PHY492: Nuclear & Particle Physics

Lecture 20

HW-5

Neutrinos and neutrino oscillations

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11.1 $J^P = 1^- : \rho^0(770) \to \pi^+ + \pi^-$, what about $\rho^0(770) \to \pi^0 + \pi^0$?

<u>Isospin spin</u>

$\rho^0(770) \to \pi^+ + \pi^- \quad : \quad 1,0 \to 1,+1; 1,-1 = \sqrt{\tfrac{1}{2}}$

$\rho^0(770) \to \pi^0 + \pi^0 \quad : \quad 1,0 \to 1,0; 1,0 = 0$

Isospin predicts $\pi^0\pi^0$ is zero.

Spin-Parity: The wave function of the $\pi^0\pi^0$ system must be symmetric, i.e., *l* even. However, to have – parity, *l* must be odd. So NO GO.

11.3 In ρ rest frame, let θ be the angle of a pion wrt the ρ momentum direction.

$J = 1,\ J_z = 0$

Wave function of two decay pions contains $Y_{10} = \cos\theta$.

$$\frac{d\sigma}{d\Omega} \sim |Y_{10}|^2 \sim \cos^2\theta$$

$J = 1,\ J_z = \pm 1$

Wave function of two decay pions contains $Y_{11} = \sin\theta$.

$$\frac{d\sigma}{d\Omega} \sim |Y_{11}|^2 \sim \sin^2\theta$$

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11.5

J^{PC} for $\gamma:1^{--}$; $\pi^0:0^{-+}$; $\omega^0:1^{--}$; $\eta':0^{-+}$; $\rho^0:1^{--}$; $J/\psi:1^{--}$

$\omega^0 \to \pi^0 + \gamma$; allowed because both sides have C = –1

$\eta' \to \rho + \gamma$ allowed because both sides have C = +

$\pi^0 \to \gamma + \gamma + \gamma$ not allowed, 3 gammas have charge conjugation –, pion is +

$J/\psi \to p + \bar{p}$ allowed. J/ψ & $p\bar{p}$ will have Parity –, state is ${}^3S_1$ (anti-symmetric)

in ${}^1S_0$, C($p\bar{p}$) = +($p\bar{p}$), while in ${}^3S_1$, C($p\bar{p}$) = –($p\bar{p}$)

$\rho^0 \to \gamma + \gamma$ not allowed, 2 gammas have charge conjugation +.

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12.1 Probability vs. time of K^0 beam yielding a $\bar{K}^0$.

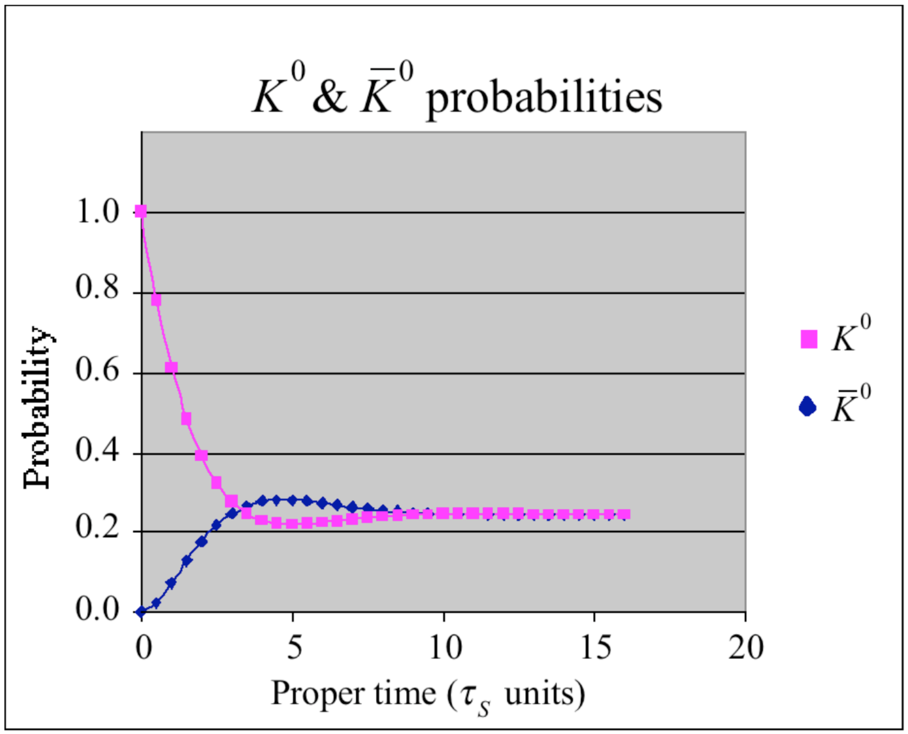
$\tau_s = 0.9\times10^{-10}\,\mathrm{s},\ \tau_L = 5\times10^{-8}\,\mathrm{s}$

