PHY492: Nuclear & Particle Physics

Lecture 13

Symmetries I

Decay of the first hadronic resonance

- Hadronic resonances (lives ~10^{-23} sec)
  - Unstable particles do not have a unique mass:

    $$\left(m_0c^2 - \Gamma/2\right) \lesssim M_{res}c^2 \lesssim \left(m_0c^2 + \Gamma/2\right)$$

  - Heisenberg U. P. with $\Delta t = 10^{-23}$s, mass width, Γ = 70MeV

    $$\Delta E \leq \frac{\hbar c}{c\Delta t} \approx 70 \text{ MeV}$$

  - Strong decay of hadrons
    - colorless & flavorless quark + anti-quark pair from the vacuum
    - mass(parent) > mass(decay particles)
  - Strong decay prevented if decay particles have a higher total mass. Only weak decays (flavor change)

$$\Lambda^0 \rightarrow \text{p} + \pi^-$$

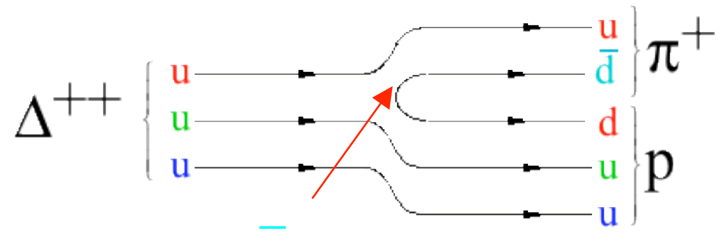
lives ~10^{-10} sec

$$\Delta^{++} \rightarrow \text{p} + \pi^+$$

energy OK

$$1238 \text{ MeV} > \left(938 + 140\right)\text{MeV}$$

quarks OK



$(\text{d}, \bar{\text{d}})$ pair created from energy

Lambda cannot strong decay

$$\Lambda^0 \rightarrow \text{p} + \text{K}^-$$

quarks OK $\left(uds\right) \rightarrow \left(udu\right) + \left(\bar{u}s\right)$

Lambda mass too small ! $1120 \text{ MeV} < \left(938 + 495\right) \text{ MeV}$

Nother's theorem

- Every symmetry of nature <------> associated with conservation law.

Symmetry	Conservation Law
Time translations	Energy
Spatial translations	Momentum
Rotations	Angular Momentum
Gauge transformation	Charge

- Q.M. spatial translations

$$\psi'(r') = \psi(r+\delta r) = \left(1 + \delta r \frac{\partial}{\partial r}\right)\psi = \left(1 - \frac{ip}{\hbar}\delta r\right)\psi = D\psi$$

$$\Delta r = n\delta r; \quad D = \left(1 - ip\delta r/\hbar\right)^n, \quad D \underset{n\to\infty}{\to} e^{-ip\Delta r/\hbar}$$

- D commutes with a Hamiltonian independent of translations
- p commutes with a Hamiltonian independent of translations
- Momentum p conserved

Groups

- Requirements of a symmetry equal the requirements on a Group
  - Closure: products of operators R_i are also legal operators
  - Identity: Element I exist such that $I R_i = R_i I$
  - Inverse; $R_i R_i^{-1} = R_i^{-1} R_i = I$
  - Associative; $(R_i R_j) R_k = R_i (R_j R_k)$
- Groups in elementary particle physics

Group	nxn matrices in group
U(n)	Unitary ($U^{T*}U = 1$)
SU(n)	Unitary (Det. = 1)
O(n)	Orthogonal ($O^T O$)
SO(n)	Orthogonal (Det.=1)

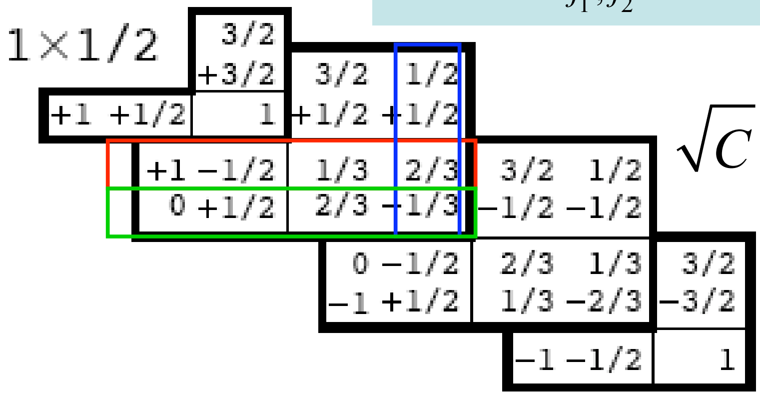
Unitary $U^{-1} = U^{T*}$

S = special (determinant =1)

