PHY492: Nuclear & Particle Physics

Lecture 7

Selected homework problems

Nuclear radiation

Alpha decay and fission

Homework problems

Problem 2.3

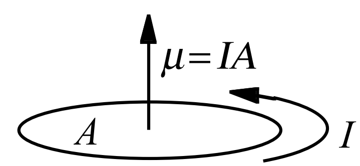
Binding Energies in MeV			
Nucleus	B.E./A	B.E.	
He-4	7.073915	28.29566	B.E. last n
He-3	2.572681	7.718043	20.577617
O-16	7.976206	127.619296	B.E. last p
N-15	7.699459	115.491885	12.127411

Problem 2.5

$$\mu = IA = \frac{e}{2\pi r / c}\pi r^2 = \frac{\left(1.6\times10^{-19}\ \text{C}\right)\left(3\times10^{8}\ \text{m/s}\right)}{2}\left(1\times10^{-15}\text{m}\right)$$

$$= 2.4\times10^{-26}\ \text{A}\cdot\text{m}^2\ \text{(Nm/T)}$$

$$= \left(6.25\times10^{12}\ \text{MeV/J}\right)\left(2.4\times10^{-26}\ \text{J/T}\right) = \underline{1.5\times10^{-13}\ \text{MeV/T}}$$



$$\mu_N = \mu_B \frac{m_e}{m_p} = \left(5.8\times10^{-11}\ \text{MeV/T}\right)\frac{0.511}{940} = 3.2\times10^{-14}\ \text{MeV/T}$$

$$\mu_p = 2.79\mu_N = \underline{8.8\times10^{-14}\ \text{MeV/T}}$$

Factor of 2 smaller - Amazing this close

Problem 2.11

$$\text{expand } e^{\frac{i}{\hbar}\vec{q}\cdot\vec{r}} = 1 + ikr\cos\theta - \tfrac{1}{2}k^2r^2\cos^2\theta; \qquad \vec{k} = \frac{1}{\hbar}\vec{q};$$

$$F(\vec{q}) = \int_{\substack{\text{all}\\ \text{space}}} d^3r\rho(\vec{r})e^{\frac{i}{\hbar}\vec{q}\cdot\vec{r}}; \qquad d^3r = r^2 dr d(\cos\theta)d\phi$$

a)

$$\rho(\vec{r}) = \frac{1}{V}\begin{cases} & r < R \\ 0 & \text{elsewhere}\end{cases}$$

$$= \int_{\substack{\text{all}\\ \text{space}}} d^3r\rho(\vec{r}) - \tfrac{1}{2}k^2 \int_{\substack{\text{all}\\ \text{space}}} r^2 d^3r\rho(\vec{r})$$

$$F(\vec{q}) = \frac{4\pi}{V}\int_0^R r^2 dr - \tfrac{1}{2}k^2\frac{2\pi}{V}\int_{-1}^{1}\cos^2\theta\, d(\cos\theta)\int_0^R (r^2)r^2 dr; \quad \int_{-1}^{1}\cos^2\theta\, d(\cos\theta) = \frac{2}{3}$$

$$= \underline{1 - \tfrac{1}{6}k^2\langle r^2\rangle}; \quad \langle r^2\rangle = \frac{4\pi}{V}\int_0^R (r^2)r^2 dr = \tfrac{3}{5}R^2$$

$$\approx e^{-\lambda q^2}; \quad \text{where } \lambda = \frac{\langle r^2\rangle}{6\hbar^2} = \frac{R^2}{10\hbar^2}$$

b)

$$\rho(\vec{r}) = \frac{1}{R^3}\frac{1}{(2\pi)^{3/2}}e^{-\frac{r^2}{2R^2}} \qquad \langle r^2\rangle = \frac{4\pi}{R^3 2\pi\sqrt{2\pi}}\int_0^\infty e^{-\frac{r^2}{2R^2}}r^4 dr = \frac{2}{R^3\sqrt{2\pi}}\left(\frac{3\sqrt{2\pi}}{2}R^5\right) = 3R^2$$

