PHY492: Nuclear & Particle Physics

Lecture 17

Quark Mixing

K^0 & $\overline{K^0}$ mixing (review)

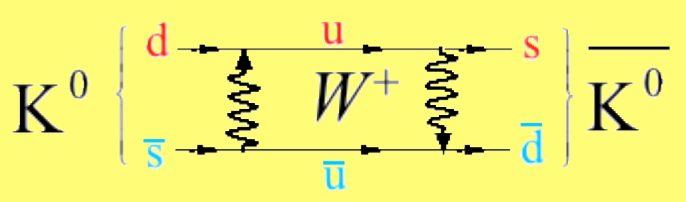
- Parity violation in K^+ decay
  - 2 pions, P = +
  - 3 pions, P = -
  - T.D. Lee, C.N. Yang (1956)

$$K^+ \to \pi^+\pi^0 \text{ or } \pi^+\pi^+\pi^- \quad (\tau = 1.2\times 10^{-8}\text{s})$$

$(\tau^+,\ \theta^+$ puzzle)

- Prediction for K^0 decays
  - M. Gell-Mann & A. Pais (1955)
  - Two decay constants
  - CP eigenstates are the key
  - Weak Interaction (2nd order) can cause particle ---> antiparticle for neutral particles

$$K^0 \to \pi^+\pi^- \quad (\tau = 0.9\times 10^{-10}\text{s}) \ \text{fast}$$
$$K^0 \to \pi^+\pi^-\pi^0 \quad (\tau = 0.5\times 10^{-7}\text{s}) \ \text{slow}$$



- K^0 is not an eigenstate of C or P.
- linear combinations of K^0/K^0-bar are eigenstates of CP

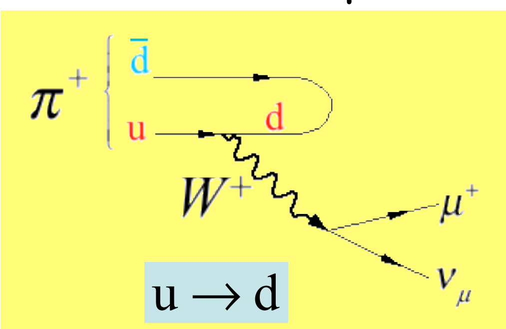
$$|K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right) \qquad CP|K_1\rangle = +|K_1\rangle$$
$$|K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right) \qquad CP|K_2\rangle = -|K_2\rangle$$

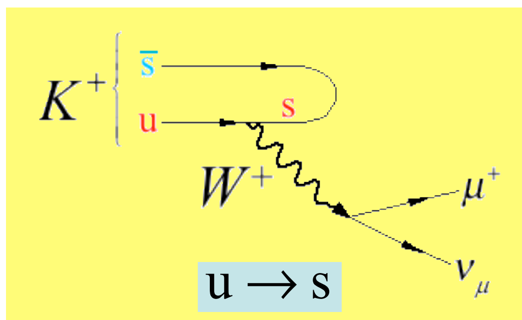
Mixing of quark generations

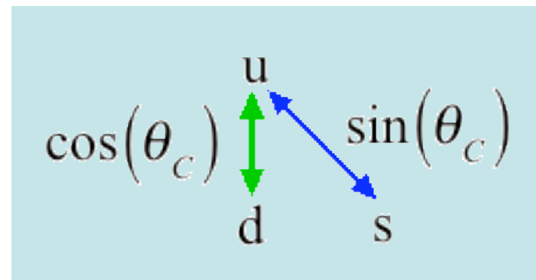
- Charged leptons don't mix (e, μ, τ)
- Cabibbo realizes an angle describes quark decays across generations
  - Cabibbo angle θ_C ~13° gives relative probability of u <--> d and u <--> s decays
  - Many decay rates explained by this angle. Example: meson leptonic decay.

