PHY492: Nuclear & Particle Physics

Lecture 14

Symmetries II

Ch. 9

9.1

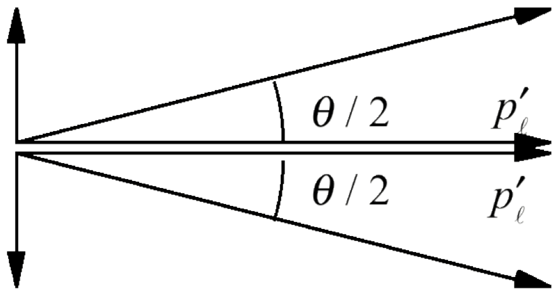
Reaction	Process
$\Omega^- \to \Xi^0 + \pi^-$	Weak decay, $\Delta S = +1(-3 \to -2)$, $\Delta I_3 = -\frac{1}{2}\left(0 \to +\frac{1}{2}, -1\right)$
$\Sigma^+ \not\to \pi^+ + \pi^0$	Violates baryon # conservation
$n \not\to p + \pi^-$	Violates energy conservation
$\pi^0 \not\to \mu^+ + e^- + \bar{\upsilon}_e$	Violates lepton number conservation
$K^0 \to K^+ + e^- + \bar{\upsilon}_e$	Allowed but never seen (but $\Sigma^+ \to \Lambda + e^+ + \upsilon,$ seen) May be related to $K^0 / \bar{K}^0$ mixing.
$\Lambda^0 \not\to p + e^-$	Violates lepton number conservation

9.3 pi-zero decay (derive opening angle)

$\gamma = 1\times 10^3, \beta \approx 1$ for a 135 GeV/c pion; $\quad t_{lab} = \gamma t_0 = 8.5\times 10^{-14}$ s,

$l = ct_{lab} = \left(3\times 10^8 \text{ m/s}\right) 8.5\times 10^{-14}\text{s} = 26\times 10^{-6} \text{ m} = 26$ microns

$E = p_t c = mc^2/2$



$E = p_t c = mc^2/2$

π^0 rest frame, $E = p_t c = mc^2/2$

lab frame, $p'_\ell c = \gamma\left(p_\ell c + \beta E\right) = \gamma\left(0 + \beta mc^2/2\right) = \gamma\beta\, mc^2/2$

$$\tan\left(\frac{\theta}{2}\right) \approx \frac{\theta}{2} = \frac{p_t}{p'_\ell} = \frac{mc^2/2}{\gamma\beta\, mc^2/2} = \frac{1}{\gamma\beta} = \frac{1}{\frac{E}{m}\frac{p}{E}} = \frac{m}{p}$$

$$\theta = \frac{2m}{p} = \frac{0.28}{135} = 2.1 \text{ mr}$$

9.5 Use $I_3 = Q - \frac{Y}{2}$

State	Q	S	$Baryon\#$	C	Bt	$\frac{Y}{2} = \frac{Ba+S+C+Bt}{2}$	I_3	I	Particle
$u\bar{s}$	+1	+1	0	0	0	$+\frac{1}{2}$	$+\frac{1}{2}$	$\frac{1}{2}$	K^+
$c\bar{d}$	+1	0	0	+1	0	$+\frac{1}{2}$	$+\frac{1}{2}$	$\frac{1}{2}$	D^+
$\bar{u}\bar{u}\bar{d}$	−1	0	−1	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2},\frac{3}{2}$	$\bar{p},\bar{\Delta}^-$
ddc	0	0	+1	+1	0	+1	−1	1	Σ_c^0
ubc	+1	0	+1	+1	−1	$+\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	Ξ_{bc}^+ (not seen)
$s\bar{s}$	0	0	0	0	0	0	0	0	η,ϕ

Baryons with only *u* and *d* quarks

$I = \frac{1}{2}$ use the N symbol (except proton is *p*, and neutron is *n*)

$I = \frac{3}{2}$ use the Δ symbol

Baryons with *s, c,* or *b* quarks

$I = 0$ use the Ω symbol, or if 3 different quarks use the Λ symbol.

