Lecture 9
Weak interactions
Radioactive dating
Mössbauer effect
Double $\beta$ decay

$^{100}\text{Mo}^{42}$ should be stable (mass $^{100}\text{Tc}^{43} > ^{100}\text{Mo}^{42}$)

Double beta decay, the correlated emission of two electrons and two anti-neutrinos, has been observed. Example: Molybdenum 100

If neutrinos and anti-neutrinos are distinct particles they are “Dirac” neutrinos.

If an anti-neutrino is just a right-handed neutrino, they are called “Majorana” neutrinos, and neutrino-less double beta decay becomes possible.

Double Beta Decay

Neutrino-less Double Beta Decay
Supernovae and weak interactions

Fusion reactions in a star

\[ p + p \rightarrow d + e^+ + \nu_e \]

\[ p + d \rightarrow ^3\text{He}^2 + \gamma + 5.49 \text{ MeV} \]

\[ ^{12}\text{C}^6 + ^{12}\text{C}^6 \rightarrow ^{24}\text{Mg}^{12} + \gamma + 13.93 \text{ MeV} \]

\[ \cdots \rightarrow ^{56}\text{Fe}^{26} \text{ (most negative B/A)} \]

Weak interactions start it

After a few \( \times 10^9 \) years

End of energy production by fusion

Thermal and Fermi pressures insufficient to resist gravitation resulting in compression of electrons and nucleons (disintegrated Iron)

\[ p + e^- \rightarrow n + \nu_e + 0.78 \text{ MeV} \]

Density is high enough that neutrinos are trapped by scattering and pick up remaining gravitational energy of the collapse.

In a few seconds a 1.4 solar mass core releases \( 3 \times 10^{46} \text{ J} \)

in \( 2 \times 10^{58} \) neutrinos (~15 MeV each)
Radioactive decay law

Decays/sec ~ # left to decay
\[-\frac{dN}{dt} = \lambda N \rightarrow N(t) = N_0 e^{-\lambda t}\]

Decay constant: \(\lambda\)

(count decays)

Decays/sec drop ~ \(\exp(-\lambda \times \text{time})\)

\[R(t) = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}\]

\[= R_0 e^{-\lambda t} \Rightarrow R_0 = \lambda N_0\]

(decay rates)

(mean) lifetime
\[\tau = 1/\lambda\]

half-life
\[t_{1/2} = (\ln 2) \tau = 0.693 \tau\]

Useful numbers (not in calculators!)

Activity Units:
1 bequerel = 1 Bq = 1 s\(^{-1}\)
1 rutherford = 1 rd = 1 \times 10^6 s\(^{-1}\)
1 curie = 1 Ci = 3.7 \times 10^{10} s\(^{-1}\)

1 yr = (365)(24)(60)(60) = 3.1 \times 10^7 sec \approx \pi \times 10^7 s
Age from sample amounts or decay rates

Method most useful when given sample amounts

\[ N(t) = N_0 e^{-\lambda t}; \text{ or } N_0 e^{-t/\tau} \text{ where } \tau = \frac{1}{\lambda} = \langle t \rangle \]

\[ t = \tau \ln \left( \frac{N_0}{N(t)} \right) \]

Method most useful when given sample decay rates

\[ R(t) = \frac{dN(t)}{dt} = -\lambda N_0 e^{-\lambda t} = -\lambda N(t); \quad R_0 = -\lambda N_0 \]

\[ R(t) = R_0 e^{-\lambda t} \]

Rate Method:

\[ t = \tau \ln \left( \frac{R_0}{R(t)} \right) \]
Carbon-14 Dating

Living Samples: \(^{14}\text{C}/^{12}\text{C} = 1.3 \times 10^{-12}\) \(^{14}\text{C} \rightarrow ^{14}\text{N} + \text{e}^{-}(156 \text{ keV}) + \nu\)

\[ \tau_{1/2} = 5730 \text{ yr} = 1.8 \times 10^{11} \text{ sec} \quad \tau = \frac{\tau_{1/2}}{0.693} = 2.6 \times 10^{11} \text{ s} \]

1g carbon sample. Rate now = 4.0 \times 10^{-12} \text{ Ci}. How old is it?

Fraction in a living sample

\(^{12}\text{C}: \quad N = A_0 \left( \frac{1 \text{ g}}{12 \text{ g}} \right) = 0.5 \times 10^{23} \)

\(^{14}\text{C}: \quad N_0 = \left( 0.5 \times 10^{23} \right) \left( 1.3 \times 10^{-12} \right) = 6.5 \times 10^{10} \)

\[ R(t) = \left( 4.0 \times 10^{-12} \text{ Ci} \right) \left( 3.7 \times 10^{10} \text{ sec}^{-1} \text{Ci}^{-1} \right) = 1.5 \times 10^{-1} \text{ sec}^{-1} \]

### Two methods of solution

**Preliminaries**

\[ \begin{align*}
N(t) &= R(t) \tau = 3.9 \times 10^{10} \\
\tau &= \ln \left( \frac{N_0}{N(t)} \right) = 1.3 \times 10^{11} \text{ s} \\
&= \text{1.3 \times 10^{11} s = 4200 yr} \\
\end{align*} \]

**Rate Method**

\[ \begin{align*}
R_0 &= \frac{N_0}{\tau} = 2.5 \times 10^{-1} \text{ sec}^{-1} \\
\tau &= \ln \left( \frac{R_0}{R(t)} \right) = 1.3 \times 10^{11} \text{ s} \\
&= \text{1.3 \times 10^{11} s} \\
\end{align*} \]
Cobalt 60 half life is 5.26y. What is the activity of a 1 gm $^{60}$Co sample?

