PHY492: Nuclear & Particle Physics

Lecture 10

Homework 3

Elementary Particle Physics

Alpha decay

4.1 Q values, T_α, T_D

Nuclei	A	Z	Mass (MeV)	BE(MeV)
α	4	2	3728.43	-28.30
^{204}Pb	204	82	190001.16	-1607.54
^{208}Po	208	84	193734.82	-1630.62
^{230}Th	230	90	214276.18	-1755.16
^{226}Ra	226	88	210543.00	-1731.63

$$Q = m_P - m_D - m_\alpha \text{, or } Q = BE_P - BE_D - BE_\alpha \text{, } T_\alpha = \frac{A-4}{A}Q, \quad T_D = \frac{4}{A}Q$$

$$^{208}\text{Po} \rightarrow {}^{204}\text{Pb} + \alpha : \quad Q = 5.22 \text{ MeV}, \quad T_D = 0.1 \text{ MeV}, \; T_\alpha = 5.12 \text{ MeV}$$

$$^{230}\text{Th} \rightarrow {}^{226}\text{Ra} + \alpha : \quad Q = 4.77 \text{ MeV}, \; T_D = 0.08 \text{ MeV}, \; T_\alpha = 4.69 \text{ MeV}$$

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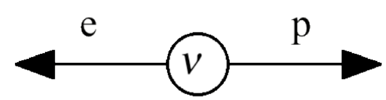
4.3 Neutron beta decay

configuration for maximum electron energy



$$E_e^2 = (T_e + m_e)^2$$

$$p_e^2 = E_e^2 - m_e^2 = T_e^2 + 2m_eT_e$$

$p_e = p_p$ momentum conservation

$$Q = T_e + T_p = T_e + \frac{p_e^2}{2m_p} = T_e + \frac{T_e^2 + 2m_eT_e}{2m_p}$$ energy conservation

$$2m_pQ = 2(m_p + m_e)T_e + T_e^2$$

$$2m_pQ = \left[2(m_p + m_e) + T_e\right]T_e; \quad \text{assume } T_e << m_p$$

$$T_e \approx Q\frac{m_p}{m_p + m_e} \approx Q; \qquad T_p \approx Q\frac{m_e}{m_p + m_e} \approx Q\frac{m_e}{m_p}$$

max. electron energy

max. proton energy

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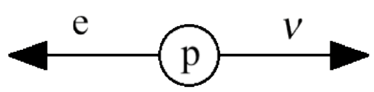
electron and neutrino energies ~ Q/2



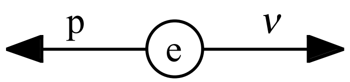
$$p_e = p_\nu$$

$$T_e + T_\nu = Q = T_e + p_\nu c = T_e + p_e c; \quad p_e c = Q - T_e$$

$$p_e^2 c^2 = Q^2 - 2QT_e + T_e^2 = E_e^2 - m_e^2 = (T_e + m_e)^2 - m_e^2$$

$$T_e = \frac{Q}{2}\frac{1}{(1 + m_e/Q)}; \quad T_\nu = Q\left[1 - \frac{1}{2(1 + m_e/Q)}\right]$$

configuration for maximum neutrino energy



$$T_p = \frac{Q}{2}\frac{1}{(1 + m_p/Q)}$$

$$T_\nu = Q\left(1 - \frac{1}{2(1 + m_p/Q)}\right) \approx Q$$

maximum neutrino energy

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4.5 Lepton number conservation

+1 for neutrinos and negative charged leptons

–1 for antineutrinos and positive charged leptons and

(a) $\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$

(b) $\tau^+ \to e^+ + \nu_e + \bar{\nu}_\tau$

(c) $e^- + {}^{A}X^{Z} \to {}^{A}Y^{Z-1} + \nu_e$

(d) $\nu_\mu + n \to \mu^- + p$

(e) ${}^{A}X^{Z} \to {}^{A}Y^{Z-1} + e^+ + \nu_e$

(f) $\bar{\nu}_e + p \to e^+ + n$

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4.7 Beta decay and S.E.M.F.