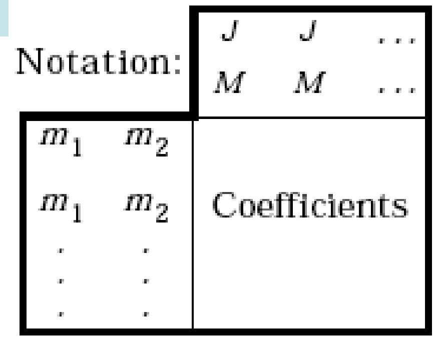
Angular momentum coupling

- Clebsh-Gordon coefficients for angular momentum coupling

$$\left|JM\right\rangle = \sum_{j_1,j_2} C^{j,j_1,j_2}_{m,m_1,m_2} \left|j_1 m_1\right\rangle \left|j_2 m_2\right\rangle$$



$\sqrt{C}$ is understood



- Spin j=1/2, m=+1/2 particle decays to a spin j_1=1 and a spin j_2=1/2
- Two possible spin configurations: $\left|1,0\right\rangle \& \left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle$ or $\left|1,1\right\rangle \& \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle$

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = -\sqrt{\tfrac{1}{3}}\left|1,0\right\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle + \sqrt{\tfrac{2}{3}}\left|1,1\right\rangle\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle; \quad \frac{P\left(\left|1,0\right\rangle\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle\right)}{P\left(\left|1,1\right\rangle\left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle\right)} = \frac{\tfrac{1}{3}}{\tfrac{2}{3}} = \frac{1}{2}$$

Spin 1/2 and Group SU(2)

- Spinors for spin 1/2

$$\left|\tfrac{1}{2},\tfrac{1}{2}\right\rangle = \begin{pmatrix}1\\0\end{pmatrix} \qquad \left|\tfrac{1}{2},-\tfrac{1}{2}\right\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

- Arbitrary state of spin 1/2

$$\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \alpha\begin{pmatrix}1\\0\end{pmatrix} + \beta\begin{pmatrix}0\\1\end{pmatrix} \qquad |\alpha|^2 + |\beta|^2 = 1$$

- Operators S_1, S_2, S_3 $\left(\text{each } \pm\tfrac{1}{2}\hbar\right)$
- Pauli spin matrices

$$\sigma_1 = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}; \quad \sigma_2 = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}; \quad \sigma_3 = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}; \qquad \sigma^2_{i=1,2,3} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$$

$$\vec{\mathbf{S}} = \frac{\hbar}{2}\vec{\sigma}; \quad \vec{\mathbf{S}}^2 = \frac{\hbar^2}{4}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2\right) = \frac{3\hbar^2}{4} \quad \left[= s(s+1)\hbar^2\right]$$

- Rotations, two dimensional representation of SU(2)

$$\begin{pmatrix}\alpha'\\\beta'\end{pmatrix} = U(\theta)\begin{pmatrix}\alpha\\\beta\end{pmatrix}; \qquad U(\theta) = e^{-i\vec{\theta}\bullet\vec{\sigma}/2} = 1 - i\vec{\theta}\bullet\vec{\sigma}/2 - \tfrac{1}{2}\left(i\vec{\theta}\bullet\vec{\sigma}/2\right)^2 - \ldots$$

Isospin and Delta cross sections

- Ratio of π^- p elastic scattering at the Delta (1232) resonance
  - Two Isospin states possible I=1/2 or I=3/2

$$\pi^- p \rightarrow \begin{cases} \Delta^0 \ (I = \tfrac{3}{2}) & \text{resonant} \\ \pi^- p(I = \tfrac{1}{2}) & \text{non resonant} \end{cases}$$

$$\left(1,-1\right)\left(\tfrac{1}{2},+\tfrac{1}{2}\right) \rightarrow \begin{matrix} \left(\tfrac{3}{2},-\tfrac{1}{2}\right) & \sqrt{\tfrac{1}{3}} \leftarrow \\ \left(\tfrac{1}{2},-\tfrac{1}{2}\right) & -\sqrt{\tfrac{2}{3}} \end{matrix} \qquad \text{CG}$$

- Only 1/3 of π^- p scattering is resonant
- Δ^0 can decay --> π^-p or π^0n

CG

$$\pi^- p \rightarrow \Delta^0 \rightarrow \pi^- p \qquad \left(\tfrac{3}{2},-\tfrac{1}{2}\right) \rightarrow \left(1,-1\right) + \left(\tfrac{1}{2},\tfrac{1}{2}\right) \qquad \sqrt{\tfrac{1}{3}} \leftarrow$$

$$\pi^- p \rightarrow \Delta^0 \rightarrow \pi^0 n \qquad \left(\tfrac{3}{2},-\tfrac{1}{2}\right) \rightarrow \left(1,0\right) + \left(\tfrac{1}{2},-\tfrac{1}{2}\right) \qquad \sqrt{\tfrac{2}{3}}$$

$$\pi^+ p \rightarrow \Delta^{++} \rightarrow \pi^+ p \qquad 1 \leftarrow$$

- Resonant cross section ratio is square of the CG product

$$\frac{\sigma\left(\pi^- p \rightarrow \pi^- p\right)}{\sigma\left(\pi^+ p \rightarrow \pi^+ p\right)} = \frac{\left(\sqrt{\tfrac{1}{3}} \cdot \sqrt{\tfrac{1}{3}}\right)^2}{1} = \frac{1}{9}$$