Homework problems

Problem 3.1 Bethe-Weisacker mass

$$M(A,Z)c^2 = \alpha A - \beta Z + \gamma Z^2 \pm a_5 A^{-\frac{3}{4}}$$

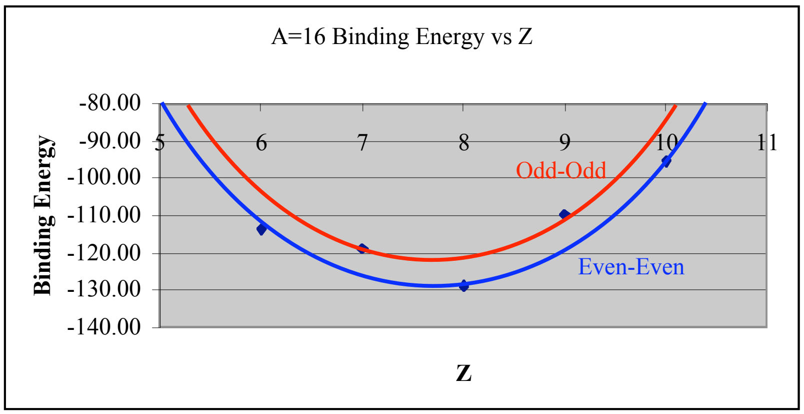
$$\frac{dM(A,Z)}{dZ} = -\beta + 2\gamma Z = 0; \quad Z_{\min} = \frac{\beta}{2\gamma}$$

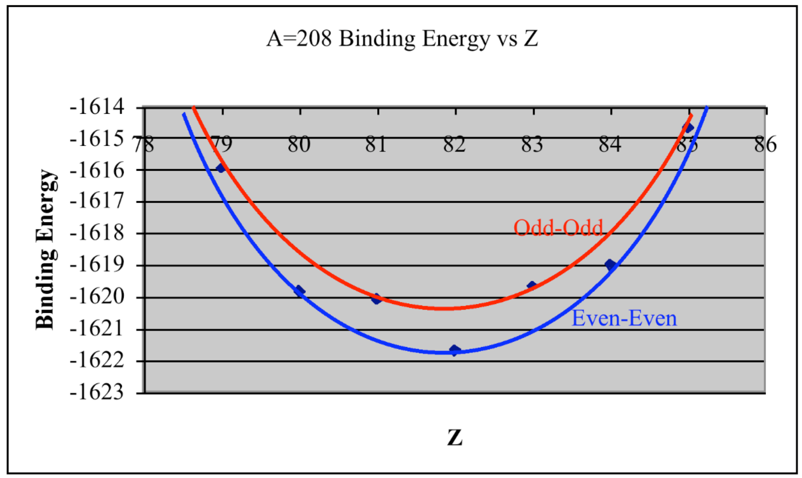
$$\alpha = m_n c^2 - a_1 + a_2 A^{-\frac{1}{3}} + a_4$$

$$\beta = 4a_4 + \left(m_n - m_p - m_e\right)c^2$$

$$\gamma = 4a_4 A^{-1} + a_3 A^{-\frac{1}{3}}$$

Binding energy for fixed A vs. Z





Homework problems

Problem 3.3 $^{8}\mathrm{Be}^{4}$ can decay to two alpha particles + ~100 keV

Problem 3.5 $^{35}\mathrm{Cl}^{17}$ 17 protons, 18 neutrons

16 protons in $(1S_{1/2})^2,(1P_{3/2})^4,(1P_{1/2})^2,(1D_{5/2})^6,(2S_{1/2})^2$ + 1 proton in $1D_{3/2}$ $j^P = \frac{3}{2}^+$

Alpha decay

$$^{A}X^{Z} \rightarrow {}^{A-4}Y^{Z-2} + {}^{4}He^{2}$$

$$^{4}He^{2} = \alpha$$

Energy available must be positive (exothermic)

$$Q = \left[M\left({}^{A}X^{Z}\right) - M\left({}^{A-4}Y^{Z-2}\right) - M\left({}^{4}He^{2}\right) \right] c^{2}$$

For nearly all light nuclei, the energy equation does not allow α decay. Most heavy nuclei can emit an α particle.

