Homework 3

**Alpha decay**

4.1 $Q$ values, $T_\alpha$, $T_D$

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>A</th>
<th>Z</th>
<th>Mass (MeV)</th>
<th>BE(MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>4</td>
<td>2</td>
<td>3728.43</td>
<td>-28.30</td>
</tr>
<tr>
<td>$^{204}$Pb</td>
<td>204</td>
<td>82</td>
<td>190001.16</td>
<td>-1607.54</td>
</tr>
<tr>
<td>$^{208}$Po</td>
<td>208</td>
<td>84</td>
<td>193734.82</td>
<td>-1630.62</td>
</tr>
<tr>
<td>$^{230}$Th</td>
<td>230</td>
<td>90</td>
<td>214276.18</td>
<td>-1755.16</td>
</tr>
<tr>
<td>$^{226}$Ra</td>
<td>226</td>
<td>88</td>
<td>210543.00</td>
<td>-1731.63</td>
</tr>
</tbody>
</table>

\[ Q = m_p - m_D - m_\alpha, \text{ or } Q = BE_p - BE_D - BE_\alpha, \quad T_\alpha = \frac{A - 4}{A}Q, \quad T_D = \frac{4}{A}Q \]

$^{208}$Po $\rightarrow$ $^{204}$Pb + $\alpha$: $Q = 5.22$ MeV, $T_D = 0.1$ MeV, $T_\alpha = 5.12$ MeV

$^{230}$Th $\rightarrow$ $^{226}$Ra + $\alpha$: $Q = 4.77$ MeV, $T_D = 0.08$ MeV, $T_\alpha = 4.69$ MeV
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4.3 Neutron beta decay

configuration for maximum electron energy

\[ p_e = p_p \quad \text{momentum conservation} \]

\[ Q = T_e + T_p = T_e + \frac{p_e^2}{2m_p} = T_e + \frac{T_e^2 + 2m_e T_e}{2m_p} \quad \text{energy conservation} \]

\[ 2m_p Q = 2 \left( m_p + m_e \right) T_e + T_e^2 \]

\[ 2m_p Q = \left[ 2 \left( m_p + m_e \right) + T_e \right] T_e; \quad \text{assume } T_e << m_p \]

\[ T_e \approx Q \frac{m_p}{m_p + m_e} \approx Q; \quad \text{max. electron energy} \]

\[ T_p \approx Q \frac{m_e}{m_p + m_e} \approx Q \frac{m_e}{m_p} \quad \text{max. proton energy} \]
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electron and neutrino energies \( \sim Q/2 \)

\[ p_e = p_\nu \]

\[ T_e + T_\nu = Q = T_e + p_\nu c = T_e + p_e c; \quad p_e c = Q - T_e \]

\[ p_e^2 c^2 = Q^2 - 2QT_e + T_e^2 = E_e^2 - m_e^2 = (T_e + m_e)^2 - m_e^2 \]

\[ T_e = \frac{Q}{2} \frac{1}{(1 + m_e/Q)}; \quad T_\nu = Q \left[ 1 - \frac{1}{2(1 + m_e/Q)} \right] \]

configuration for maximum neutrino energy

\[ T_p = \frac{Q}{2} \frac{1}{(1 + m_p/Q)} \quad T_\nu = Q \left[ 1 - \frac{1}{2(1 + m_p/Q)} \right] \approx Q \]

maximum neutrino energy
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4.5 Lepton number conservation

+1 for neutrinos and negative charged leptons
-1 for antineutrinos and positive charged leptons and

(a) $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$
(b) $\tau^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\tau$
(c) $e^- + ^AX^Z \rightarrow ^AY^{Z-1} + \nu_e$
(d) $\nu_\mu + n \rightarrow \mu^- + p$
(e) $^AX^Z \rightarrow ^AY^{Z-1} + e^+ + \nu_e$
(f) $\bar{\nu}_e + p \rightarrow e^+ + n$
4.7 Beta decay and S.E.M.F.