$I = \frac{1}{2}$ use the Ξ (cascade) symbol

$I = 1$ use the Σ symbol

10.1 Schmushkevich method ($\rho \to \pi\pi$ decay, both orderings of the π's)

Reaction	Rate
$\rho^0 \to \pi^0 + \pi^0$	x
$\rho^0 \to \pi^+ + \pi^-$	$(1-x)$
$\rho^+ \to \pi^0 + \pi^+$	1
$\rho^- \to \pi^0 + \pi^-$	1

rate for π^0 is $2x+2$; rate for π^+ is $(1-x)+1$; rate for π^- is $(1-x)+1$

Rate for π^0 = Rate for π^+; $2x+2=2-x$; $x=0$ and $\rho^0 \not\to \pi^0 + \pi^0$

Clebsch-Gordon Method

$|1,0\rangle \to |1,0\rangle|1,0\rangle$ use Clebsch-Gordon coefficient 1×1 table $J=1, M=0, m_1=m_2=0$

Coefficient = 0 ! requires Isospin violation.

10.3 $N^*: I=\frac{1}{2};\quad \Delta: I=\frac{3}{2};\quad p: I=\frac{1}{2};\quad \pi: I=1$

Schmushkevich Method

$N^{*+}\to p\pi^0 \quad x \quad =\frac{1}{3} \quad ; \quad \Delta^+\to p\pi^0 \quad =\frac{2}{3}$ (text)

$N^{*+}\to n\pi^+ \quad 1-x \quad =\frac{2}{3} \quad ; \quad \Delta^+\to n\pi^+ \quad =\frac{1}{3}$ "

$N^{*0}\to p\pi^- \quad 1-y \quad =\frac{2}{3} \quad ; \quad \Delta^0\to p\pi^- \quad =\frac{1}{3}$ "

$N^{*0}\to n\pi^0 \quad y \quad =\frac{1}{3} \quad ; \quad \Delta^0\to n\pi^0 \quad =\frac{2}{3}$ "

$(p,n): x+1-y=1-x+y \quad \therefore \quad x=y; \quad (\pi^+,\pi^0): 1-x=x+y \quad \therefore \quad x=y=\frac{1}{3}$

Clebsch-Gordon Method

$N^{*+}\to p\pi^0 \quad \frac{1}{2},+\frac{1}{2}\to+\frac{1}{2},0=\frac{1}{3} \quad ; \quad \Delta^+\to p\pi^0 \quad \frac{3}{2},+\frac{1}{2}\to+\frac{1}{2},0=\frac{2}{3}$

$N^{*+}\to n\pi^+ \quad \frac{1}{2},+\frac{1}{2}\to-\frac{1}{2},+1=\frac{2}{3} \quad ; \quad \Delta^+\to n\pi^+ \quad \frac{3}{2},+\frac{1}{2}\to-\frac{1}{2},+1=\frac{1}{3}$

$N^{*0}\to p\pi^- \quad \frac{1}{2},-\frac{1}{2}\to+\frac{1}{2},-1=\frac{2}{3} \quad ; \quad \Delta^0\to p\pi^- \quad \frac{3}{2},-\frac{1}{2}\to+\frac{1}{2},-1=\frac{1}{3}$

$N^{*0}\to n\pi^0 \quad \frac{1}{2},-\frac{1}{2}\to-\frac{1}{2},0=\frac{1}{3} \quad ; \quad \Delta^0\to n\pi^0 \quad \frac{3}{2},-\frac{1}{2}\to-\frac{1}{2},0=\frac{2}{3}$

Rates for the final states are 1/3 and 2/3 but reversed for the $I=\frac{3}{2}$ and $I=\frac{1}{2}$ particles .