	Generation		
Charge	1	2	3
+2/3	**u**	**c**	**t**
–1/3	**d**	**s**	**b**

Decays within a generation are favored over across generations







s/d quark mixing

- π decay rate is proportional to $\cos^2(\theta_C)$
- K decay rate is proportional to $\sin^2(\theta_C)$
- Ratio of the rates: $\frac{\Gamma(K \to \mu\nu)}{\Gamma(\pi \to \mu\nu)} \sim \tan^2(\theta_c)$

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} \\ s \end{pmatrix} \to \begin{pmatrix} u \\ d \cdot \cos\theta_C + s \cdot \sin\theta_C \end{pmatrix}$$

strong interaction — weak interaction

with mass effect corrections, agreement is excellent

Cabibbo's headache decays

- The Cabibbo theory worked -- almost !

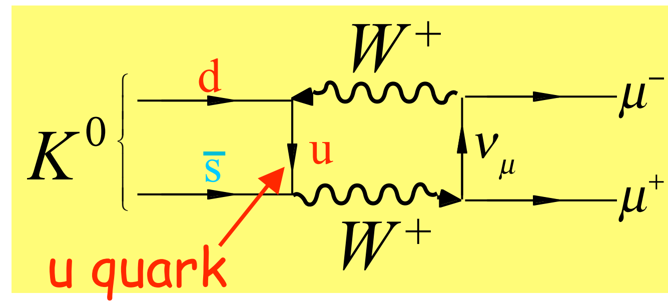
$$\Gamma\left(K_L^0 \to \mu^+\mu^-\right) \text{ should be } \sim \cos\left(\theta_c\right)\sin\left(\theta_c\right)$$

$$\begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix}\begin{pmatrix} \\ \mathrm{s} \end{pmatrix} \to \begin{pmatrix} \mathrm{u} \\ \mathrm{d}\cdot\cos\theta_C + \mathrm{s}\cdot\sin\theta_C \end{pmatrix}$$

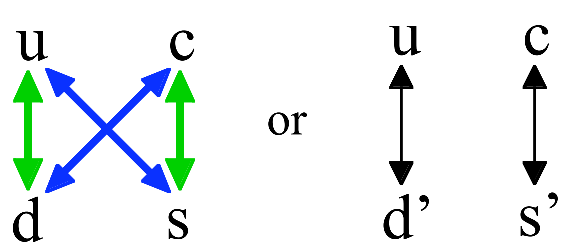
2nd order weak decay

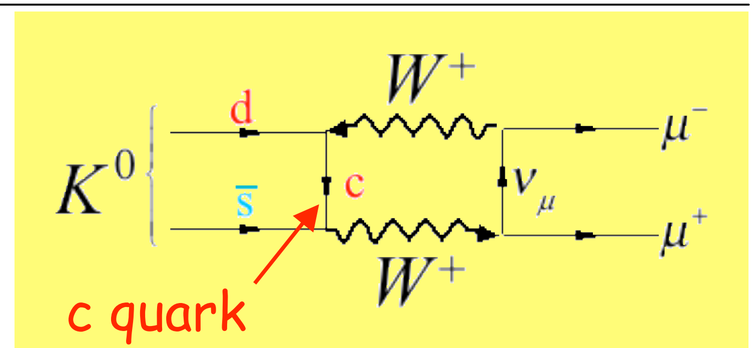
- Many experiments try to measure $K_L \to \mu^+\mu^-$ but rate was very small

$$\frac{\Gamma\left(K_L^0 \to \mu^+\mu^-\right)}{\Gamma\left(K^+ \to \mu^+\nu_\mu\right)} \approx 3\times10^{-9}$$



- Glashow, Illiopoulos, Maiani (GIM) suggest a new quark (charm, +2/3)





$$\begin{pmatrix} \mathrm{c} \\ \mathrm{s} \end{pmatrix} \to \begin{pmatrix} \mathrm{c} \\ \mathrm{s}' \end{pmatrix} = \begin{pmatrix} \mathrm{c} \\ -\mathrm{d}\cdot\sin\theta_C + \mathrm{s}\cdot\cos\theta_C \end{pmatrix}$$

Note:<d|u><u|s> cancelled by <d|c><c|s>

Verification of the charm quark hypothesis

- Initial searches for mesons with a charm quark were negative.
- Mesons with quark pairs of the same flavor
  - $\pi^0 = \frac{1}{\sqrt{2}}\left(\mathrm{u\bar{u}} - \mathrm{d\bar{d}}\right)$ $m_{\pi^0} = 135\ \mathrm{MeV/c^2}$, lightest meson.