\[ M(A,Z)c^2 = \alpha A - \beta Z + \gamma Z^2 \pm a_3 A^{-\frac{3}{4}} \]

\[
\frac{dM}{dZ}c^2 = -\beta + 2\gamma Z = 0
\]

\[ Z = \frac{\beta}{2\gamma} \approx \frac{4a_4}{2(4a_4 + a_3 A^{-\frac{1}{3}})} \]

\[
= \frac{1}{2 + \frac{a_3}{2a_4} A^{-\frac{1}{3}}} = \left[ 2 + \left(1.5 \times 10^{-2}\right) A^{-\frac{1}{3}} \right]^{-1}
\]

Shell Model would favor \( Z=126, N=184, A=Z+N=310 \)

Very close to the \( \frac{Z^2}{A} = 51 \) (>47) susceptible to fission
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5.1 Neutron energy loss in scattering
(see Problem 2.9)

\[
e_1 = \left( \frac{A-1}{A+1} \right)^2 e_0
\]

energy of neutron after one backscatter off A.

\[
e_1 = \left( \frac{A-1}{A+1} \right)^2 e_0 = 0; \quad A = 1
\]

one backscatter off \(^1\text{H} \).

\[
e_1 = \left( \frac{A-1}{A+1} \right)^2 e_0 = \left( \frac{26}{28} \right)^2 e_0 = 0.86e_0
\]

one backscatter off \(^{27}\text{Al} \).
5.3 Radioactive decay rate

\[ R(t) = A \exp\left(\frac{t}{\tau}\right); \]
\[ A = 96 \text{ / min} \]
\[ \frac{1}{\tau} = 0.0031 \text{ min}^{-1} \quad \tau = 322 \text{ min} \]

5.5 Proton decay, 10^3 metric tons of H_2O

\[ N - N_0 = N_0 \left(1 - e^{-t/\tau}\right) \approx \frac{N_0 t}{\tau} \]
\[ \frac{N_0}{\tau} \text{ is the rate per year, } N = \frac{N_0 t}{\tau} \quad \tau = 10^{33} \text{ yrs} \]

\[ m_0 = 10^9 \text{ gm, } N_0 = \left(\frac{2}{18}\right)10^9 \left(6 \times 10^{23}\right) = 6.7 \times 10^{31} \]

\[ R_0 = \frac{N_0}{\tau} = \frac{6.7 \times 10^{31}}{10^{33} \text{ yr}} = 6.7 \times 10^{-2} \text{ yr}^{-1}; \quad N = \left(6.7 \times 10^{-2} \text{ yr}^{-1}\right)(50 \text{ yr}) = 3.3 \]
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5.7 Power Plant: 500MW, $^{235}\text{U}$, 5% efficiency

500MW = $5 \times 10^8$ J/s / $1.6 \times 10^{-13}$ J/MeV = $3.0 \times 10^{21}$ MeV/s

$Q / \text{nucleus} \approx 200$ MeV/nucleus (see D&F calculation 5.2)

$R_{nuclei} = \left( 3 \times 10^{21} \text{ MeV/s} \right) / (200 \text{ MeV/nucleus}) = 1.5 \times 10^{19}$ nuclei/s

$R_{mass} = \frac{(235 \text{ gm/mole}) \left( 1.5 \times 10^{19} \text{ nuclei/s} \right)}{6.02 \times 10^{23} \text{ nuclei/mole}} = 6.0 \times 10^{-3} \text{ gm/s}$

= $(6.0 \times 10^{-3} \text{ gm/s})(86400 \text{ s/day}) = 520$ gm/day

but, only 5% conversion of heat to electricity

$R_{elec} = \frac{520 \text{ gm/day}}{.05} = 10,000 \text{ gm/day} = 10 \text{ kg/day}$
Exam 1, Wednesday

Don’t forget Martinus Veltman’s book, chapters 1-4
Das & Ferbel, chapters 1-5, homework 1,2,3
Bromberg, lectures 1-10,
Elementary particle physics

• **Old question:** what are the “fundamental” constituents of matter?
  
