
PHY492: Nuclear & Particle Physics

Lecture 16

CP Violation
Weak Boson

K^0 & \bar{K}^0 mixing

- Parity violation in K^+ decay

- 2 pions, $P = +$
- 3 pions, $P = -$
- T.D. Lee, C.N. Yang (1956)

$$K^+ \rightarrow \pi^+ \pi^0 \text{ or } \pi^+ \pi^+ \pi^-$$

$$(\tau = 1.2 \times 10^{-8} \text{ s})$$

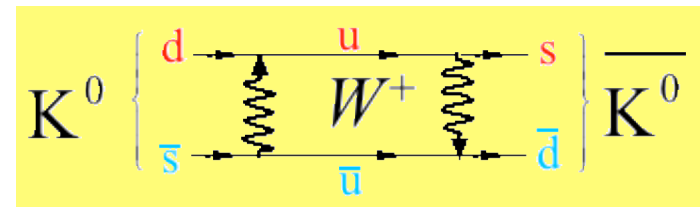
$(\tau^+, \theta^+ \text{ puzzle})$

- Prediction for K^0 decays

- M. Gell-Mann & A. Pais (1955)
- Two decay constants
- CP eigenstates are the key
- Weak Interaction (2nd order) can cause particle \rightarrow antiparticle for neutral particles

$$K^0 \rightarrow \pi^+ \pi^- \quad (\tau = 0.9 \times 10^{-10} \text{ s}) \text{ fast}$$

$$K^0 \rightarrow \pi^+ \pi^- \pi^0 \quad (\tau = 0.5 \times 10^{-7} \text{ s}) \text{ slow}$$



- K^0 is not an eigenstate of C or P .

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \quad CP|K_1\rangle = +|K_1\rangle$$

- linear combinations are eigenstates of CP

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \quad CP|K_2\rangle = -|K_2\rangle$$

K^0 & \bar{K}^0 oscillations

- Neutral K decays

- $\pi^+\pi^-$, $P = +1$, $C = +1$, $CP = +1$

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle) \quad \text{fast decay}$$

- $\pi^+\pi^-\pi^0$, $P = -1$, $C = +1$, $CP = -1$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) \quad \text{slow decay}$$

Invert



- K^0 & \bar{K}^0 created by strong interactions.

- weak interactions select K_1 & K_2 and have slightly different masses ($\Delta m = m_2 - m_1$)

- Mix of K_1 and K_2 will change as particle propagates

- time dependence almost straight from QM text book.

$$|K^0\rangle = \frac{1}{\sqrt{2}}(|K_1\rangle + |K_2\rangle)$$

$$|\bar{K}^0\rangle = -\frac{1}{\sqrt{2}}(|K_1\rangle - |K_2\rangle)$$

$$|K_1(t)\rangle = K_1(0) \left[e^{im_1 c^2 t / \hbar} e^{-\Gamma_1 t / 2\hbar} \right]$$

$$|K_2(t)\rangle = K_2(0) \left[e^{im_2 c^2 t / \hbar} e^{-\Gamma_2 t / 2\hbar} \right]$$

- Starting with pure K^0 , the \bar{K}^0 intensity **grows then oscillates**

$$I(\bar{K}^0) = \frac{1}{4} \left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-(\Gamma_1 + \Gamma_2)/2 t} \cos(\Delta m c^2 t / \hbar) \right]$$

$$\tau = 2\pi\hbar / \Delta m c^2 = 1.2 \times 10^{-9} \text{ s}$$

$$\Delta m c^2 = 3.52 \times 10^{-6} \text{ eV}$$

CP violation in K decays

- 1964 Christenson, Cronin, Fitch, & Turlay discover $\sim 3 \times 10^{-3}$ of K_2 beam decays to $\pi^+\pi^-$ or $\pi^0\pi^0$

K^0 and K^0 -bar are not pure mixtures of CP eigenstates, K_1 & K_2

- Short lived version is called K_S^0 and long lived called K_L^0

K_L^0 = mostly K_2 with a little K_1 , so causes "indirect" CP violation

$$|K_L^0\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}} (|K_2\rangle + \epsilon |K_1\rangle)$$

- Also H_{weak} has a very small direct CP violating term
 - Parity violation in weak interactions is maximal.
No right handed neutrinos have ever been found.
Relativistic quantum mechanics can handle this well.
 - CP violation is a whole other story, that continues to dominate about 1/2 of all HEP to this day.

