PHY492: Nuclear & Particle Physics

Lecture 8

Fusion

Nuclear Radiation: β decay

Energy released in nuclear fission and fusion

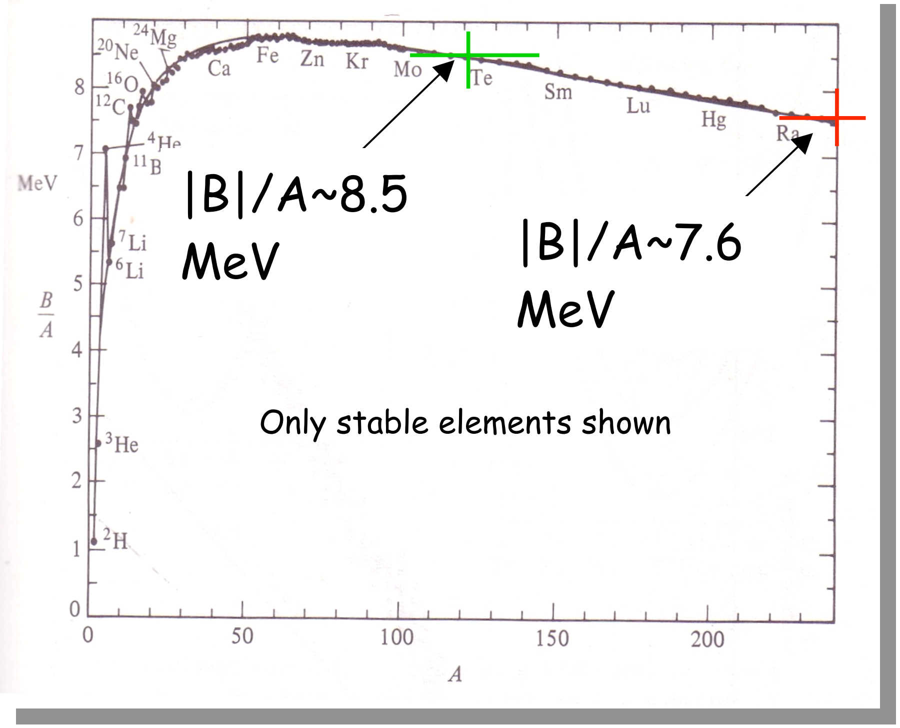
Fission

Nucleus A=236 fissions into two nuclei with A~118

$$Q \approx 236\left[\left(\frac{B}{A}\right)_{A=236} - \left(\frac{B}{A}\right)_{A=118}\right]$$

$$\approx 236\left[\left(-7.6+8.5\right)\text{MeV}\right]$$

$$\approx 210\ \text{MeV}\ \ (\sim 10^7\ \times\ \text{chemical reaction})$$



Solar fusion

A young star is nearly all ionized hydrogen atoms: p + e^-

Energy released by fusing the protons (p) to form Helium ($^4He^2$), B = -28 MeV

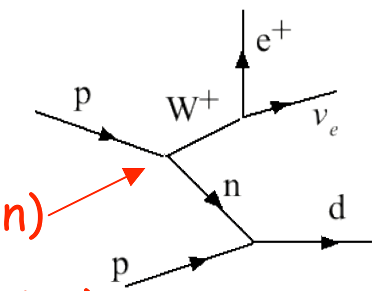
BUT, Helium is (2p,2n). Who supplied the neutrons?

This question separates the majors in physics from "pseudo-majors".

Solar fusion: First the Weak Interaction

Solar energy comes from fusion of Hydrogen nuclei ($^1H^1$, 1 proton) to Helium ($^4He^2$, 2 protons, 2 neutrons) in the sun's core (T = 15x10^{6}K).