Energy and momentum conservation (N.R.) yields for the energy of the emitted α

$$T_{\alpha} = \frac{Q}{\left(1 + \frac{m_{\alpha}}{m_{D}}\right)}$$

m_{α} is alpha mass

m_{D} is Daughter mass

Energy of small nucleus "emitted" from ^{235}U

Nucleus emitted	Q (MeV)
n	-5.30
p	-6.70
^{2}H	-9.71
^{3}H	-9.97
^{3}He	-9.46
^{4}He	4.68
^{6}Li	-3.85
^{7}Li	-2.88
^{7}Be	-3.79

Only alpha is exothermic

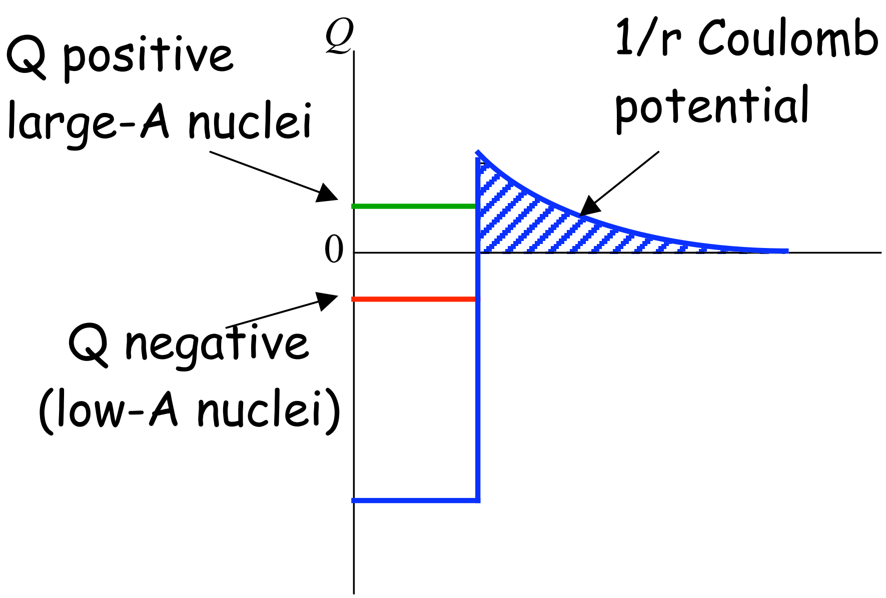
Nuclear potential well

Consider an alpha particle headed toward a Uranium nucleus

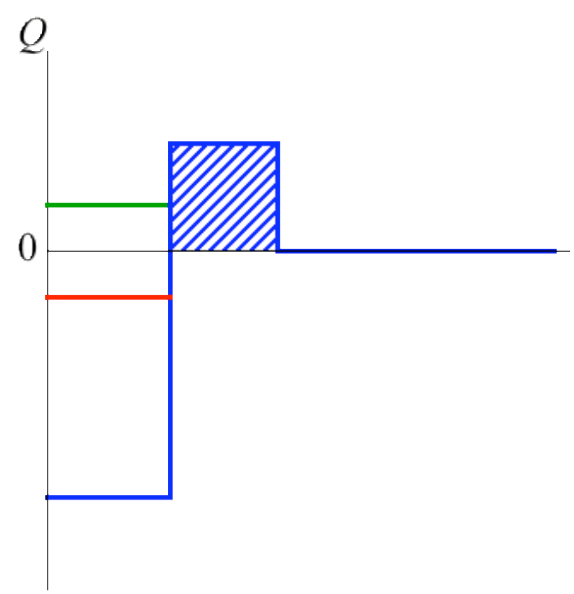
An α with sufficient kinetic energy to get over the coulomb barrier, by emitting photons will "fall" into the nuclear potential well to become bound.