What's all the fuss about CP violation?

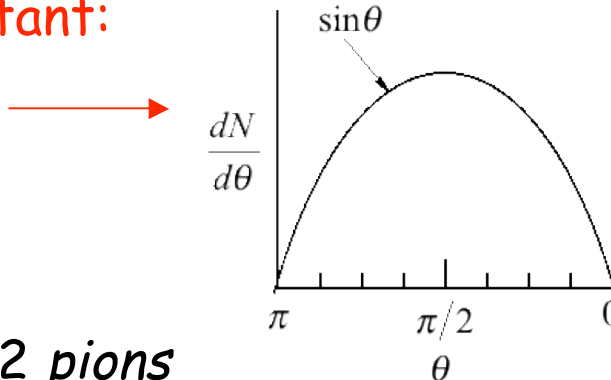
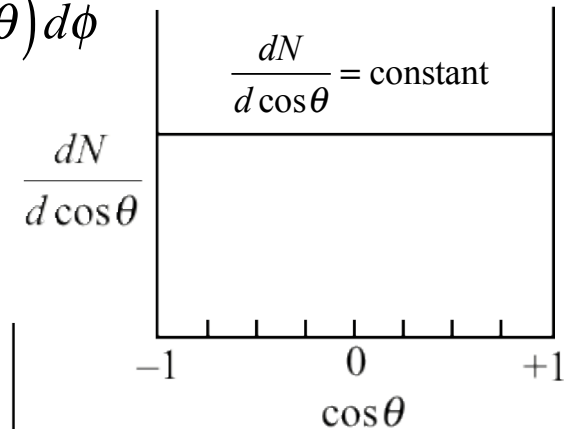
- Semi-leptonic decays of K_L^0 :
$$\begin{aligned} |K_L\rangle &\rightarrow \pi^+ + e^- + \bar{\nu}_e \\ |K_L\rangle &\rightarrow \pi^- + e^+ + \nu_e \end{aligned}$$
- If CP were a good symmetry decays should be identical (decay fraction, etc.)
- Experiments show the e^+ decay greater than e^- decay by 3.3 parts in 1000, \rightarrow **CP violation**.

Consequences of CP violation

1. Makes absolute distinction of matter (e^-) from anti-matter (e^+).
2. Provides unambiguous definition of "positive" charge.
3. Since CPT is good symmetry, CP violation implies Time reversal symmetry (T) is also broken - though never seen.
4. May be responsible for matter-antimatter asymmetry in nature.

How to present angular data

- Differential solid angle: $d\Omega = \sin\theta d\theta d\phi = d(\cos\theta) d\phi$
- Isotropic distribution yields equal intensity in each solid angle element.
- Plotting a isotropic distribution vs θ does not yield a constant:

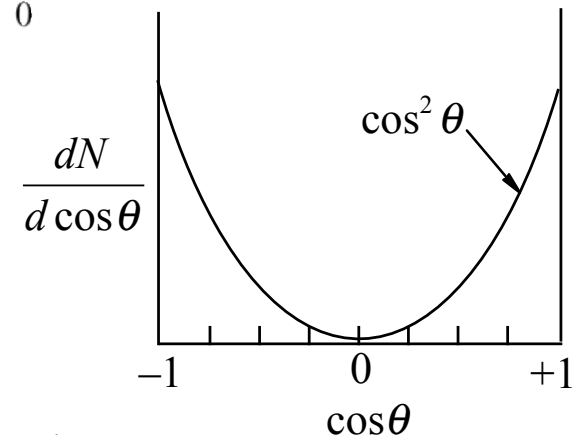


- Rho meson decay to 2 pions*

$$\rho^0(J=1, J_z=0) \rightarrow \pi^+\pi^- \text{ (both } J=0\text{)}$$

- Angular distribution:

$$\frac{d\sigma}{d\cos\theta} \sim |Y_{10}|^2 \sim \cos^2\theta$$

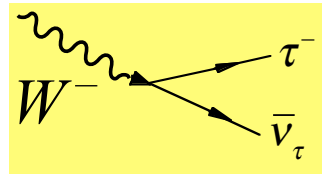
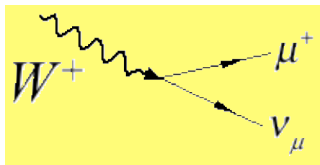


Actual data has some contributions from $L=0,2$ distorting the distribution

Weak interactions and W boson

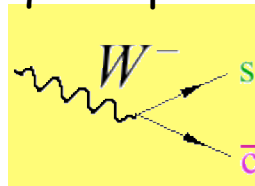
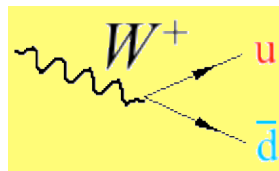
- Leptonic weak decays
 - quark color unchanged, W colorless
 - quark flavor change: e.g., $d \rightarrow u$
 - W^+ & W^- mediate flavor change
 - electron and antineutrino originate in W boson "decay"

- Given **enough energy** W can "decay" to
 - leptons heavier than an electron

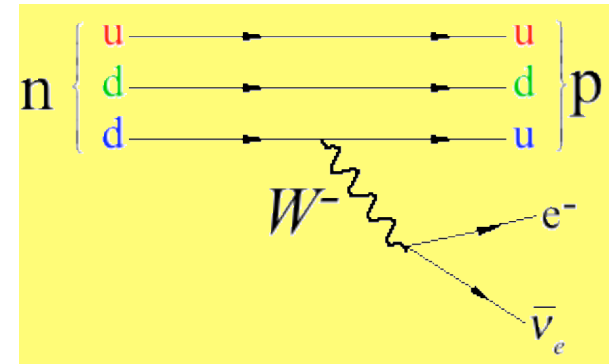


- Hadrons, via colorless $q \bar{q}$ pairs

Σ (quark charges)
= W charge