$$M(A,Z)c^2 = \alpha A - \beta Z + \gamma Z^2 \pm a_5 A^{-\frac{3}{4}}$$

$$\alpha = m_n c^2 - a_1 + a_2 A^{-\frac{1}{3}} + a_4$$

$$\beta = 4a_4 + \left(m_n - m_p\right)c^2 \approx 4a_4$$

$$\gamma = 4a_4 + a_3 A^{-\frac{1}{3}}$$

$$\frac{a_3}{2a_4} = \frac{0.72}{46.6} \approx 1.5\times 10^{-2}$$

$$\frac{dM}{dZ}c^2 = -\beta + 2\gamma Z = 0$$

$$Z = \frac{\beta}{2\gamma} \approx \frac{4a_4}{2\left(4a_4 + a_3 A^{-\frac{1}{3}}\right)}$$

$$= \frac{1}{\left(2 + \frac{a_3}{2a_4}A^{-\frac{1}{3}}\right)} = \frac{1}{\left[2 + \left(1.5\times 10^{-2}\right)A^{-\frac{1}{3}}\right]}$$

Shell Model would favor Z=126, N=184, A=Z+N=310

Very close to the Z^2/A =51 (>47) susceptible to fission

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<u>5.1 Neutron energy loss in scattering</u>

(see Problem 2.9)

$$E_1 = \left(\frac{A-1}{A+1}\right)^2 E_0$$

energy of neutron after one backscatter off A.

$$E_1 = \left(\frac{A-1}{A+1}\right)^2 E_0 = 0; \quad A = 1$$

one backscatter off ^{1}H.

$$E_1 = \left(\frac{A-1}{A+1}\right)^2 E_0 = \left(\frac{26}{28}\right)^2 E_0 = 0.86 E_0$$

one backscatter off ^{27}Al.

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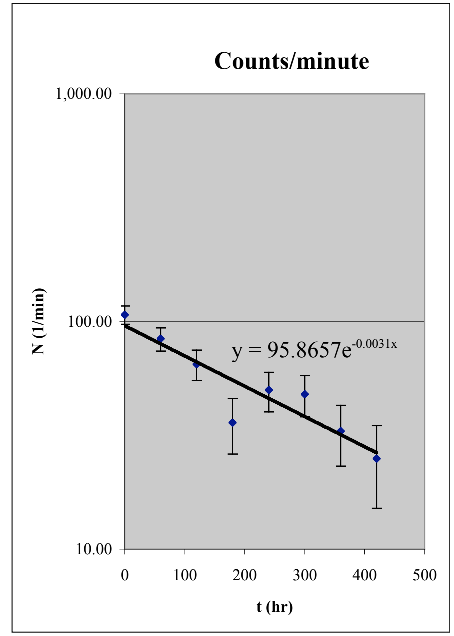
5.3 Radioactive decay rate

$$R(t) = A\exp[t/\tau];$$

$$A = 96\ /\ \text{min}$$

$$1/\tau = .0031\ \text{min}^{-1} \qquad \tau = 322\,\text{min}$$



5.5 Proton decay, 10^3 metric tons of H_2O

$$N - N_0 = N_0\left(1 - e^{-t/\tau}\right) \approx \frac{N_0}{\tau}t$$

$$\frac{N_0}{\tau} \text{ is the rate per year, } N = \frac{N_0}{\tau}t \quad \tau = 10^{33}\ \text{yrs}$$

$$m_0 = 10^9\ \text{gm},\ N_0 = \left(\frac{2}{18}\right)10^9\left(6\times10^{23}\right) = 6.7\times10^{31}$$

$$R_0 = \frac{N_0}{\tau} = \frac{6.7\times10^{31}}{10^{33}\ \text{yr}} = 6.7\times10^{-2}\ \text{yr}^{-1}; \quad N = \left(6.7\times10^{-2}\ \text{yr}^{-1}\right)\left(50\ \text{yr}\right) = 3.3$$