Classically the alpha is permanently bound below the peak of the coulomb potential, but Q.M. tunneling allows barrier penetration if Q is positive.

approximate potential



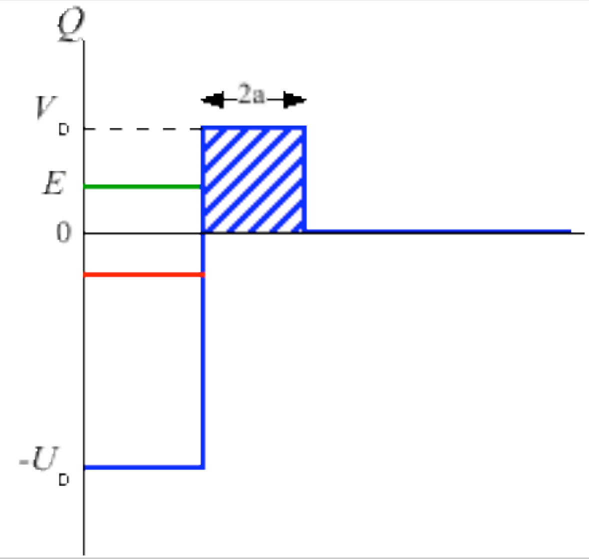
calculation potential



Alpha decay lifetimes

Wave functions are periodic inside well and beyond the barrier, and exponential decay within the *barrier*. Match boundary conditions.

Transmission coefficient depends on the barrier width 2a, the difference between alpha energy E and the barrier height V_0, and well depth U_0.



$$T \approx \frac{4\left(V_0 - E\right)}{V_0}\left(\frac{E}{E + U_0}\right)^{\frac{1}{2}} 4e^{-\frac{4a}{\hbar}\left[2m\left(V_0 - E\right)\right]^{\frac{1}{2}}}$$

$$= 4 \times 10^{-40} \text{ probability of transmission}$$

$V_0 = 14$ MeV barrier height

$E = 4$ MeV kinetic energy of α outside well

$U_0 + E \approx 44$ MeV kinetic energy of α inside well

$v_\alpha \approx 0.15c$ speed of alpha

$f = \frac{v_\alpha}{R} \approx 6 \times 10^{21}\ \text{s}^{-1}$ frequency hitting the barrier.