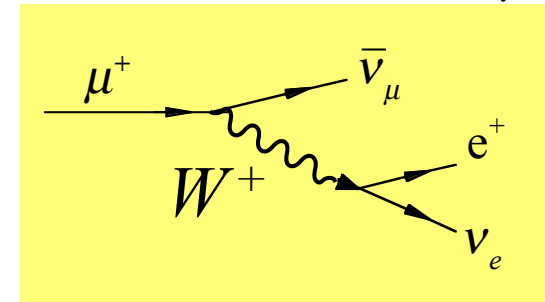


- Neutron β decay



$$n \rightarrow p + e^- + \bar{\nu}_e$$

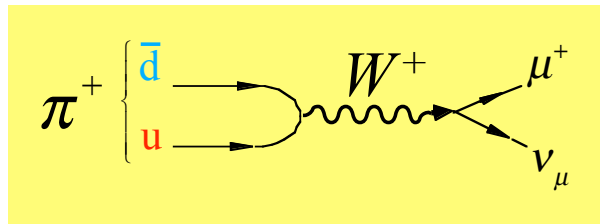
- Positive muon decay



$$\mu^+ \rightarrow \bar{\nu}_\mu + e^+ + \nu_e$$

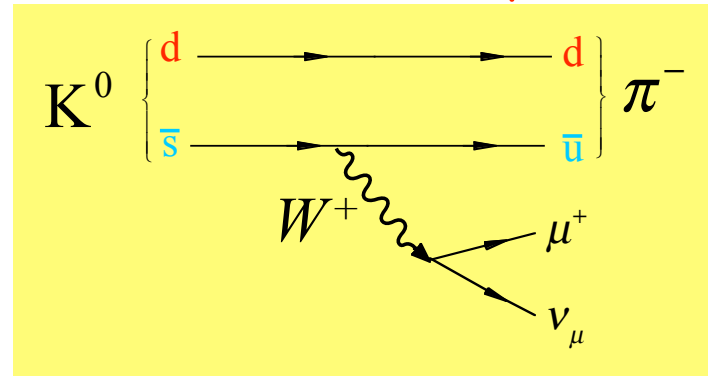
K^0 decays and the W boson

- π^+ meson **leptonic** decay



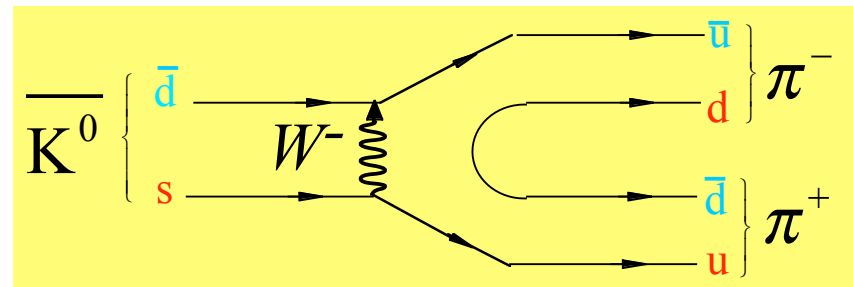
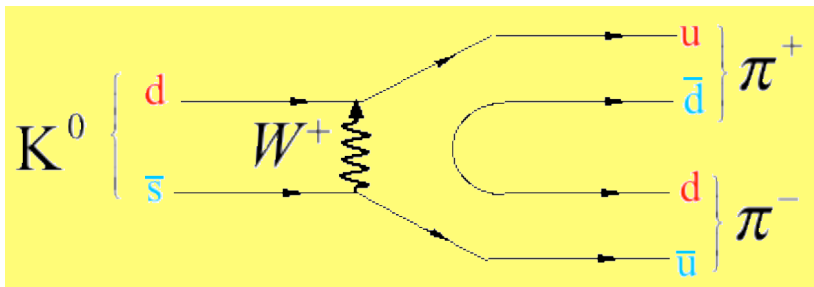
$$u \rightarrow d [W^+ \rightarrow \mu^+ + \nu_\mu]$$

- K^0 meson **semi-leptonic** decay



$$\bar{s} \rightarrow \bar{u} [W^+ \rightarrow e^+ + \nu_e]$$

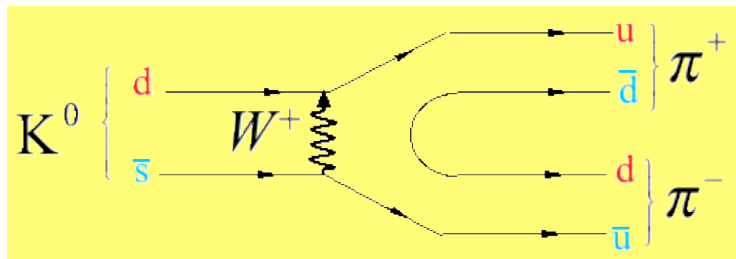
K^0 & \bar{K}^0 have hadronic decays to the same state $\pi^+\pi^-$



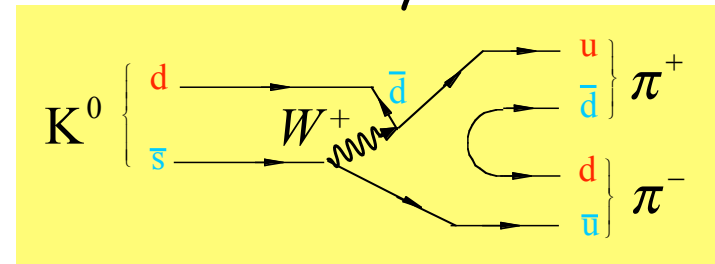
Why don't these diagrams look like the leptonic decay of W ?