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5.7 Power Plant: 500MW, ^{235}U, 5% efficiency

$$500\text{MW} = 5\times10^{8}\ \text{J/s} / 1.6\times10^{-13}\ \text{J/MeV} = 3.0\times10^{21}\ \text{MeV/s}$$

$$Q\ /\ \text{nucleus}\ \approx\ 200\ \text{MeV/nucleus}\ \ \text{(see D\&F calculation 5.2)}$$

$$R_{nuclei} = \left(3\times10^{21}\ \text{MeV/s}\right)/\ (200\ \text{MeV/nucleus}) = 1.5\times10^{19}\ \text{nuclei/s}$$

$$R_{mass} = \frac{(235\ \text{gm/mole})\left(1.5\times10^{19}\ \text{nuclei/s}\right)}{6.02\times10^{23}\ \text{nuclei/mole}} = 6.0\times10^{-3}\ \text{gm/s}$$

$$= \left(6.0\times10^{-3}\ \text{gm/s}\right)\left(86400\ \text{s/day}\right) = 520\ \text{gm/day}$$

but, only 5% conversion of heat to electricity

$$R_{elec} = \frac{520\ \text{gm/day}}{.05} = 10{,}000\ \text{gm/day} = 10\ \text{kg/day}$$

Exam 1, Wednesday

Don’t forget Martinus Veltman’s book, chapters 1-4
Das & Ferbel, chapters 1-5, homework 1,2,3
Bromberg, lectures 1-10,

Elementary particle physics

- Old question: what are the “fundamental” constituents of matter?
  - Matter is constructed from the (elementary) elements of the periodic table
    - Atoms; H, He, ..., C, ..., O, ... U, ...
    - Molecules: $H_2 0$, CO_2, O_2, ...
  - Atoms have internal structure
    - Atomic nucleus: positive charge, mass ~ A x 931.5 MeV/c^2
    - Atomic electrons: negative charge, small mass 0.51 MeV/c^2
  - Nuclei have internal structure
    - Protons, charge +1e, mass, ~ 938.3MeV/c^2
    - Neutrons, charge 0, mass ~ 939.6MeV/c^2
    - Minus |binding energy|
  - Nucleons and other “hadrons” have internal structure
    - Three charge $+2/3e$ quarks, masses ~ 50, 1500, 175,000 MeV/c^2
    - Three charge $-1/3e$ quarks, masses ~ 5, 500, 5100 MeV/c^2
- Does this sequence of ever finer elemental structure continue forever?
  - Do quarks have substructure, and higher energies needed to see it?
  - Or develop a new paradigm

New paradigm of elementary particle physics

- The "Standard Model" of elementary particle physics
  - Quarks and leptons are unique disturbances in the "fabric" of space-time. The number of disturbance types is limited.
  - Quarks and leptons obtain their unique masses through interactions of the disturbance with a hypothetical "Higgs" field.
  - Gauge particles (photon, gluon, and weak bosons) mediate interactions between the quarks and leptons
- What the standard model does not predict
  - Where does gravity fit into the picture
  - Origin of the particle - antiparticle asymmetry in the universe
  - Mixing of mass and flavor states for quarks and leptons
  - The relationship between quarks and leptons
  - The nature of "dark matter" and "dark energy"
  - Surprises of unknown origin

Interactions

- Interactions (forces) occur between (act on) "charges"
  - Strong (color charge, QCD)
    - quarks carry color charge, (Q_c = r, g, b)
  - Electromagnetic (electric charge)
    - quarks (Q_e =-1/3, +2/3)
    - charged leptons (Q_e = -1)
  - Weak (weak charge)
    - quark flavors: *up*, *charm*, *top* (Q_w = +1/2)
      down, *strange*, *bottom* (Q_w = -1/2)
    - leptons: neutrinos - ν_e, ν_μ, ν_τ (Q_w = +1/2)
      charged - e^-, μ^-, τ^- (Q_w = -1/2)

- Quantities conserved by the interactions
  - Electric Charge, Baryon (3 quarks) number, Lepton number,
  - Quark flavor (Can be violated by Weak Interactions)
  - Isospin (Conserved only by Strong Interactions)

Leptonic (point-like) particles of the Standard Model

- Lepton "flavors" (spin = 1/2)
  - Electric charge Q = $-e$
    - e^-, electron, mass = 0.511 MeV/c^2
    - μ^-, muon, mass = 105 MeV/c^2
    - τ^-, tau, mass = 1.77 GeV/c^2
  - Electric charge Q = 0 (left-handed)
    - Flavor states (same as the charged leptons)
      - ν_e, electron neutrino
      - ν_μ, muon neutrino
      - ν_τ, tau neutrino
    - Mass states: ν_1, ν_2, ν_3 ($m < 1$ eV/c^2)
- Anti-leptons
  - charged: $e^+,\ \mu^+,\ \tau^+$ neutral: $\bar{\nu}_e,\ \bar{\nu}_\mu,\ \bar{\nu}_\tau$