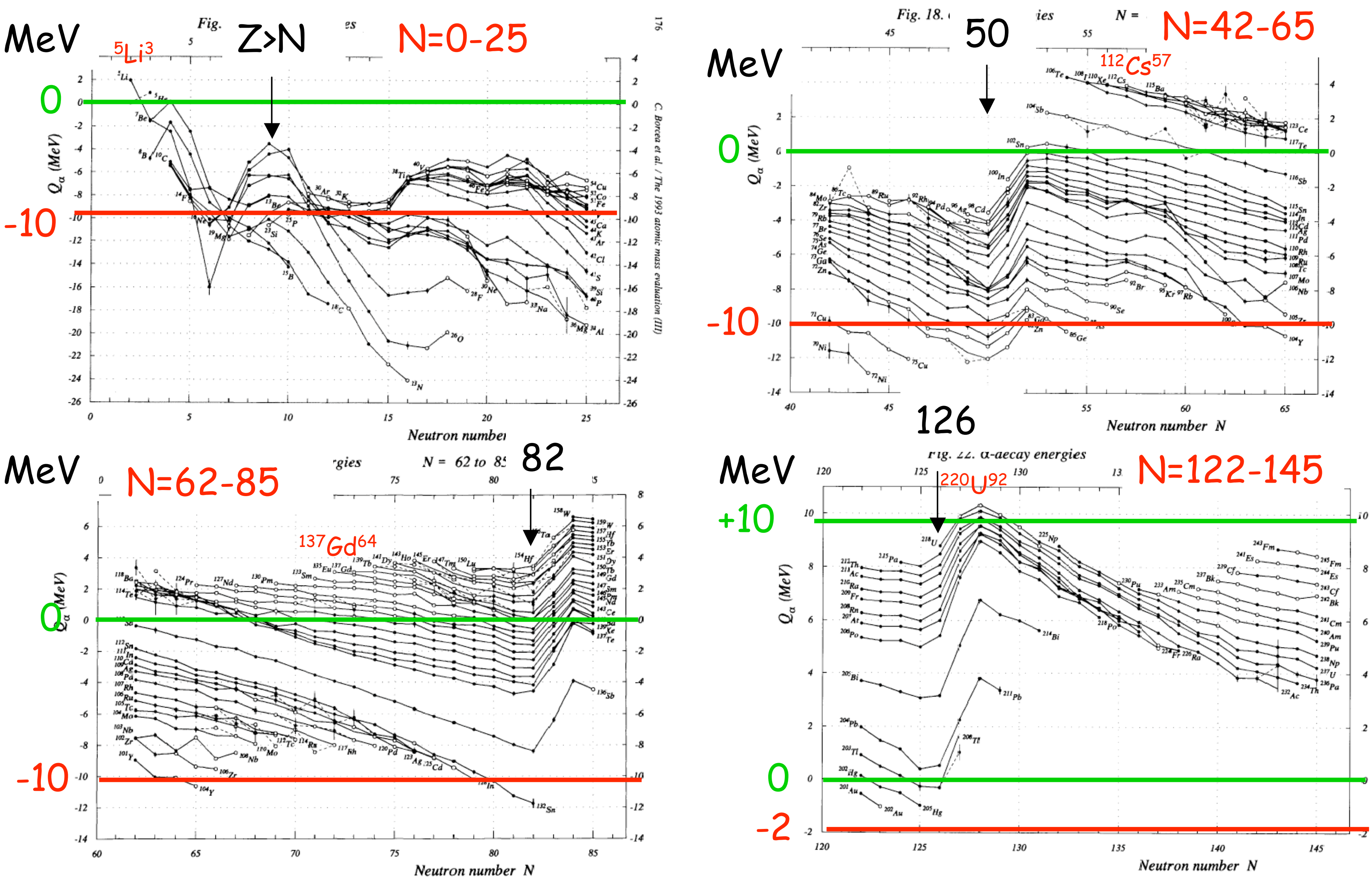
$$\lambda = Tf = 2.4 \times 10^{-18}\,\text{s}^{-1} \quad \text{tunneling rate}$$

$$\tau = 1/\lambda = 0.4 \times 10^{18}\,\text{s} = 13 \times 10^{9} \text{ years; lifetime}$$

Nuclear lifetime for 1/r potential

$$\tau \sim \tau_0 e^{C/\sqrt{E_\alpha}}$$

Alpha decay energies vs. N (magic numbers)



Shell effects on alpha decay lifetimes

- Even/Odd character of the parent and daughter nuclei
  - Alpha decay does not change Even/Odd character of Z or N
  - Even-Even nuclei easily form α 's in outer shells and decay to ground states of the Even-Even daughter
  - Even-even nuclei will have the shortest lifetimes for a given α energy.
  - Transitions of Odd Z or N nuclei to excited daughter states yield smaller α energies and longer lifetimes
- Angular momentum barrier ~ $\ell(\ell+1)$ and parity restrictions
  - Angular momentum of emitted α in transition from parent to daughter raises effective potential
  - Parity conservation may force decay into higher angular momentum transitions