Coulomb barrier is 1.2 MeV = 14x10^{9}K
Fusion due to tail of the temperature distribution!



$$^1\text{H} + {}^1\text{H} + e^- \rightarrow {}^2\text{H} + e^+ + e^- + \nu_e + 0.42\text{ MeV}$$ (Weak interaction)

$$e^+ + e^- \rightarrow \quad + 1.02\text{ MeV } (2\gamma)$$ (EM interaction)

$$^1\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + 5.49\text{ MeV } (\gamma)$$ (Strong followed by EM interaction)

$$^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + 2\left({}^1\text{H}\right) + 12.86\text{ MeV}$$ (Strong interaction)

Net effect: $$4\left({}^1\text{H}\right) + 2e^- \rightarrow {}^4\text{He} + 2\nu_e + 26.72\text{ MeV}$$

Other reactions important for neutrino physics

$$^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e + 0.383\text{ or }0.861\text{ MeV}$$

$$^8\text{B} \rightarrow {}^8\text{Be} + e^+ + \nu_e + (\sim 15\text{ MeV})$$

Difficult but convincing experiments have shown that the number of solar electron neutrinos is about 1/2 of that predicted.

Fusion reactors

Likely research reaction: $d + t \to {}^4\mathrm{He} + \mathrm{n} + 17.6\ \mathrm{MeV}$ must make the Tritium

Q- energy of He nucleus (3.5 MeV)

σ- cross section for *d+t* reaction

Three critical parameters:

n - nuclei number density

τ- time large n can be maintained

T- ion temperature

Lawson criterion for "ignition"

$$n\tau > L = \frac{12kT}{\langle \sigma v \rangle Q}$$

Lasers: Currently $n\tau = 10^{21}$ reached at $T = 3\times10^6$K, but $L \sim 10^{24}$.

$$\text{where } \langle \sigma v \rangle \sim T^4 \quad \to \quad L \sim T^{-3}$$

Need to keep $n\tau$ constant while raising temperature a factor of 10 !

Tokamak: Currently $n\tau = 3\times10^{19}$ reached at $T = 3\times10^8$K, but $L_{min} \sim 3\times10^{20}$.

$$L \sim \text{constant near this temperature}$$

Need to raise $n\tau$ a factor of 10 at the same temperature !

ITER: International Tokomak to be built in Cadarache, France.

β^- radiation

Free neutron "β decay"

$n \to p\, e^- \,\bar{\nu}_e$ anti electron-neutrino: $\bar{\nu}_e$ with a negligible mass.

Half-life

$$N = N_0 e^{-t/\tau};\ \text{lifetime } \tau = 886\,\text{s};$$
$$\text{half-life: } t_{\frac{1}{2}} = \ln(2)\tau = 0.693\tau = 614\,\text{s}$$

Energy available

$$Q = \left(m_n - m_p - m_e - m_\nu\right)c^2$$
$$= (939.5653 - 938.2723 - 0.5110)\ \text{MeV}$$
$$= 0.782\ \text{MeV}$$

Free protons

$p \not\to n\, e^+\, \nu_e$

energy conservation prevents this decay

nuclear binding energy may allow β^+ decay

Nuclear β decays

$${}^{A}X^{Z} \to {}^{A}Y^{Z+1} + e^- + \bar{\nu}_e$$
$${}^{A}X^{Z} \to {}^{A}Y^{Z-1} + e^+ + \nu_e$$

Atomic electron capture (EC)

$${}^{A}X^{Z} + e^- \to {}^{A}Y^{Z-1} + \nu_e$$

These decay reactions are possible if they do not violate energy conservation. The total energy of the final state products must not exceed the total energy of the initial state.

β-decay energy considerations

In β decay, the initial state nucleus is called the Parent (P) while the final state nucleus is called the Daughter (D). The energy available in the decay, Q, is distributed among the final state products

$$Q = \left(M_P - M_D - m_e - m_\nu\right)c^2 \geq 0$$
$$= T_D + T_e + T_\nu$$

Every particle carries momentum, but M_D is large compared to Q or m_e, so that $T_D \sim 0$.

