PHY492: Nuclear & Particle Physics

Lecture 9

Weak interactions

Radioactive dating

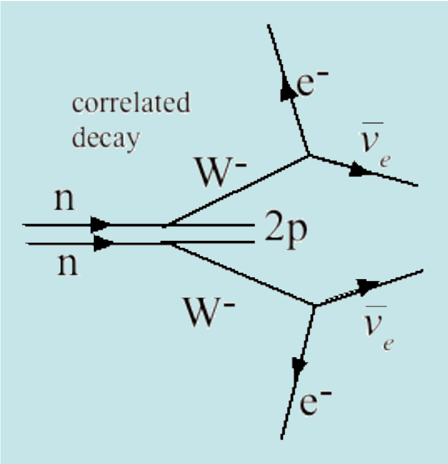
Mössbauer effect

Double β decay

$^{100}Mo^{42}$ should be stable (mass $^{100}Tc^{43} > {}^{100}Mo^{42}$)

Double beta decay, the *correlated* emission of two electrons and *two anti-neutrinos*, has been observed. Example: Molybdenum 100

$$^{100}Mo^{42} \rightarrow {}^{100}Ru^{44} + 2(e^- \bar{\nu}_e) + 3\text{ MeV}$$

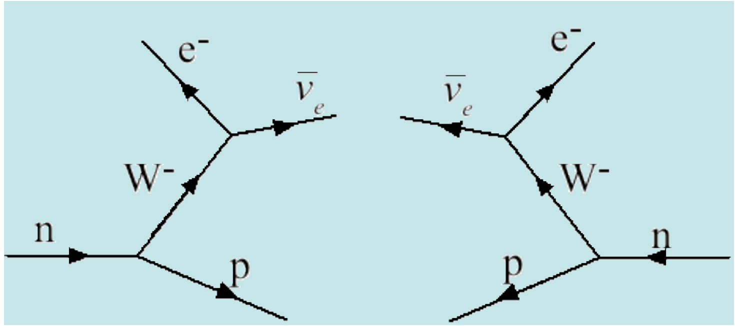


If neutrinos and anti-neutrinos are distinct particles they are "Dirac" neutrinos.

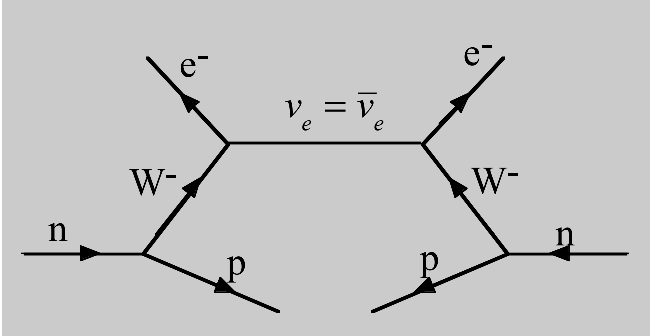
If an anti-neutrino is just a right-handed neutrino, they are called "Majorana" neutrinos, and

<u>neutrino-less</u> double beta decay becomes possible.

<u>Double Beta Decay</u>



<u>Neutrino-less Double Beta Decay</u>



Supernovae and weak interactions

Fusion reactions in a star

$$p + p \rightarrow d + e^{+} + \nu_e$$

Weak interactions start it

$$p + d \rightarrow {}^{3}He^{2} + \gamma + 5.49 \text{ MeV}$$

$$\vdots$$

After a few x 10^9 years

$${}^{12}C^{6} + {}^{12}C^{6} \rightarrow {}^{24}Mg^{12} + \gamma + 13.93 \text{ MeV}$$

$$\vdots$$

$$\cdots \rightarrow {}^{56}Fe^{26} \text{ (most negative B/A)}$$

End of energy production by fusion

Thermal and Fermi pressures insufficient to resist gravitation resulting in compression of electrons and nucleons (disintegrated Iron)

$$p + e^{-} \rightarrow n + \nu_e + 0.78 \text{ MeV}$$

Weak interactions end it

Density is high enough that neutrinos are trapped by scattering and pick up remaining gravitational energy of the collapse.