$p = 1+\varepsilon;\quad q = -(1-\varepsilon)$

$\varepsilon = 2.29\times10^{-3}$

$\Delta mc^2 = 3.5\times10^{-12}$ MeV

$$P\left(\bar{K}^0, t\right) = \left|\frac{q}{p}\right|^2 \left[\frac{1}{4}\exp\left(-\frac{t}{\tau_S}\right) + \frac{1}{4}\exp\left(-\frac{t}{\tau_L}\right) - \frac{1}{2}\exp\left(-\frac{1}{2}\left(\frac{1}{\tau_S}+\frac{1}{\tau_L}\right)t\right)\cos\frac{\Delta mc^2}{\hbar}t\right]$$



Neutrino mass

- Many attempts to see if neutrinos have mass
  - Tritium beta decay end-point
    - electron neutrino mass < 3 eV/c^2
  - Meson decays
    - muon neutrino mass < 200 keV/c^2
    - tau neutrino mass < 18 MeV/c^2
  - Cosmological limits
    - lots of assumptions about big bang
    - nuclear-synthesis, more assumptions
    - (number density)x(mass) in universe constraint
    - neutrino masses < 1 eV
- Theoretical bias for non-zero neutrino mass
  - Why are all neutrinos left-handed?
  - What happened to all the right hand neutrinos?
  - "See-Saw" mechanism
    - right hand neutrinos forced to be VERY MASSIVE
    - and left hand neutrinos to be very light (but not zero).

Propagation of neutrino mass states

- Remember, Schrodinger wave equation solutions ?
- Remember, time dependent Schrodinger wave equation solutions ?

$$e^{-i(\omega t - kx)}; \quad \omega = \frac{E}{\hbar}; \quad k = \frac{p}{\hbar}$$

$$\psi(x,t) = \psi(x,0)e^{-\frac{i}{\hbar}(Et - px)}; \quad \text{let } x = L,\ t = L/c$$

$$\psi(L) = \psi(0)e^{-\frac{i}{\hbar c}(E - pc)L}$$

$$\psi(L) = \psi(0)e^{-\frac{i}{\hbar c}\frac{m^2c^4}{2E}L}$$

$$pc = E\sqrt{1 - \frac{m^2c^4}{E^2}} \approx E - \frac{m^2c^4}{2E}, \quad E >> mc^2$$

$$E - pc \approx \frac{m^2c^4}{2E}$$

- Phase factor $e^{-\frac{i}{\hbar c}\frac{m^2c^4}{2E}L}$ depends on distance L from production, particle energy, and mass squared !!

Neutrino masses and flavors

- Neutrino mass states (H_{free}) and flavor states (H_{weak}) are different!
- Remember Cabibbo mixing of charge -1/3 quark states?

weak eigenstates
$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$
mass eigenstates

Cabibbo angle θ

- A neutrino flavor state is a mixture of 3 mass states (simplify: use 2)

weak eigenstates
$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$
mass, m_1, m_2 eigenstates

Mixing angle θ

- At x = 0, t = 0 an electron or muon neutrino is a mass state mixture.

$$\left|\nu_e\right\rangle = \left|\nu_1\right\rangle\cos\theta + \left|\nu_2\right\rangle\sin\theta$$
$$\left|\nu_\mu\right\rangle = \left|\nu_2\right\rangle\cos\theta - \left|\nu_1\right\rangle\sin\theta$$

$$K^+ \to \pi^0 + e^+ + \nu_e$$
$$\pi^+ \to \mu^+ + \nu_\mu$$

ν's produced in these decays

- Attempts (really can't do it) to measure the mass of the electron neutrino near t = 0 would give one of two possible answers:

$$m_1\text{, with probability } \cos^2\theta, \text{ or } \quad m_2\text{, with probability } \sin^2\theta.$$