The nucleon's spin/flavor quark wave function

- Proton; spin 1/2 combination of 3 quarks: wave-function with $\uparrow\uparrow\downarrow$

$$\chi_p\left(\tfrac{1}{2},\tfrac{1}{2}\right) = \sqrt{\tfrac{2}{3}}\,\chi_{uu}(1,1)\,\chi_d\left(\tfrac{1}{2},-\tfrac{1}{2}\right) - \sqrt{\tfrac{1}{3}}\,\chi_{uu}(1,0)\,\chi_d\left(\tfrac{1}{2},\tfrac{1}{2}\right)$$

CG coefficients

$$\chi_{uu}(1,1) = \uparrow\uparrow; \quad \chi_{uu}(1,0) = \tfrac{1}{\sqrt{2}}\left(\uparrow\downarrow + \downarrow\uparrow\right)$$

$$\left|\mathrm{p}^{\uparrow}\right\rangle = \sqrt{\tfrac{2}{3}}\left|\mathrm{u}^{\uparrow}\mathrm{u}^{\uparrow}\mathrm{d}^{\downarrow}\right\rangle - \sqrt{\tfrac{1}{3}}\left|\sqrt{\tfrac{1}{2}}(\mathrm{u}^{\uparrow}\mathrm{u}^{\downarrow} + \mathrm{u}^{\downarrow}\mathrm{u}^{\uparrow})\mathrm{d}^{\uparrow}\right\rangle$$

(now symmetrize terms)

$$= \sqrt{\tfrac{1}{18}}\Big[2\left(\left|\mathrm{u}^{\uparrow}\mathrm{u}^{\uparrow}\mathrm{d}^{\downarrow}\right\rangle + \left|\mathrm{u}^{\uparrow}\mathrm{d}^{\downarrow}\mathrm{u}^{\uparrow}\right\rangle + \left|\mathrm{d}^{\downarrow}\mathrm{u}^{\uparrow}\mathrm{u}^{\uparrow}\right\rangle\right)$$

$$-\left(\left|\mathrm{u}^{\uparrow}\mathrm{u}^{\downarrow}\mathrm{d}^{\uparrow}\right\rangle + \left|\mathrm{u}^{\uparrow}\mathrm{d}^{\uparrow}\mathrm{u}^{\downarrow}\right\rangle + \left|\mathrm{d}^{\uparrow}\mathrm{u}^{\uparrow}\mathrm{u}^{\downarrow}\right\rangle + \left|\mathrm{u}^{\downarrow}\mathrm{u}^{\uparrow}\mathrm{d}^{\uparrow}\right\rangle + \left|\mathrm{u}^{\downarrow}\mathrm{d}^{\uparrow}\mathrm{u}^{\uparrow}\right\rangle + \left|\mathrm{d}^{\uparrow}\mathrm{u}^{\downarrow}\mathrm{u}^{\uparrow}\right\rangle\right)\Big]$$

- Ξ and Σ, Λ obtained the same way with 1 strange and 2 light quarks

Continuous parity

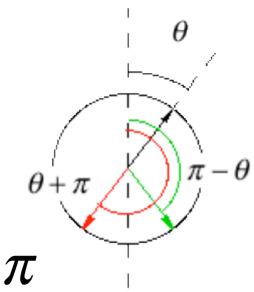
- Continuous parity transformations
  - Bound state wave functions (H: harmonic oscillator, square well, ...)

$$H\psi(x) = E\psi(x)$$

  - Parity transformation of x --> -x, identifies wave functions as either positive or negative parity

$$P\psi_e(x) = \psi_e(-x) = +\psi_e(x) \quad \text{(Even parity)}$$
$$P\psi_o(x) = \psi_o(-x) = -\psi_o(x) \quad \text{(Odd parity)}$$

  - Orbital angular momentum parity: $(\vec{r} \rightarrow -\vec{r})$



Note: $x \rightarrow -x,\ y \rightarrow -y,\ z \rightarrow -z \quad = \underline{r \rightarrow r,}\ \theta \rightarrow \pi - \theta,\ \phi \rightarrow \phi + \pi$

$$P\left[Y_{\ell m}(\theta,\phi)\right] = Y_{\ell m}(\pi - \theta, \phi + \pi) = (-1)^{\ell}\, Y_{\ell m}(\theta,\phi)$$