$Q \approx T_e + T_\nu$ ignoring T_D

β decay changes the Z of the nucleus but A remains constant.

$${}^{A}X^{Z} \rightarrow {}^{A}Y^{Z+1} + e^- + \bar{\nu}_e \qquad {}^{A}X^{Z} \rightarrow {}^{A}Y^{Z-1} + e^+ + \nu_e$$

β decay & the semi-empirical mass formula

$$M(A,Z)c^2 = \alpha A - \beta Z + \gamma Z^2 \pm a_5 A^{-\frac{3}{4}}$$

$$\alpha = m_n c^2 - a_1 + a_2 A^{-\frac{1}{3}} + a_4$$
$$\beta = 4a_4 + \left(m_n - m_p - m_e\right)c^2$$
$$\gamma = 4a_4 + a_3 A^{-\frac{1}{3}}$$

At fixed A the mass is quadratic in Z. At fixed A, a plot of M(A,Z) vs. Z is parabolic!

Beta stability

Even-A results from: even-Z & even-N (e-e), or odd-Z & odd-N (o-o)
At constant A, SEMF is parabolic in Z, with minimum at Z_{min}
Pairing term, $\pm a_5 A^{-3/4}$, makes e-e nuclei have a lower minima than o-o

β^- raises Z, β^+ or EC lowers Z

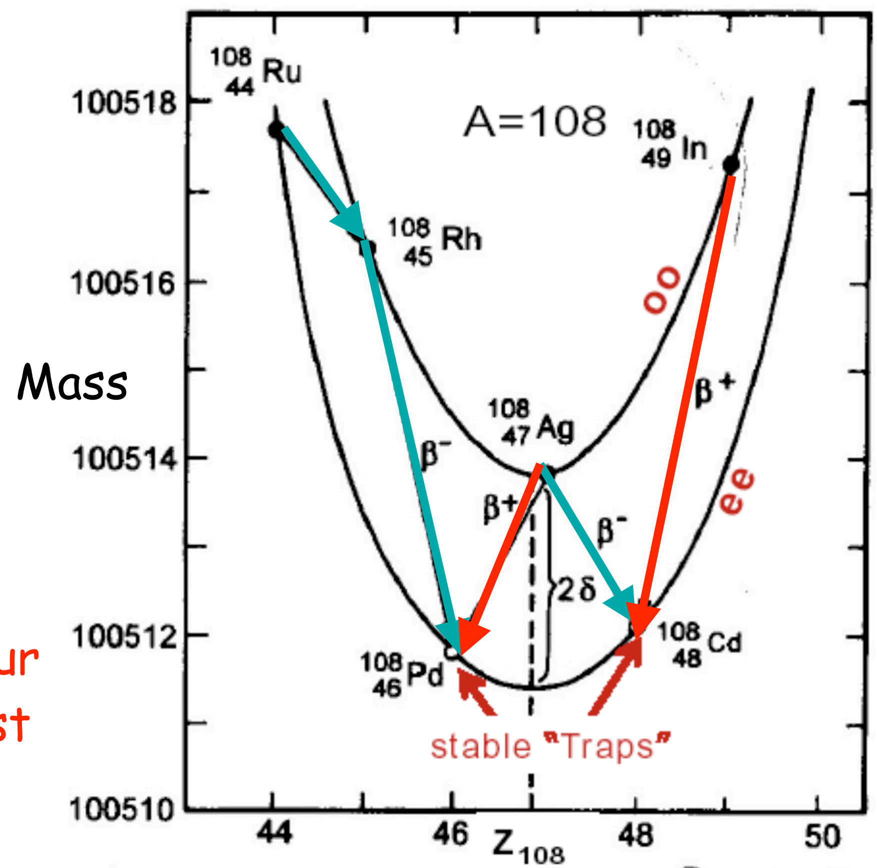
<u>Allowed β decays (short lifetimes)</u>

Parent and Daughter nuclei have the same parity, and differ in spin by no more than 1 unit ($\Delta j = j_D - j_P = 0, \pm 1$)