    $\rightarrow \gamma + \gamma$ EM decay in 10^{-16}s (Γ~10 eV/c^2)
  - $\phi = \left(\mathrm{s\bar{s}}\right)$ $m_\phi = 1020\ \mathrm{MeV/c^2}$, more mass than two K mesons.

    $\rightarrow \left(\mathrm{s\,\bar{u}}\right) + \left(\mathrm{u\,\bar{s}}\right) = K^- + K^+$

    $\rightarrow \left(\mathrm{s\,\bar{d}}\right) + \left(\mathrm{d\,\bar{s}}\right) = K_L^0 + K_S^0$

    Strong decay in 10^{-22}s (Γ~ 4 MeV/c^2, narrow resonance)

- 1974, Ting (pN) & Richter (e^+e^-) found a **very** narrow resonance

more later

  - $J/\psi = \left(\mathrm{c\,\bar{c}}\right)$ bound state $m_\psi = 3097\ \mathrm{MeV/c^2}$

    less mass than two charm mesons.

    $\rightarrow \ell^+ + \ell^-$; $\ell = e, \mu$

    $\rightarrow q + \bar{q}$; $q = \mathrm{u,\ d,\ s,}\ \neq \mathrm{c}$

    EM & hadron decays (Γ~ 91 keV/c^2)
- Soon came discovery of D mesons: $D^{0,+}\left(\mathrm{c\bar{u}}\right),\left(\mathrm{c\bar{d}}\right) \,\&\, \bar{D}^{0,-}\left(\mathrm{\bar{c}\,u}\right),\left(\mathrm{\bar{c}\,d}\right)$

Quark mixing

- Cabibbo-GIM scheme: W's couple quarks (u,c) --> (d',s') rotated (d,s) states, exactly like W's couple leptons (ν_e,ν_μ) --> (e,μ).

Mixing of -1/3 quarks

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

2x2 W boson coupling matrix

$$W^+ = (u,c) \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

- Kobayashi & Maskawa incorporate CP violation
  - required a third generation of quarks (top, bottom; before discovery)
  - 3x3 matrix, three angles (θ_1=θ_C, θ_2, θ_3) and a complex phase, $e^{i\delta}$, $\delta \cong 60°$

Mixing of -1/3 quarks

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

3x3 W boson (CKM) coupling matrix

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ -s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

$$c_{ij} = \cos\theta_{ij} \; ; \; s_{ij} = \sin\theta_{ij}$$

General character of CKM matrix

- Mixing of 1st & 2nd generation with the 3rd is very small

$$|V_{CKM}| = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 0.975 & 0.22 & \sim 0.004 \\ 0.22 & 0.975 & 0.04 \\ < 0.01 & 0.04 & 0.999 \end{pmatrix} \approx \begin{pmatrix} \cos\theta_1 & \sin\theta_1 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- $V_{tb} = 1$ means t-quark (m_t~175 GeV/c^2) decays to b-quarks (m_b~5 GeV/c^2), and almost never to s-quarks or d-quarks

- If V_{cb} & V_{ub} were ZERO, bottom mesons would be STABLE.
  - B meson decays restricted by small (but dominant) V_{cb}, resulting in a lifetime much longer than expected for a quark with mass ~5 GeV/c^2. In fact τ_B~4τ_D, bottom-mesons live longer than charmed-mesons !!
  - B meson decays are primarily to charm mesons, m_D ~ 1.8 GeV/c^2
  - However, many interesting, but very rare, B decays skip 2nd generation, e.g., B -> $\pi^+\pi^-$, a CP eigenstate.

