar}{2m_e}$ $\quad \mu_B = \dfrac{e\hbar}{2m_e c}$

$(F = qvB)_{\text{SI}}$ $\quad \left(F = qv\dfrac{B}{c}\right)_{\text{cgs}}$

SI:
$$\mu_B = \frac{e\hbar}{2m_e} = \frac{e\hbar c}{2m_e c}$$
$$= \frac{(1.6\times10^{-19}\ \text{C})\left(197\times10^{-15}\,\text{MeV}\cdot\text{m}\right)}{2\left(9.1\times10^{-31}\ \text{kg}\right)\left(3\times10^{8}\ \text{m/s}\right)}$$
$$= 5.8\times10^{-11}\ \text{MeV/T}$$
$$= 9.3\times10^{-24}\,\text{A}\cdot\text{m}^2\ \ \text{(MKS)}$$

Nuclear magneton: $\mu_N = \dfrac{e\hbar}{2m_p} = \dfrac{m_e}{m_p}\mu_B$ 2000 times smaller than Bohr magneton

<u>Magnetic dipole moment associated with point-like particles with spin</u>

$$\vec{\mu} = \frac{e}{2m}g\vec{S}$$

Landé g factor: $gS = \hbar$

$S = \hbar/2$ $\quad g = 2$

A spin 1/2 particle with a magnetic moment (Landé) g-factor not close to 2, implies internal structure.

Anomalous magnetic moments of proton and neutron

Spin of proton and neutron are 1/2

<u>Prediction for point-like particles</u>

$$\mu_p = \mu_N = 3.15 \times 10^{-14}\ \text{MeV/T}$$
$$\mu_n = 0\ \text{(no charge)}$$

<u>Measured values</u>

$$\mu_p = 2.79\ \mu_N$$
$$\mu_n = -1.91\ \mu_N$$

Mighty anomalous!

Quark model of nucleons

u-quark charge = +2/3 e

d-quark charge = -1/3 e

Nucleon quark content

proton = 2u + 1d

neutron = 1u + 2d

<u>Assuming magnetic moments of the u and d quarks are</u>

u-quark $\mu_u = +1.86\mu_N$

d-quark $\mu_d = -0.93\mu_N$

<u>Predictions of quark model</u>

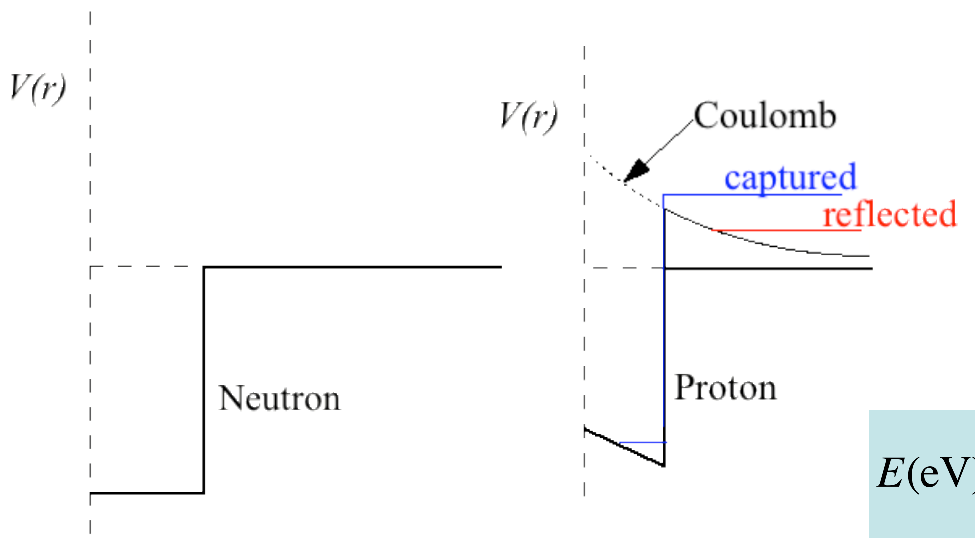
$$\mu_p = \tfrac{4}{3}\mu_u - \tfrac{1}{3}\mu_d = 2.79$$
$$\mu_n = \tfrac{4}{3}\mu_d - \tfrac{1}{3}\mu_u = -1.86$$

Nuclear phenomenology

- Strong vs. Coulomb forces
- Liquid drop model
  - Binding energy parameterization
- Fermi-gas effects
  - Justification for terms in Liquid Drop, and Shell models
- Shell model potential
  - Infinite square well
  - Harmonic oscillator
  - Spin-Orbit
- Collective model
  - Heavy nuclei

The nuclear potential

Character of the nuclear potential



Coulomb force due to exchange of a γ

$$V(r) \propto \frac{1}{r}$$

Long range

Coulomb Potential energy of two protons 1 fm apart

$$E(\mathrm{eV}) = \frac{e}{4\pi\varepsilon_0 r} = \left(9\times10^{9}\ \mathrm{Nm}^2/\mathrm{C}^2\right)\frac{1.6\times10^{-19}\ \mathrm{C}}{1\times10^{-15}\ \mathrm{m}}$$
$$= 1.4\times10^{6}\ \mathrm{eV} = 1.4\ \mathrm{MeV}$$

Yukawa (strong nuclear) potential

$$V(r) \propto \frac{e^{-\frac{mc}{\hbar}r}}{r}$$

Short range: down by 1/*e* at the nucleon surface

Strong nuclear force results from the exchange of a massive particle

$$mc^2 = \frac{\hbar c}{r} = \frac{200\ \mathrm{MeV}\cdot10^{-15}\mathrm{m}}{1.2\times10^{-15}\mathrm{m}}$$
$$\approx 160\ \mathrm{MeV}$$

Liquid drop model

Five terms (+ means weaker binding) in a prediction of the B.E.