  - **Matter is constructed from the (elementary) elements of the periodic table**
    
    • Atoms: H, He, ..., C, ..., O, ... U, ...
    
    • Molecules: H₂O, CO₂, O₂, ...
  
  - **Atoms have internal structure**
    
    • Atomic nucleus: positive charge, mass \( \sim A \times 931.5 \text{ MeV/c}^2 \)
    
    • Atomic electrons: negative charge, small mass 0.51 MeV/c²
  
  - **Nuclei have internal structure**
    
    • Protons, charge +1e, mass, \( \sim 938.3 \text{MeV/c}^2 \)
    
    • Neutrons, charge 0, mass \( \sim 939.6 \text{MeV/c}^2 \)
    
    • Minus |binding energy|
  
  - **Nucleons and other “hadrons” have internal structure**
    
    • Three charge \(+2/3e\) quarks, masses \( \sim 50, 1500, 175,000 \text{ MeV/c}^2 \)
    
    • Three charge \(-1/3e\) quarks, masses \( \sim 5, 500, 5100 \text{ MeV/c}^2 \)
  
• **Does this sequence of ever finer elemental structure continue forever?**
  
  - **Do quarks have substructure, and higher energies needed to see it?**
  
  - **Or develop a new paradigm**
New paradigm of elementary particle physics

• The “Standard Model” of elementary particle physics
  - Quarks and leptons are unique disturbances in the “fabric” of space-time. The number of disturbance types is limited.
  - Quarks and leptons obtain their unique masses through interactions of the disturbance with a hypothetical “Higgs” field.
  - Gauge particles (photon, gluon, and weak bosons) mediate interactions between the quarks and leptons

• What the standard model does not predict
  - Where does gravity fit into the picture
  - Origin of the particle - antiparticle asymmetry in the universe
  - Mixing of mass and flavor states for quarks and leptons
  - The relationship between quarks and leptons
  - The nature of “dark matter” and “dark energy”
  - Surprises of unknown origin
Interactions

- Interactions (forces) occur between (act on) “charges”
  - Strong (color charge, QCD)
    • quarks carry color charge, \( Q_c = r, g, b \)
  - Electromagnetic (electric charge)
    • quarks \( Q_e = -1/3, +2/3 \)
    • charged leptons \( Q_e = -1 \)
  - Weak (weak charge)
    • quark flavors: up, charm, top \( Q_w = +1/2 \)
      down, strange, bottom \( Q_w = -1/2 \)
    • leptons: neutrinos - \( \nu_e, \nu_\mu, \nu_\tau \) \( Q_w = +1/2 \)
      charged - \( e^-, \mu^-, \tau^- \) \( Q_w = -1/2 \)
- Quantities conserved by the interactions
  - Electric Charge, Baryon (3 quarks) number, Lepton number,
  - Quark flavor (Can be violated by Weak Interactions)
  - Isospin (Conserved only by Strong Interactions)
Leptonic (point-like) particles of the Standard Model

- Lepton “flavors” (spin = 1/2)
  - Electric charge $Q = -e$
    - $e^-$, electron, mass = 0.511 MeV/$c^2$
    - $\mu^-$, muon, mass = 105 MeV/$c^2$
    - $\tau^-$, tau, mass = 1.77 GeV/$c^2$
  - Electric charge $Q = 0$ (left-handed)
    - Flavor states (same as the charged leptons)
      - $\nu_e$, electron neutrino
      - $\nu_\mu$, muon neutrino
      - $\nu_\tau$, tau neutrino
    - Mass states: $\nu_1, \nu_2, \nu_3$ ($m < 1$ eV/$c^2$)
- Anti-leptons
  - charged: $e^+, \mu^+, \tau^+$
  - neutral: $\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$
Hadronic (point-like) particles of the Standard Model