Propagation of mass eigenstates

$$e^{-\frac{i}{\hbar c}\frac{m^2c^4}{2E}L}$$

- What happens to pure muon neutrinos a distance L away?

$$\left|\nu_\mu(L)\right\rangle = e^{-\frac{i}{\hbar c}\frac{m_2^2c^4}{2E}L}\left|\nu_2\right\rangle\cos\theta - e^{-\frac{i}{\hbar c}\frac{m_1^2c^4}{2E}L}\left|\nu_1\right\rangle\sin\theta$$

$$\left|\nu_1\right\rangle = \left|\nu_e\right\rangle\cos\theta - \left|\nu_\mu\right\rangle\sin\theta$$

$$\left|\nu_2\right\rangle = \left|\nu_\mu\right\rangle\cos\theta + \left|\nu_e\right\rangle\sin\theta$$

- Masses are different, so phases will be different!
- Neutrinos become a mixture of muon and electron neutrinos.

$$\left|\nu_\mu(L)\right\rangle = \left(e^{-\frac{i}{\hbar c}\frac{m_2^2c^4}{2E}L}\cos^2\theta + e^{-\frac{i}{\hbar c}\frac{m_1^2c^4}{2E}L}\sin^2\theta\right)\left|\nu_\mu\right\rangle + (\quad)\left|\nu_e\right\rangle$$

$$\left\langle\nu_\mu\middle|\nu_\mu(L)\right\rangle = \left(e^{-\frac{i}{\hbar c}\frac{m_2^2c^4}{2E}L}\cos^2\theta + e^{-\frac{i}{\hbar c}\frac{m_1^2c^4}{2E}L}\sin^2\theta\right)$$

$\sin^2 2\theta$ is amplitude

wavelength
$\lambda(\text{km}) = 2\pi E/(1.27\Delta m^2)$

surviving muon neutrinos

$$\left|\left\langle\nu_\mu\middle|\nu(L)\right\rangle\right|^2 = 1 - \sin^2 2\theta\sin^2\left(1.27\Delta m^2 L/E\right)$$

appearing electron neutrino

$$\left|\left\langle\nu_e\middle|\nu(L)\right\rangle\right|^2 = \sin^2 2\theta\sin^2\left(1.27\Delta m^2 L/E\right)$$

Δm^2 in eV^2
L in km
E in GeV

Calculate muon neutrino survival probability

Messy trigonometry and algebra:

$$\left|\left\langle \nu_\mu \middle| \nu_\mu(L)\right\rangle\right|^2 = \left(e^{-\frac{i}{\hbar c}\frac{m_2^2 c^4}{2E}L}\cos^2\theta + e^{-\frac{i}{\hbar c}\frac{m_1^2 c^4}{2E}L}\sin^2\theta\right)$$

$$\times\left(e^{+\frac{i}{\hbar c}\frac{m_2^2 c^4}{2E}L}\cos^2\theta + e^{+\frac{i}{\hbar c}\frac{m_1^2 c^4}{2E}L}\sin^2\theta\right)$$

$$= \cos^4\theta + \sin^4\theta + \left(e^{+\frac{i}{\hbar c}\frac{\left(m_2^2 - m_1^2\right)c^4}{2E}L} + e^{-\frac{i}{\hbar c}\frac{\left(m_2^2 - m_1^2\right)c^4}{2E}L}\right)\cos^2\theta\sin^2\theta$$

$$= \cos^2\theta\left(1-\sin^2\theta\right) + \sin^2\theta\left(1-\cos^2\theta\right) + \frac{\left(\;\right)}{2}2\cos^2\theta\sin^2\theta$$

$$= 1 - 2\sin^2\theta\cos^2\theta\left(1-\cos\left(\frac{\left(m_2^2-m_1^2\right)c^4 L}{2E\hbar c}\right)\right)$$

$$= 1 - \sin^2 2\theta\left(\sin^2\left(\frac{\left(m_2^2-m_1^2\right)c^4 L}{4E\hbar c}\right)\right)$$