Feynman diagram wizardry

- W changes **both quark** flavors
- **One** quark flavor change + W "decay"

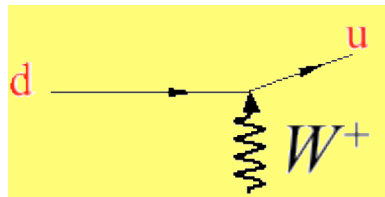


$$K^0 \rightarrow \pi^+ \pi^-$$



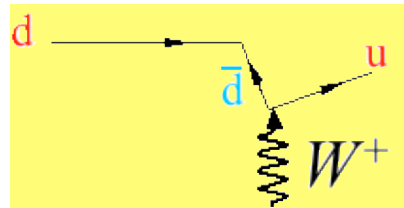
$$K^0 \rightarrow \pi^+ \pi^-$$

$$d + W^+ \rightarrow u$$



Quark "absorbs" a W and changes flavor

$$W^+ \rightarrow \bar{d} + u$$

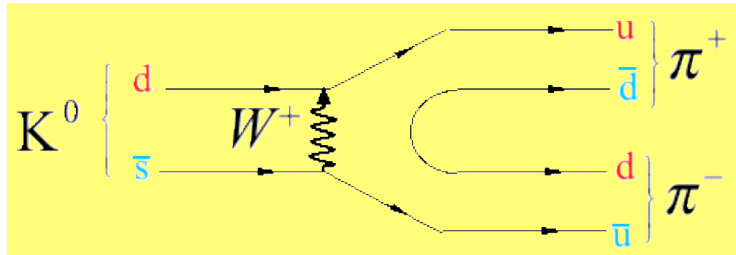


W "decay" with quark + anti-quark annihilation

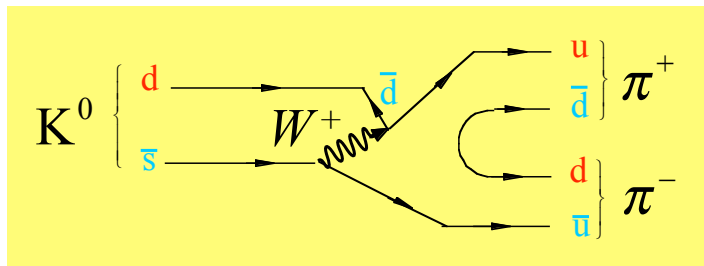
Equivalent

- Changing direction of a quark line turns it into an anti-quark line

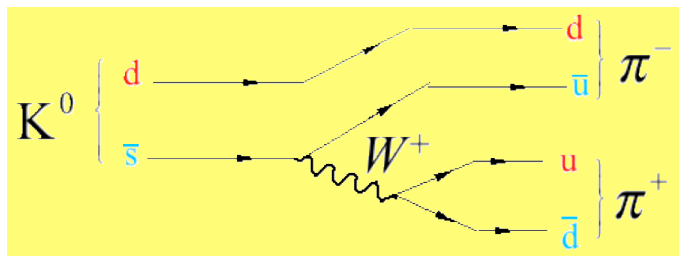
Weak interaction Feynman diagrams



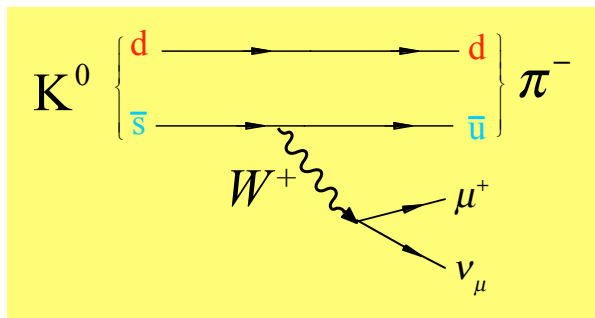
- W changes **both quark** flavors ?