- Quark “flavors” (spin = 1/2)
  - Electric charge $Q = +2/3e$
    - $u$, up, mass ~ 3 MeV$/c^2$
    - $c$, charm, mass ~ 1.2 GeV$/c^2$
    - $t$, top, mass ~ 175 GeV$/c^2$
  - Electric charge $Q = -1/3e$
    - $d$, down, mass ~ 7 MeV$/c^2$
    - $s$, strange, mass ~ 120 MeV$/c^2$
    - $b$, bottom, mass ~ 4.2 GeV$/c^2$

- Anti-quark “flavors” (spin = 1/2)
  - $Q = -2/3e$: $\bar{u}, \bar{c}, \bar{t}$
  - $Q = -2/3e$: $\bar{d}, \bar{s}, \bar{b}$

- Hadrons (colorless quark combinations)
  - Baryons: three quarks, one of each color ($Q = -1$ to $+2e$, spin = 1/2, 3/2)
    - light: proton ($uud$), neutron ($udd$), $\Delta^+$ ($uuu$), ...
    - strange, charm, bottom: $\Xi_c (csd)$, $\Lambda_b (bdu)$, ... no top baryons

  - Mesons: quark & antiquark pair (color/anti-color, colorless, spin = 0, 1)
    - light: $\pi^+(u\bar{d})$, $K^-(s\bar{u})$, ...
    - strange, charm, bottom: $D^+(cd\bar{d})$, $B^0 (bd\bar{d})$, ..., no top mesons
Hadron masses and binding energy

- **$^3$He Nucleus (ppn)**
  - proton, mass = 938.27 MeV/$c^2$
  - neutron mass = 939.57 MeV/$c^2$

\[
(2m_p + m_n) c^2 = 2816.11 \text{ MeV}
\]

\[
(m_{\text{He}}^3) c^2 = 2808.39 \text{ MeV}
\]

\[
BE = (m_{\text{He}}^3 - 2m_p - m_n) c^2 = -7.72 \text{ MeV}
\]

BE is negative. Energy is radiated to form the nucleus. i.e., energy is needed to break it up

- **Proton (uud)**
  - $u$, up, mass ~ 3 MeV/$c^2$
  - $d$, down, mass ~ 7 MeV/$c^2$

\[
(2m_u + m_d) c^2 \approx 13 \text{ MeV}
\]

\[
m_p c^2 = 938.27 \text{ MeV}
\]

\[
BE = (m_p - 2m_u - m_d) c^2 = + 925 \text{ MeV}
\]

BE is POSITIVE? Is this “binding”? Proton should blow apart!

Quantum Chromo-Dynamics (QCD) resolves the paradox
QCD allows quarks to remain low mass (asymptotic freedom) when inside a hadron. However, the color force increases the quark mass greatly ($m_{\text{quark}} \gg m_{\text{hadron}}$) if separated >1 fm from others. Quarks confined within hadrons, there are NO free quarks.
Baryon quantum number

• Baryon number (B)
  
  B = +1 for three quarks \((qqq)\) in a color singlet \((r, g, b)\)
  
  B = −1 for three anti-quarks \((\bar{q} \bar{q} \bar{q})\) in a color singlet \((\bar{r}, \bar{g}, \bar{b})\)
  
  B = 0 for meson \((q, \bar{q})\) color singlet states
  
  Note: some recent evidence for \((qqqq\bar{q})\) baryon states

• Baryon number conservation
  
  - Experiment finds B conserved in all interactions
  
  - Baryon number violation required to generate matter asymmetry
  
  - Unseen transformation lepton \(\leftrightarrow\) quark violates B conservation
  
  - Proton decay violates B but may conserve B - L (e.g., \(p \rightarrow e^+ \pi^0\))

Examples

<table>
<thead>
<tr>
<th>Decay: (p \not\rightarrow \pi^+ \pi^0)</th>
<th>B = +1 (\not\rightarrow) B = 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collision: (pp \rightarrow ppp\bar{p})</td>
<td>B = +2 (\rightarrow) B = +3 − 1 = +2</td>
</tr>
</tbody>
</table>