**Preliminaries**

$$\tau = \frac{t_{1/2}}{\ln 2} = 7.59 \text{ yr} \left(3.1 \times 10^7 \text{ s/yr}\right) = 2.35 \times 10^8 \text{ s}$$

$$\lambda = \frac{1}{\tau} = 4.25 \times 10^{-9} \text{ s}^{-1}$$

$$N_0 = A_0 \frac{1}{60} = 1 \times 10^{22}$$

$$R_0 = -\lambda N_0 = -4.25 \times 10^{13} \text{ s}^{-1}$$

$$= \left(4.25 \times 10^{13} \text{ s}^{-1}\right) / \left(3.7 \times 10^{10} \text{ s}^{-1} / \text{Ci}\right) = 1.15 \times 10^3 \text{ Ci}$$

$$= \left(4.25 \times 10^{13} \text{ s}^{-1}\right) / \left(1 \times 10^6 \text{ s}^{-1} / \text{ru}\right) = 4.25 \times 10^7 \text{ ru}$$
Decay chains

The rate at which the amount of an isotope changes (called activity, \( A \))

\[
\frac{dN_1}{dt} = -\lambda_1 N_1; \quad \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2; \quad \ldots
\]

\[
A_i = \frac{dN_i}{dt} = \lambda_{i-1} N_{i-1} - \lambda_i N_i
\]

\[
A_n = N_0 \sum_{i=1}^{n} c_i e^{(-\lambda_i t)}; \quad N_0 = N_1(0)
\]

where \( c_m = \frac{\lambda_1 \lambda_2 \ldots \lambda_n}{(\lambda_1 - \lambda_m)(\lambda_2 - \lambda_m)\ldots(\lambda_n - \lambda_m)} \),

omit \( (\lambda_m - \lambda_m) \) in the denominator
Radioactive equilibrium

Assume a single parent nucleus with a long lifetime generates a series of short lived daughters from sequential decays

\[ X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow \ldots \]

The activity of each is given by the (feed-in rate) - (feed-out rate).

\[
\frac{dN_1}{dt} = -\lambda_1 N_1; \quad \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2; \quad \frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3; \quad \ldots
\]

After a long time one finds an equilibrium such that the number of each nucleus stops changing, i.e., the activity of each is constant.

\[
\frac{dN_1}{dt} = \frac{dN_2}{dt} = \frac{dN_3}{dt} = \ldots = 0 \quad \text{also,} \quad \lambda_1 N_1 = \lambda_2 N_2 = \lambda_3 N_3 = \ldots
\]
Measuring photon line widths

A nucleus that decays by photon emission with energy $E_\gamma$, should readily absorb these photons via resonant absorption. Nuclear recoil energies ruin this expectation for narrow line widths.

Consider $^{57}$Fe* decay to a 14.4 keV photon (lifetime $\sim 10^{-7}$s)

Energy of gamma: \[
(M_i - M_f)c^2 = E_\gamma + E_R \approx E_\gamma \approx 14.4 \text{ keV}
\]

Recoil energy: \[
E_R \approx \frac{E_\gamma^2}{2 M_R c^2} = 1 \times 10^{-3} \text{ eV}
\]

$\Delta E$ (line-width): \[
\Delta E \sim \frac{\hbar}{\Delta t} = \Gamma = 1 \times 10^{-8} \text{ eV}
\]

Make recoil energy much smaller by trapping Fe in a cold crystal so that about $10^5$ atoms move together as a single mass.

\[
E_R \approx \frac{E_\gamma^2}{2 M_R c^2} = 1 \times 10^{-8} \text{ eV}
\]

Now photons from a cold source will be absorbed by cold target.
Mössbauer effect

Motion of the source will Doppler shift gamma energies off resonance.

\[ E'_\gamma - E_\gamma = \Delta E_\gamma = \beta_S E_\gamma \]
\[ \beta_S = 10^{-8} \text{eV}/10^4 \text{eV} = 10^{-12} \]
\[ v_S \sim 10^{-12} c = 0.3 \text{ mm/s} \]

Nuclear line widths measured to $10^{-7}$ eV
and level spacing to 1 part in $10^{12}$.
Hyperfine level splitting is observable.

Gravitational “red shift” was measured using Mössbauer effect.
Light pointed down has a higher frequency that when pointed up
by 5 parts in $10^{15}$. 