ℓ even: parity +

ℓ odd: parity –

Check these out: $Y_{10}(\theta,\phi) \sim \cos\theta;$ $\quad Y_{11}(\theta,\phi) \sim \sin\theta e^{i\phi}$

Parity

- Discrete parity transformations
  - Photons

$$\nabla \cdot E(x,t) = \frac{1}{\varepsilon_0}\rho(x,t)$$

from Maxwell's equations

$$-\nabla \cdot E(-x,t) = \frac{1}{\varepsilon_0}\rho(-x,t)$$

therefore photons have have negative parity

  - Particles and antiparticles (*f* = fermion, *b* = boson)

fermion/anti-fermion $P(f)P(\bar{f}) = -1$ opposite parity

boson/anti-boson $P(b)P(\bar{b}) = +1$ same parity

  - Photon pairs: Parity requires analysis of photon polarization vectors

pi-zero decay $\pi^0 \to 2\gamma$ $\vec{k} \cdot (\vec{\varepsilon}_1 \times \vec{\varepsilon}_2)$ changes sign under Parity

$$P(\pi^0) = P(\gamma)P(\gamma)P(\vec{k} \cdot (\vec{\varepsilon}_1 \times \vec{\varepsilon}_2)) = (-1)^3 = -1$$

π^0 has negative parity

Also quarks: $|\pi^0\rangle = \sqrt{\tfrac{1}{2}}\left(|u\bar{u}\rangle - |d\bar{d}\rangle\right) \quad \Rightarrow \quad P = (-1)(-1)^\ell \quad \ell = 0 \text{ (S-state)}$

Parity conservation

- Parity conserved in electromagnetic interactions
  - Electromagnetic decay (lifetime t ~ 10^{-16} s)

Sigma baryon decay
to Lambda baryon + gamma

$$\Sigma^0 \rightarrow \Lambda^0 + \gamma$$
$$(uds) \rightarrow (uds) + \gamma$$

quark content
the same

Baryons are mostly spin 1/2

conceptually

ψ_{spin} two quarks w/spin up
one quark w/spin down

$$(u^{\uparrow} d^{\uparrow} s^{\downarrow}) \rightarrow (u^{\uparrow} d^{\downarrow} s^{\downarrow}) + \gamma$$

spin of d-quark flips

Spin wave-functions are considerably more complicated
(see earlier slide) but the essence of the situation is the same.

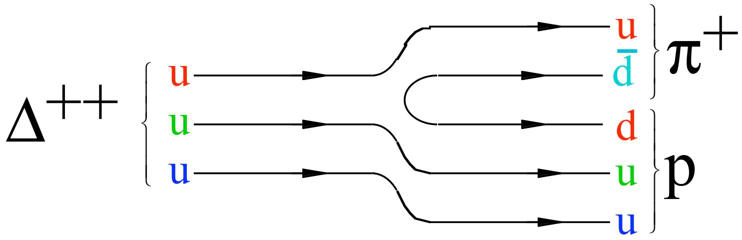
Baryon
wavefunction

$$\psi = \psi(\text{space})\psi(\text{spin})\psi(\text{flavor})\psi(\text{color})$$

whole works must be anti-symmetric (usually only color part is anti-symmetric)

Parity conservation

- Parity conserved in strong interactions
  - Strong decay (lifetime t ~ 10^{-23} s)



$$\Delta : J^P = \tfrac{3}{2}^+$$
$$p : J^P = \tfrac{1}{2}^+$$
$$\pi : J^P = 0^-$$

$$\Delta^{++} \to p + \pi^+$$

$$\text{spin } \tfrac{3}{2} \to \tfrac{1}{2} + 0$$

$$\ell = 1$$

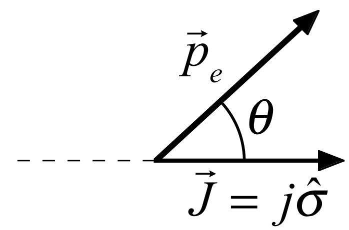
angular momentum conservation requires orbital angular momentum

$$P(\Delta) \to P(p)P(\pi)(-1)^{\ell}$$
$$+1 \to (+1)(-1)(-1) \text{ parity is conserved}$$