In a few seconds a 1.4 solar mass core releases 3×10^{46} J in 2×10^{58} neutrinos (~15 MeV each)

Radioactive decay law

Decays/sec ~ # left to decay

$$-\frac{dN}{dt} = \lambda N \quad \rightarrow \quad N(t) = N_0 e^{-\lambda t}$$

count decays

Decays/sec drop ~ exp(-λ x time)

$$R(t) = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$$

$$= R_0 e^{-\lambda t} \qquad R_0 = \lambda N_0$$

decay rates

Decay constant: λ

(mean) lifetime

$$\tau = 1/\lambda$$

half-life

$$t_{\frac{1}{2}} = (\ln 2)\tau = 0.693\tau$$

Useful numbers (not in calculators!)

Activity Units:

$$1 \text{ bequerel} = 1 \text{ Bq} = 1 \text{ s}^{-1}$$

$$1 \text{ rutherford} = 1 \text{ rd} = 1 \times 10^6 \text{ s}^{-1}$$

$$1 \text{ curie} = 1 \text{ Ci} = 3.7 \times 10^{10} \text{ s}^{-1}$$

$$1 \text{ yr} = (365)(24)(60)(60) = 3.1 \times 10^7 \text{ sec} \quad \approx \pi \times 10^7 \text{ s}$$

Age from sample amounts or decay rates

Method most useful when given sample amounts

$$N(t) = N_0 e^{-\lambda t}; \quad \text{or} \quad N_0 e^{-t/\tau} \text{ where } \tau = \frac{1}{\lambda} = \langle t \rangle$$

\# Method:

$$t = \tau \ln\left[\frac{N_0}{N(t)}\right]$$

Method most useful when given sample decay rates

$$R(t) = \frac{dN(t)}{dt} = -\lambda N_0 e^{-\lambda t} = -\lambda N(t); \quad R_0 = -\lambda N_0$$

$$R(t) = R_0 e^{-\lambda t}$$

Rate Method:

$$t = \tau \ln\left[\frac{R_0}{R(t)}\right]$$

Carbon-14 Dating

Living Samples: $^{14}C/^{12}C = 1.3\times10^{-12}$ $^{14}C \rightarrow {}^{14}N + e^{-}$ (156 keV) + ν

$\tau_{1/2} = 5730 \text{ yr} = 1.8\times10^{11} \text{ sec}$ $\tau = \tau_{\frac{1}{2}}/0.693 = 2.6\times10^{11} \text{s}$

1g carbon sample. Rate now = 4.0×10^{-12} Ci. How old is it?

Preliminaries

$$^{12}\text{C:}\ \ N = A_0\left(\frac{1\,\text{g}}{12\,\text{g}}\right) = 0.5\times10^{23}$$

fraction in a living sample

$$^{14}\text{C:}\ \ N_0 = \left(0.5\times10^{23}\right)\left(1.3\times10^{-12}\right) = 6.5\times10^{10}$$

$$R(t) = \left(4.0\times10^{-12}\ \text{Ci}\right)\left(3.7\times10^{10}\ \text{sec}^{-1}\text{Ci}^{-1}\right) = 1.5\times10^{-1}\ \text{sec}^{-1}$$

Two methods of solution

Method

$$N(t) = R(t)\tau = 3.9\times10^{10}$$

$$t = \tau\ln\left[\frac{N_0}{N(t)}\right] = 1.3\times10^{11}\ \text{s}$$

Rate Method

$$R_0 = \frac{N_0}{\tau} = 2.5\times10^{-1}\ \text{sec}^{-1}$$

$$t = \tau\ln\left[\frac{R_0}{R(t)}\right] = 1.3\times10^{11}\ \text{s}$$

$$1.3\times10^{11}\ \text{s} = 4200\ \text{yr}$$

Generic example

Cobalt 60 half life is 5.26y. What is the activity of a 1 gm ^{60}Co sample?