Bottom meson and CP violation in CKM matrix

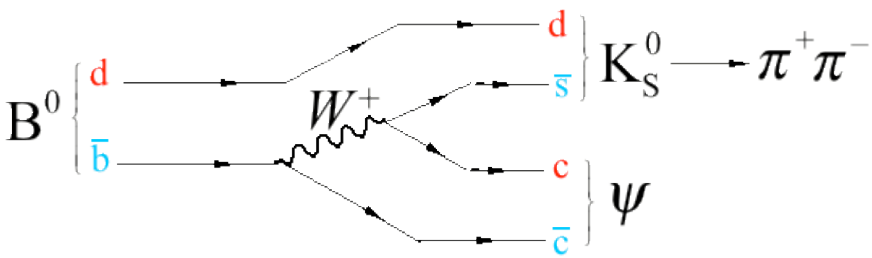
- b with 1st generation; b with 2nd generation

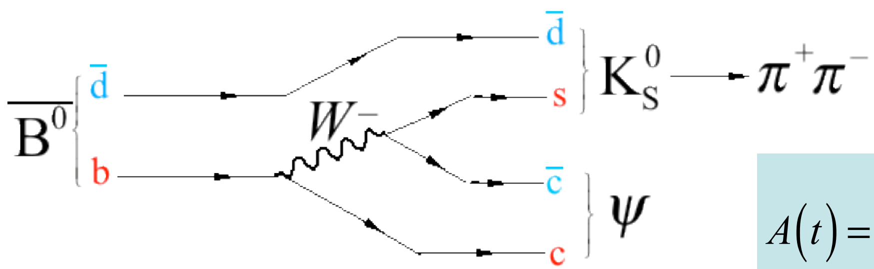
$$B^+(\mathrm{u}\,\bar{\mathrm{b}});\ B^0(\mathrm{d}\,\bar{\mathrm{b}})$$
$$B^-(\bar{\mathrm{u}}\,\mathrm{b});\ \overline{B^0}(\bar{\mathrm{d}}\,\mathrm{b})$$

$$B_c^+(\mathrm{c}\,\bar{\mathrm{b}});\ B_s^0(\mathrm{s}\,\bar{\mathrm{b}})$$
$$B_c^-(\bar{\mathrm{c}}\,\mathrm{b});\ \overline{B_s^0}(\bar{\mathrm{s}}\,\mathrm{b})$$

- B^0 & $\overline{B}^0$ decay to same ~CP eigenstate: ψK_S

Any difference in the decays violates CP



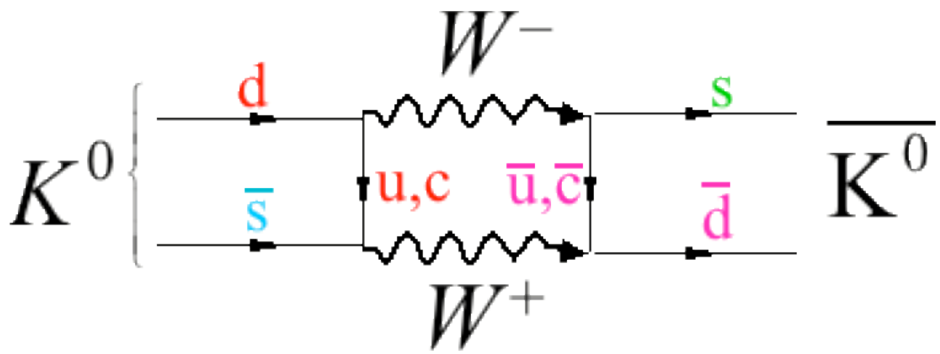


B and $\overline{\mathrm{B}}$ created in pairs in e^+e^- collisions and at hadron colliders: Identify one by a definitive decay, then the other is tagged as B or $\overline{\mathrm{B}}$