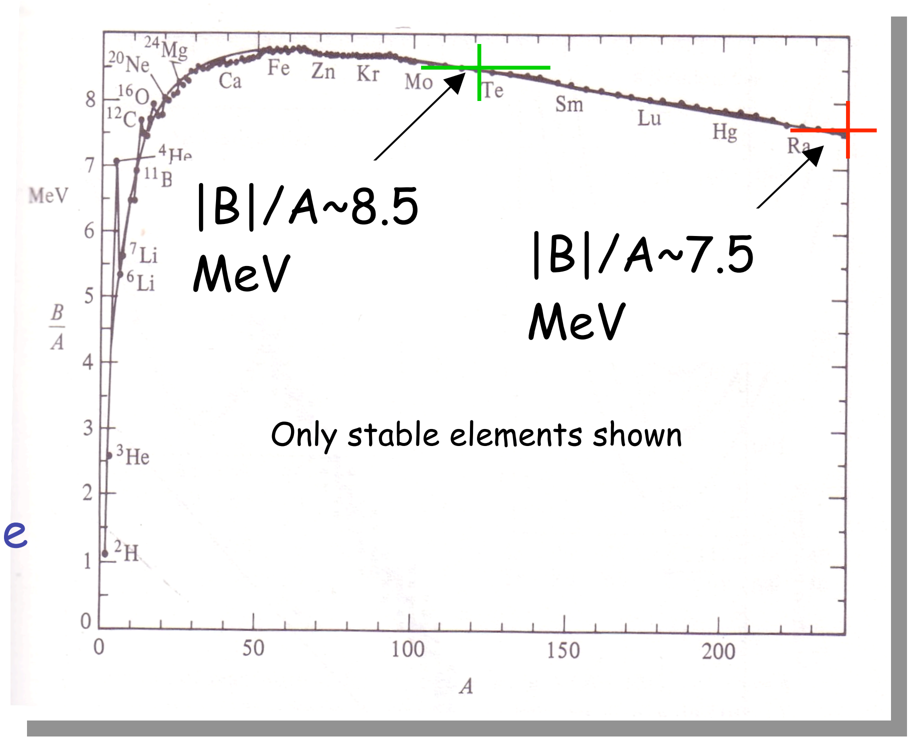
Fission

Consider a nucleus A=236 which can fission to two nuclei with A~118

$$Q \approx 236\left[\left(\frac{B}{A}\right)_{A=236} - \left(\frac{B}{A}\right)_{A=118}\right]$$

$$\approx 236\left[(8.5 - 7.6)\,\mathrm{MeV}\right]$$

$$\approx 210\ \mathrm{MeV}\ \ (\sim 10^7\ \times\ \text{chemical reaction})$$

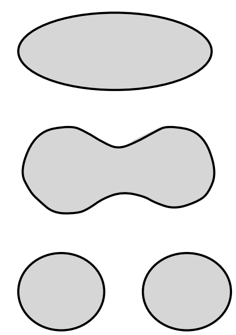


Fission typically to unequal masses in the ratio of 3/2. The asymmetric fission is *attributed* to shell model corrections

$$\text{e.g.,}\quad n + {}^{235}_{92}\mathrm{U} \rightarrow \underbrace{{}^{132}_{50}\mathrm{Sn} + {}^{100}_{42}\mathrm{Mo}}_{\text{fast beta decays}} + 4n$$

Binding energy release overcomes fission barrier

How large can the nucleus get before it spontaneously fissions?



Spontaneous fission

BE surface and coulomb terms: $BE_S = a_2 A^{\frac{2}{3}}; \quad BE_C = a_3 Z^2 A^{-\frac{1}{3}}$

How does the BE change for a Spherical (s) or Ellipsoidal (e) nucleus with the same volume?

Ellipsoid (eccentricity ε) surface area: $S_\varepsilon = S_s\left(1 + \tfrac{2}{5}\varepsilon^2 + \ldots\right)$

Coulomb energy ε dependence: $E_\varepsilon \sim \dfrac{R}{\langle r_{12}\rangle} \sim \dfrac{(S_S)^{\frac{1}{2}}}{(S_\varepsilon)^{\frac{1}{2}}} = \dfrac{1}{\left(1+\tfrac{2}{5}\varepsilon^2\right)^{\frac{1}{2}}} \approx 1 - \tfrac{1}{5}\varepsilon^2$

Binding energy difference

$$\Delta BE = \tfrac{2}{5}\varepsilon^2 BE_S - \tfrac{1}{5}\varepsilon^2 BE_C = \left(\tfrac{2}{5} a_2 A^{\frac{2}{3}} - \tfrac{1}{5} Z^2 A^{-\frac{1}{3}}\right)\varepsilon^2$$

ΔBE becomes positive (nucleus becomes unstable) $\dfrac{Z^2}{A} > \dfrac{2a_2}{a_3} = 47$

Heavy nucleus Z/A ~ 0.4 $Z > 118$

Maximum atomic number that is stable w.r.t. spontaneous fission