- $r \sim A^{1/3}$, Binding is short ranged, depending only on nearest neighbors. This leads to a B.E. term proportional to A: $-a_1 A$.
- The surface nucleons are not surrounded by others. This leads to a term proportional to $A^{2/3}$ that weakens the B.E. : $+a_2 A^{2/3}$.
- Coulomb repulsion of the protons. This leads to a term proportional to Z^2/r that weakens the binding energy: $+a_3 (Z^2/A^{1/3})$.
- Orderly mix of p and n favors equal number of nucleons, but dilutes at big A. This leads to a B.E. term: $+a_4 (N-Z)^2/A$.
- Spin effects favor even numbers of protons or neutrons, but dilutes at big A. This leads to a term: $\pm a_5 \, 1/A^{3/4}$ (Z,N)

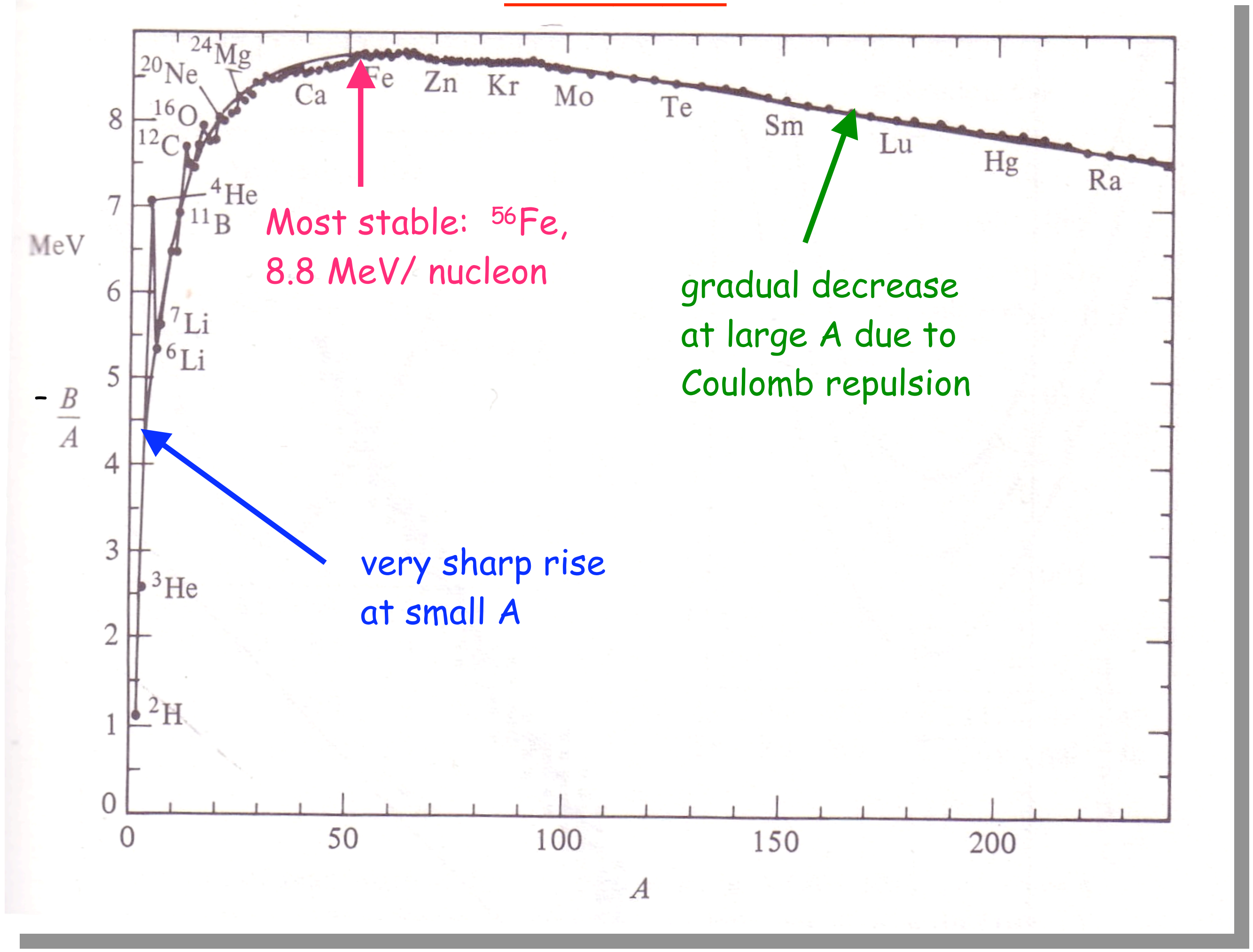
  + (odd,odd), − for (even,even), 0 for (even,odd) or (odd,even)

$$B.E.(A,Z) = -a_1 A + a_2 A^{2/3} + a_3 Z^2 A^{-1/3} + a_4 \left(2Z - A\right)^2 A^{-1} \pm a_5 A^{-3/4}$$

$$\text{in MeV: } a_1 \approx 15.6, \quad a_2 \approx 16.8, \quad a_3 \approx 0.72, \quad a_4 \approx 23.3, \quad a_5 \approx 34$$

Binding energy per nucleon

B/A vs. A



tabularized binding energies and masses - http://ie.lbl.gov/toimass.html