Preliminaries

$$\tau = \frac{t_{1/2}}{\ln 2} = 7.59 \text{ yr}\left(3.1\times10^{7} \text{ s/yr}\right) = 2.35\times10^{8}\text{ s}$$

$$\lambda = \frac{1}{\tau} = 4.25\times10^{-9}\text{ s}^{-1}$$

$$N_0 = A_0 \frac{1}{60} = 1\times10^{22}$$

$$R_0 = -\lambda N_0 = -4.25\times10^{13}\text{ s}^{-1}$$

$$= \left(4.25\times10^{13}\text{ s}^{-1}\right)/\left(3.7\times10^{10}\text{ s}^{-1}/\text{Ci}\right) = 1.15\times10^{3}\text{ Ci}$$

$$= \left(4.25\times10^{13}\text{ s}^{-1}\right)/\left(1\times10^{6}\text{ s}^{-1}/\text{ru}\right) = 4.25\times10^{7}\text{ ru}$$

Decay chains

The rate at which the amount of an isotope changes (called activity, A)

= (feed in rate) – (feed out rate).

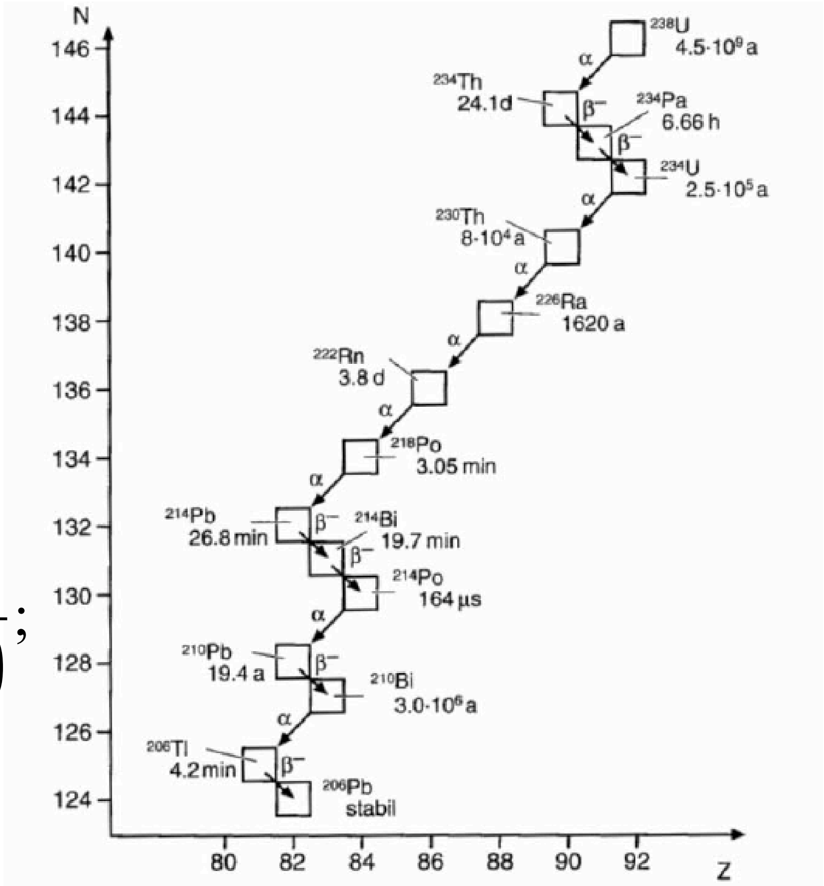
$$\frac{dN_1}{dt} = -\lambda_1 N_1; \quad \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2; \quad \ldots$$

$$A_i = \frac{dN_i}{dt} = \lambda_{i-1} N_{i-1} - \lambda_i N_i$$

$$A_n = N_0 \sum_{i=1}^{n} c_i e^{(-\lambda_i t)}; \quad N_0 = N_1(0)$$

$$\text{where } c_m = \frac{\lambda_1 \lambda_2 \ldots \lambda_n}{(\lambda_1 - \lambda_m)(\lambda_2 - \lambda_m)\ldots(\lambda_n - \lambda_m)};$$

omit $(\lambda_m - \lambda_m)$ in the denominator



Radioactive equilibrium

Assume a single parent nucleus with a long lifetime generates a series of short lived daughters from sequential decays

$$X_1 \rightarrow X_2 \rightarrow X_3 \rightarrow \ldots$$

The activity of each is given by the (feed-in rate) - (feed-out rate).