Parity violation in weak interactions

- Parity violated in weak interactions
  - Beta decay of polarized Co-60

$$^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni} + \mathrm{e}^- + \bar{\upsilon}_e$$



(Co-60)

Rate vs. angle must be a scalar function

Only scalar using the vector variables is

$$\hat{\sigma} \bullet \vec{p}$$

(pseudo)scalar combination of vector quantities

Most general linear form

$$I(\theta) = 1 + \alpha\left(\frac{\hat{\sigma}\bullet\vec{p}c}{E}\right)$$

$$= 1 + \alpha\beta\cos\theta$$

$\hat{\sigma}$ is "axial" vector $(\hat{r} \times \hat{q})$

$$P(\hat{\sigma}) = +\hat{\sigma}; \quad P(\vec{p}) = -\vec{p}$$

$$P(\hat{\sigma} \cdot \vec{p}) = -\hat{\sigma} \cdot \vec{p}$$

Any angular asymmetry is reversed in a parity inverted world.
If parity is conserved the angular dependence must be symmetric w.r.t. spin

Strong angular asymmetry seen ---> Parity Violated

Tau-Theta puzzle resolved

- Two particles same mass and decay lifetime, and spin = 0.
- Only difference: one was negative parity, the other positive parity.

$$\tau^+ \rightarrow \pi^+ + \pi^0$$
$$\theta^+ \rightarrow \pi^+ + \pi^+ + \pi^-$$

$$P(\tau^+) = P(\pi^+)P(\pi^0) = +1$$
$$P(\theta^+) = P^2(\pi^+)P(\pi^-) = -1$$

- This led Lee and Yang to propose that parity might be violated in weak interactions ---> particle is K^+ with two decay modes.
- Stimulated Wu's experiment (previous slide) with ^{60}Co
- Stimulated Lederman's experiment with muon decay
  - polarized muons produced in pi-meson decay
  - Polarized muons stopped in carbon block (kept polarization)
  - Observe direction of positrons w.r.t. muon polarization

$$\frac{dN}{d\Omega} = 1 - \frac{\alpha}{3}\cos\theta$$

angular dependence is asymmetric
parity violated in muon decay.

Time reversal invariance

- Newtonian mechanics is invariant to t --> -t.
- Maxwell's Equations are invariant to t --> -t.
- Quantum mechanical wave functions almost invariant

$$\psi(\vec{r},t) \xrightarrow{T} \psi^*(\vec{r},-t)$$

$$\text{Probability} = \psi^* \psi \text{ is invariant}$$

- An electric dipole moment of the neutron would violate T.

$$\vec{s} \xrightarrow{T} -\vec{s}$$

$$\vec{\mu}_E \xrightarrow{T} \vec{\mu}_E$$

$$\vec{\mu}_E \cdot \vec{s} \xrightarrow{T} -\vec{\mu}_E \cdot \vec{s}$$

- Maximum expectation is qr with q~e, and r~10^{-13}cm: $\mu_E \sim 10^{-13}$ e-cm

Experimental limit

$$\mu_E < 10^{-25} \text{ e-cm}$$

Charge conjugation

- Charge conjugation operator (*C*) makes particle <---> anti-particle
- All charges (electric charge, lepton #, baryon #, etc.,) must be zero for particle to be eigenstate of *C* operator.
- Photon has *C*-parity = –1
- Neutral pion (π^0 --> $\gamma+\gamma$)
  - *C*-parity = +1
  - No π^0 decays to odd number of photons
- *C*-parity is conserved in strong and electromagnetic interactions
- *C*-parity is violated in weak interactions
  - Only left-handed neutrinos and right-handed anti-neutrinos exist.
  - *C*(left-handed neutrino) --> left-handed anti-neutrino (but none exist)
- Combination of *C* & P is symmetry of weak interaction (*ALMOST*)
  - CP(left-handed neutrino) --> right-handed anti-neutrinos (OK)
  - CP(right-handed anti-neutrinos) --> left-handed neutrino (OK)
  - K^0 system violates even this CP symmetry