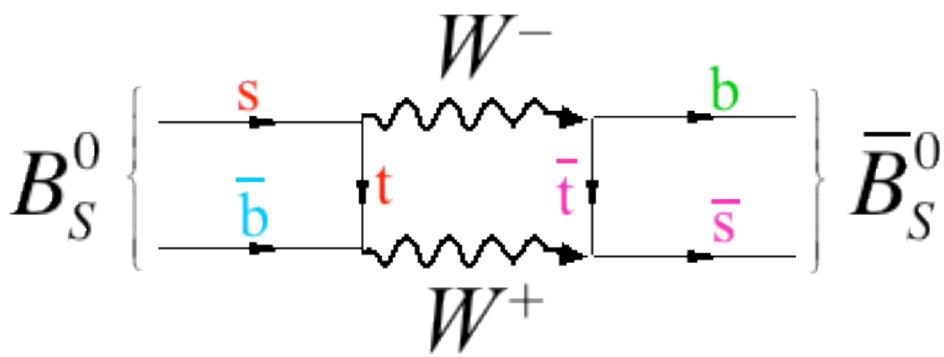
$$A(t) = \frac{\Gamma'\left[\bar{B}(t)\to\psi K_s\right] - \Gamma'\left[B(t)\to\psi K_s\right]}{\Gamma'\left[\bar{B}(t)\to\psi K_s\right] + \Gamma'\left[B(t)\to\psi K_s\right]} = 0.731 \pm 0.056$$

Matter <-> Antimatter oscillations

$K^0 \leftrightarrow \bar{K}^0$ oscillations



$B_S^0 \leftrightarrow \bar{B}_S^0$ oscillations



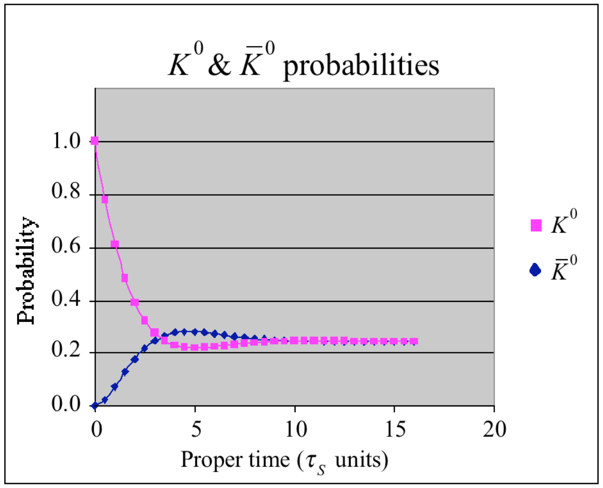
$$e^{i\phi} \sim V_{tb}^* V_{ts} / V_{tb} V_{ts}^*$$

$B_S^0 \leftrightarrow \bar{B}_S^0$ Oscillations

$K^0 \leftrightarrow \bar{K}^0$ oscillations   Homework problem 12.1

$$P\left(\bar{K}^0, t\right) \approx \tfrac{1}{4} e^{-t/\tau_S} + \tfrac{1}{4} e^{-t/\tau_L} - \tfrac{1}{2} e^{-\frac{1}{2}\left(\frac{1}{\tau_S}+\frac{1}{\tau_L}\right)t} \cos \frac{\Delta m c^2}{\hbar} t$$

$$\tau_S = 0.9 \times 10^{-10}\,\text{s},\ \tau_L = 5 \times 10^{-8}\,\text{s}\ ,\ \Delta m c^2 = 3.5 \times 10^{-12}\ \text{MeV}$$



$B_S^0 \leftrightarrow \bar{B}_S^0$ oscillations

$$P\left(\bar{B}_S^0, t\right) \approx \tfrac{1}{2} e^{-t/\tau} \left[1 - \cos \frac{\Delta m c^2}{\hbar} t \right]$$

$$\tau_L \approx \tau_H ; \tau = 1.5 \times 10^{-12}\,\text{s},\ \Delta m c^2 = 1.2 \times 10^{-8}\ \text{MeV}$$

CDF announcement of last year!

Fourier analysis of oscillations