(A=108, e-e are 0^+, o-o are 1^+ or 2^+)

<u>"Forbidden" β decays</u>

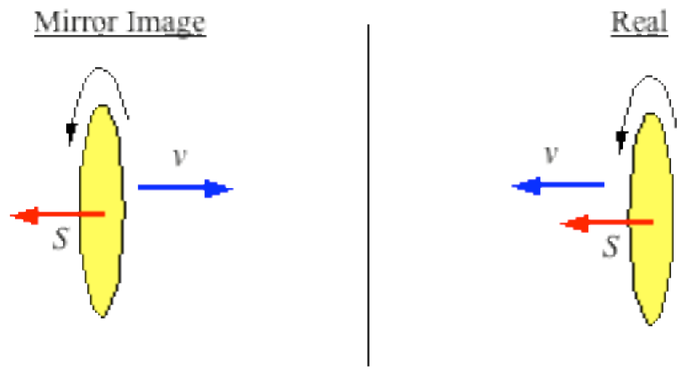
If allowed decays are absent, larger values of Δj or parity changes will occur but with much longer lifetimes. Largest Δj will have longest lifetime.



Parity violation in beta decay

The direction of travel reverses in a mirror (parity transformation)

Rotating disk moves **right**, spin points **left**



Rotating disk moves **left**, spin points **left**

Rotation direction doesn't change in a mirror and therefore spin direction doesn't change in a mirror

If the disk is an anti-neutrino emitted in a beta decay, one better not find more anti-neutrinos with spin along, rather than opposite to, the velocity. In a mirror velocity reverses, but spin would not: parity would be violated !

Experiment shows that all anti-neutrinos are emitted with their spin in the direction of motion. All the anti-neutrinos are "RIGHT HANDED"

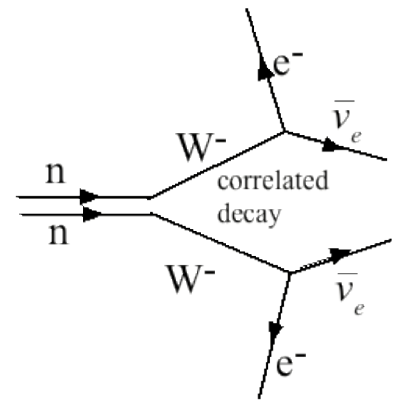
Parity is VIOLATED (maximally) in Weak Interactions

Double β decay

$^{100}Mo^{42}$ should be stable (mass $^{100}Tc^{43} > {}^{100}Mo^{42}$)

Double beta decay, *correlated* emission of two electrons and two anti-neutrinos, has been observed. Example: Molybdenum 100

$$^{100}Mo^{42} \rightarrow {}^{100}Ru^{44} + 2e^{-} + 2\bar{\nu}_e + 3\text{ MeV}$$

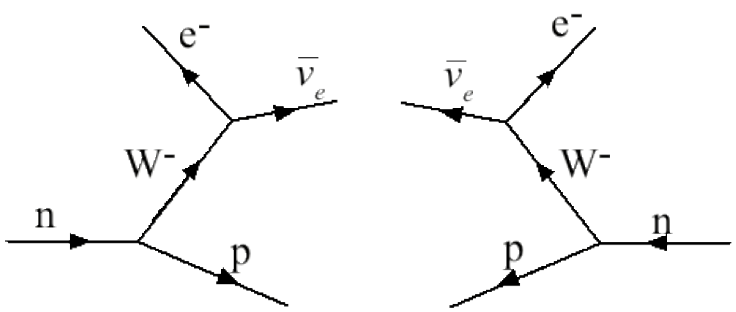


If neutrinos and anti-neutrinos are distinct particles they are "Dirac" neutrinos.

If an anti-neutrino is just a right-handed neutrino, they are called "Majorana" neutrinos.

Then neutrino-less double beta decay becomes possible.

Double Beta Decay



Neutrino-less Double Beta Decay