Hadronic (point-like) particles of the Standard Model

- Quark "flavors" (spin = 1/2)
  - Electric charge Q = +2/3e
    - *u*, up, mass ~ 3 MeV/c^2
    - *c*, charm, mass ~ 1.2 GeV /c^2
    - *t*, top, mass ~ 175 GeV /c^2
  - Electric charge Q = −1/3e
    - *d*, down, mass ~ 7 MeV/c^2
    - *s*, strange, mass ~ 120 MeV/c^2
    - *b*, bottom, mass ~ 4.2 GeV/c^2
- Anti-quark "flavors" (spin = 1/2)
  - Q = −2/3e: $\bar{u}, \bar{c}, \bar{t}$ Q = −2/3e: $\bar{d}, \bar{s}, \bar{b}$
- Hadrons (colorless quark combinations)
  - Baryons: three quarks, one of each color (Q = −1 to +2e, spin = 1/2, 3/2)
    - light: proton (*uud*), neutron (*udd*), Δ^{++} (*uuu*), ...
    - strange, charm, bottom: Ξ_c (*csd*), Λ_b(*bdu*), ... no top baryons
  - Mesons: quark & antiquark pair (color/anti-color, colorless, spin = 0, 1)
    - light: $\pi^+(u\bar{d})$, $K^-(s\bar{u})$, ...
    - strange, charm, bottom: $D^+ (c\bar{d})$, $B^0 (b\bar{d})$, ..., no top mesons

Hadron masses and binding energy

- ^{3}He Nucleus (ppn)
  - proton, mass = 938.27 MeV/c^2
  - neutron mass = 939.57 MeV/c^2

$$\left(2m_p + m_n\right)c^2 = 2816.11\ \text{MeV}$$
$$\left(m_{^3\text{He}}\right)c^2 = 2808.39\ \text{MeV}$$
$$BE = \left(m_{^3\text{He}} - 2m_p - m_n\right)c^2 = -7.72\ \text{MeV}$$

BE is negative. Energy is radiated to form the nucleus. i.e., energy is needed to break it up

- Proton (uud)
  - *u*, up, mass ~ 3 MeV/c^2
  - *d*, down, mass ~ 7 MeV/c^2

$$\left(2m_u + m_d\right)c^2 \approx 13\ \text{MeV}$$
$$m_p c^2 = 938.27\ \text{MeV}$$
$$BE = \left(m_p - 2m_u - m_d\right)c^2 = +\ 925\ \text{MeV}$$

BE is POSITIVE ? Is this "binding"? Proton should blow apart!

Quantum Chromo-Dynamics (QCD) resolves the paradox

QCD allows quarks to remain low mass (asymptotic freedom) when inside a hadron. However, the color force increases the quark mass greatly ($m_{quark} \gg m_{hadron}$) if separated >1 fm from others. Quarks confined within hadrons, there are NO free quarks.

Baryon quantum number

- Baryon number (**B**)

  $\mathrm{B} = +1$ for three quarks (qqq) in a color singlet (r, g, b)

  $\mathrm{B} = -1$ for three anti-quarks $(\bar{q}\bar{q}\bar{q})$ in a color singlet $(\bar{r}, \bar{g}, \bar{b})$

  $\mathrm{B} = 0$ for meson $(q, \bar{q})$ color singlet states

  Note: some recent evidence for $(qqqq\bar{q})$ baryon states

- Baryon number conservation
  - Experiment finds *B* conserved in all interactions
  - Baryon number violation required to generate matter asymmetry
  - Unseen transformation lepton <--> quark violates *B* conservation
  - Proton decay violates *B* but may conserve *B* - *L* (e.g., $p \to e^+ \pi^0$)

Examples

Decay:	$p \not\to \pi^+\pi^0$	$\mathrm{B} = +1$	$\not\to$	$\mathrm{B} = 0$
Collision:	$pp \to ppp\bar{p}$	$\mathrm{B} = +2$	$\to$	$\mathrm{B} = +3 - 1 = +2$