$$\frac{dN_1}{dt} = -\lambda_1 N_1; \quad \frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2; \quad \frac{dN_3}{dt} = \lambda_2 N_2 - \lambda_3 N_3; \quad \ldots$$

After a long time one finds an equilibrium such that the number of each nucleus stops changing, i.e., the activity of each is constant.

$$\frac{dN_1}{dt} = \frac{dN_2}{dt} = \frac{dN_3}{dt} = \ldots = 0 \qquad \text{also,}\ \ \lambda_1 N_1 = \lambda_2 N_2 = \lambda_3 N_3 = \ldots$$

Measuring photon line widths

A nucleus that decays by photon emission with energy E_γ, should readily absorb these photons via resonant absorption. Nuclear recoil energies ruin this expectation for narrow line widths.

Consider $^{57}Fe^{*}$ decay to a 14.4 keV photon (lifetime ~10^{-7}s)

Energy of gamma: $\left(M_i - M_f\right)c^2 = E_\gamma + E_R \approx E_\gamma \approx 14.4\text{ keV}$

Recoil energy: $E_R \approx E_\gamma^2 / 2M_R c^2 = 1\times 10^{-3}\text{eV}$

ΔE (line-width): $\Delta E \sim \hbar/\Delta t = \Gamma = 1\times 10^{-8}\text{eV}$

} no resonant absorption

Make recoil energy much smaller by trapping Fe in a cold crystal so that about 10^5 atoms move together as a single mass.

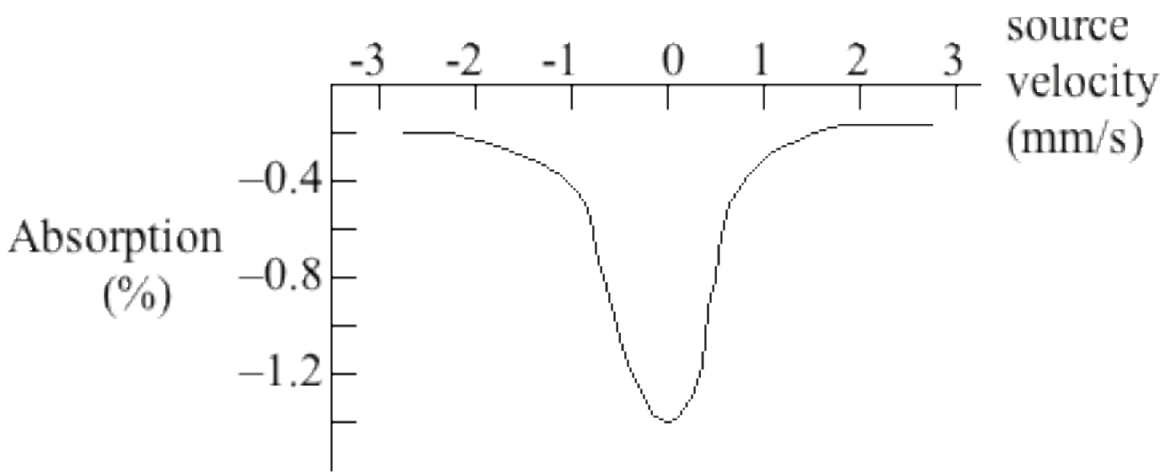
$$E_R \approx E_\gamma^2 / 2M_R c^2 = 1\times 10^{-8}\text{eV}$$

Now photons from a cold source will be absorbed by cold target

Mössbauer effect

Motion of the source will Doppler shift gamma energies off resonance .

$$E'_\gamma - E_\gamma = \Delta E_\gamma = \beta_S E_\gamma$$
$$\beta_S = 10^{-8}\,\text{eV}/10^4\,\text{eV} = 10^{-12}$$
$$v_S \sim 10^{-12} c = 0.3\ \text{mm/s}$$



Nuclear line widths measured to 10^{-7} eV
and level spacing to 1 part in 10^{12}.
Hyperfine level splitting is observable.

Gravitational "red shift" was measured using Mössbauer effect.
Light pointed down has a higher frequency that when pointed up
by 5 parts in 10^{15}.