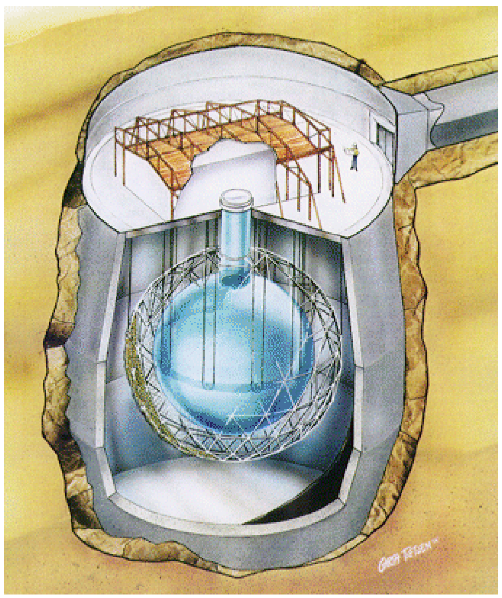
Solar and cosmic ray oscillation experiments

- Three neutrino flavors, three mass states, the mixing matrix is 3x3
- Three Δm^2 and two angles (one can't be measured this way) determined by solar and cosmic ray neutrino experiments

solar neutrinos

Sudbury (SNO)

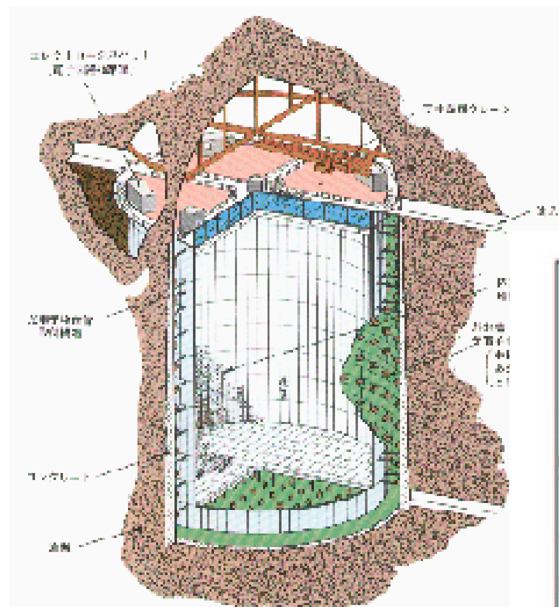
1kT D_2O

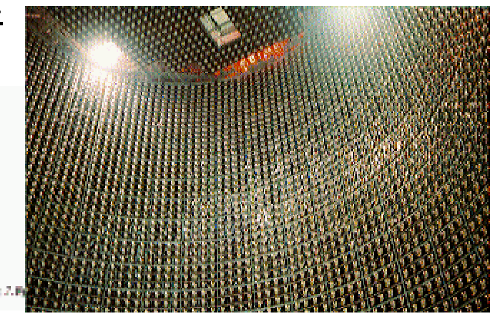


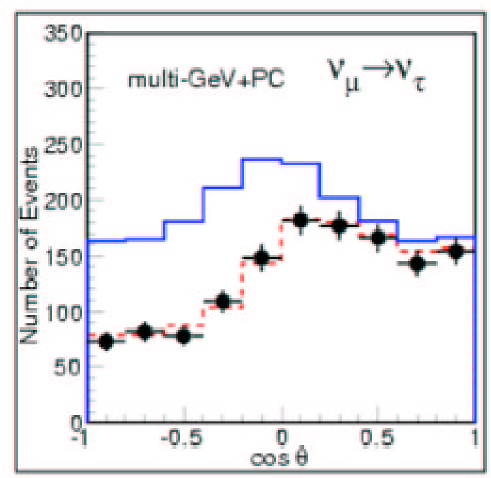
solar and cosmic ray

Super-kamiokanda

50 kT H_2O







Three ν flavors and three ν masses

- As is the case for quarks, the mixing matrix is 3x3
- Very different from quarks, off-diagonal mixing angles are LARGE.