- **One** quark flavor change
+ W "decay"



- **One** quark flavor change
+ W "decay"



- **One** quark flavor change
+ W leptonic decay

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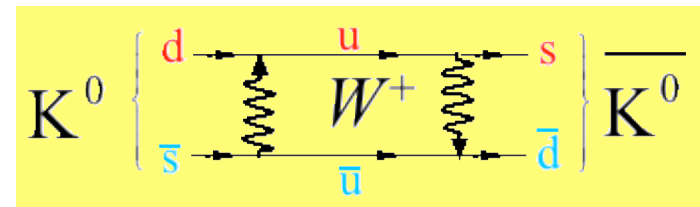
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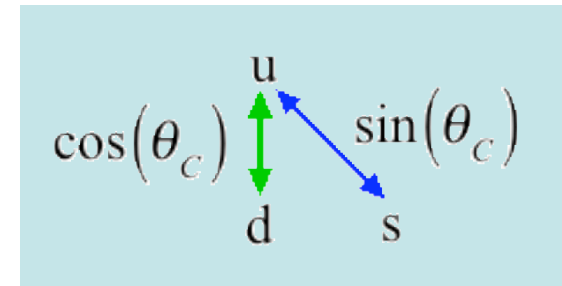
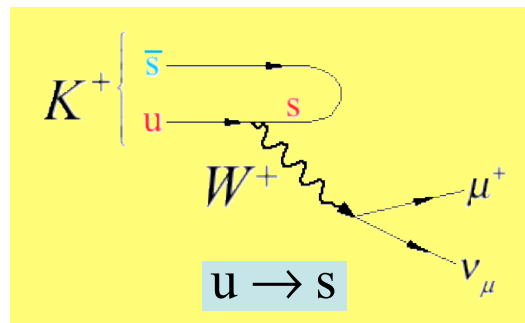
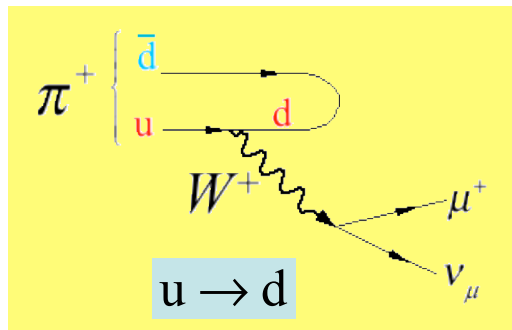
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Mixing of quark generations

- Leptons **don't cross generations** (e, μ, τ)
- Cabibbo realizes an angle describes quark decays across generations
 - Cabibbo angle $\theta_c \sim 13^\circ$ gives relative probability of $u \leftrightarrow d$ and $u \leftrightarrow s$ decays
 - Many decay rates explained by this angle. Example: meson leptonic decay.

	Generation		
Charge	1	2	3
+2/3	u	c	t
-1/3	d	s	b

Decays within a generation are favored over across generations



s/d quark mixing

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} s \end{pmatrix} \rightarrow \begin{pmatrix} u \\ d \cdot \cos\theta_c + s \cdot \sin\theta_c \end{pmatrix}$$

strong interaction

weak interaction

- π decay rate is proportional to $\cos^2(\theta_c)$
- K decay rate is proportional to $\sin^2(\theta_c)$
- Ratio of the rates: $\frac{\Gamma(K \rightarrow \mu\nu)}{\Gamma(\pi \rightarrow \mu\nu)} \sim \tan^2(\theta_c)$

with mass effect corrections, agreement is excellent