Missing one crucial angle, sinθ_{13}, and a CP violating phase δ

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = M \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$|M| \approx \begin{pmatrix} 0.83 & 0.55 & s_{13}e^{i\delta} \\ 0.39 & 0.59 & .7 \\ 0.39 & 0.59 & .7 \end{pmatrix}; \quad \begin{matrix} s_{13} < 0.25 \\ \delta \sim ? \end{matrix}$$

- Result for muon neutrinos oscillating to electron neutrinos is the same as in the two neutrino case.

appearing electron neutrinos

$$\left|\left\langle \nu_e \middle| \nu(L) \right\rangle\right|^2 = \sin^2 2\theta_{13} \sin^2\left(1.27\Delta m_{23}^2 L / E\right)$$

$$\Delta m_{23}^2 \sim 3\times 10^{-3}\ \text{eV}^2; \quad \text{pick } E \sim 2\ \text{GeV}$$

$$\lambda = \frac{2\pi E}{1.27\Delta m_{23}^2} = \frac{4\pi}{4\times 10^{-3}}\text{km} \ \approx 3000\ \text{km}$$

First maximum at $\lambda / 4 \approx 750$ km

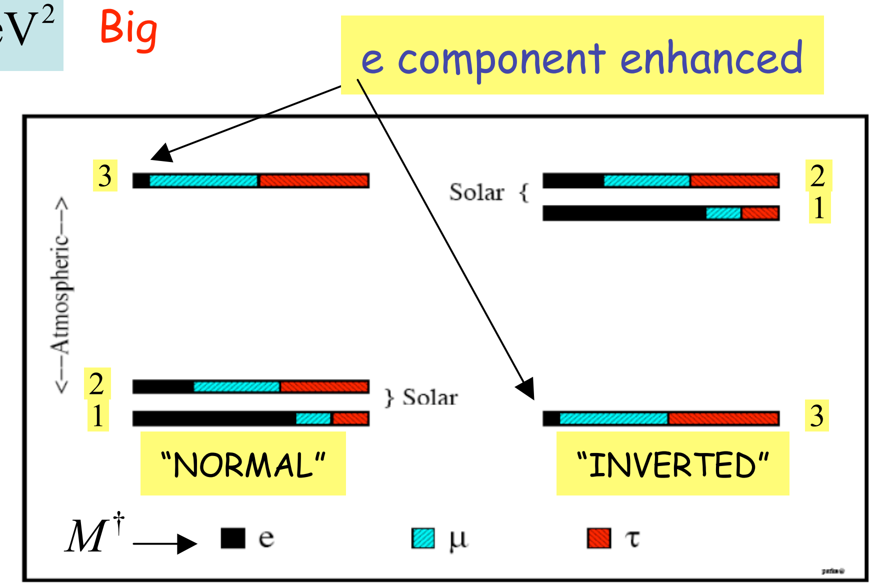
Fermilab neutrino beam points to Sudan MN which is 750 km away

Mass hierarchy of three mass states

- Mass splitting $\Delta m_{ij}^2 = \left| m_j^2 - m_i^2 \right|$

$$\text{Solar: } \Delta m_{12}^2 \sim 5\times 10^{-5}\ \text{eV}^2 \quad \text{Small}$$
$$\text{Atmospheric: } \Delta m_{23}^2 \sim 2.5\times 10^{-3}\ \text{eV}^2 \quad \text{Big}$$

- Mass hierarchy unknown
  - "NORMAL" as in quark masses
  - "INVERTED" as in P-states?
  - Can't do CP violation on earth without hierarchy resolution



- MSW effect
  - In a long baseline experiment ν_e and $\bar{\nu}_e$ passing through the earth have oscillation probability affected with opposite signs.
  - Good for determination of mass hierarchy, but complicates CP violation analysis. Unavoidable, even in Japan.

Rate for muon to electron neutrinos

Appearing electron neutrino

$$\left|\langle \nu_e | \nu(L) \rangle\right|^2 = \sin^2 2\theta_{13} \sin^2\left(1.27\Delta m_{23}^2 L / E\right)$$

- Muon neutrino (anti-neutrino) beam from Chicago to northern MN
- Put 20 kT of liquid scintillator there in the beam
- Look for neutrino interactions that produce electrons.

$$\nu_e + N \to e^- + hadrons \qquad \bar{\nu}_e + N \to e^+ + hadrons$$

  - Will need 5 years to do R&D, design, and construct detector
  - Will need 3-4 years to accumulate data on neutrinos
  - Will need 5 years to accumulate data on anti-neutrinos
  - So look for results on CP violation in leptons 2020.

- New technology: Liquid Argon Time Projection Chamber
  - May be able to do it faster -- I hope.