PHY492: Nuclear & Particle Physics

Lecture 23

HW

Particle Detectors

Homework

13.1

$$x_{\min} = \left(\frac{m\omega^2}{\lambda}\right)^{\frac{1}{2}} \quad x^2_{\min} = \frac{m\omega^2}{\lambda}; \quad x^3_{\min} = \left(\frac{m\omega^2}{\lambda}\right)^{\frac{3}{2}}; \quad x^4_{\min} = \frac{m^2\omega^4}{\lambda^2}$$

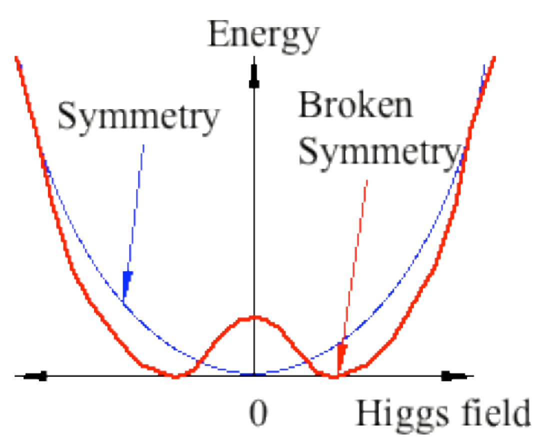
$$V\left(x_{\min}+x,y\right) = -\frac{1}{2}m\omega^2\left[\left(x_{\min}+x\right)^2+y^2\right]+\frac{\lambda}{4}\left[\left(x_{\min}+x\right)^2+y^2\right]^2$$

$$= -\frac{1}{2}m\omega^2\left[x^2_{\min}+2x_{\min}x+x^2+y^2\right]+\frac{\lambda}{4}\left[\begin{array}{c} x^4_{\min}+4x^3_{\min}x+6x^2_{\min}x^2+4x_{\min}x^3+x^4 \\ +2\left(x^2_{\min}+2x_{\min}x+x^2\right)y^2+y^4 \end{array}\right]$$

$$= \left[-\frac{1}{2}\frac{m^2\omega^4}{\lambda}-m\omega^2\left(\frac{m\omega^2}{\lambda}\right)^{\frac{1}{2}}x-\frac{1}{2}m\omega^2x^2+\ldots\right]$$

$$+\left[\frac{1}{4}\frac{m^2\omega^4}{\lambda}+m\omega^2\left(\frac{m\omega^2}{\lambda}\right)^{\frac{1}{2}}x+\frac{3}{2}m\omega^2x^2+\ldots\right]$$

$$= \underline{-\frac{1}{4}\frac{m^2\omega^4}{\lambda}+m\omega^2x^2+\ldots} \quad \text{Q.E.D.}$$



Homework

14.1 a) Top lifetime

$$\delta E \delta t \sim \hbar; \quad \tau \sim \frac{\hbar}{\Gamma} = \frac{6.6\times 10^{-22}\ \text{MeV}\cdot\text{s}}{1.5\times 10^{3}\ \text{MeV}} = 4.4\times 0^{-25}\,\text{s}$$

b) No time to interact

$$t_i = 1\ \text{fm}/c = 3.3\times 10^{-24}\,\text{s}$$

$$\frac{\tau}{t_i} = \frac{4.4\times 10^{-25}\,\text{s}}{3.3\times 10^{-24}\,\text{s}} = 1.3\times 10^{-1}$$

c) Top quark decay $t \to W + b$

Energy Cons.: $E_W + E_b = m_t; \quad E_W^2 = m_t^2 - 2m_t E_b + E_b^2$

Momentum Cons.: $E_W^2 - m_W^2 = p^2 = E_b^2 - m_b^2; \quad E_W^2 = E_b^2 - m_b^2 + m_W^2$

$$E_b = \left(m_t^2 + m_b^2 - m_W^2 / 2m_t\right) = 69\ \text{GeV}; \quad p_b = \left(E_b^2 - m_b^2\right)^{\frac{1}{2}} = 69\ \text{GeV/c}$$

d) Parton-parton collision

$$p_a = x_a \frac{\sqrt{s}}{2c} \qquad p_b = -x_b \frac{\sqrt{s}}{2c}$$

$$E_a = x_a \frac{\sqrt{s}}{2}; \quad p_a = x_a \frac{\sqrt{s}}{2c}; \quad E_b = x_b \frac{\sqrt{s}}{2}; \quad p_b = -x_b \frac{\sqrt{s}}{2c}$$

$$\hat{s} = \left(E_a + E_b\right)^2 - \left(p_a c + p_b c\right)^2 = \left(x_a^2 + x_b^2\right)\frac{s}{4} + x_a x_b \frac{s}{2} - \left(x_a^2 + x_b^2\right)\frac{s}{4} + x_a x_b \frac{s}{2}$$

$$= x_a x_b s$$

top mass = 0.175 TeV $\quad s_{TeV}$ =(2 TeV)^2 = 4 TeV^2 $\quad s_{LHC}$ =(14 TeV)^2 = 200 TeV^2

$$(.350)^2 = x_a x_b (4); \quad x_a x_b = 0.03; \quad x_a \sim x_b = \sqrt{.03} = 0.18$$

$$x_a \sim x_b = 0.025$$

Homework

14.3

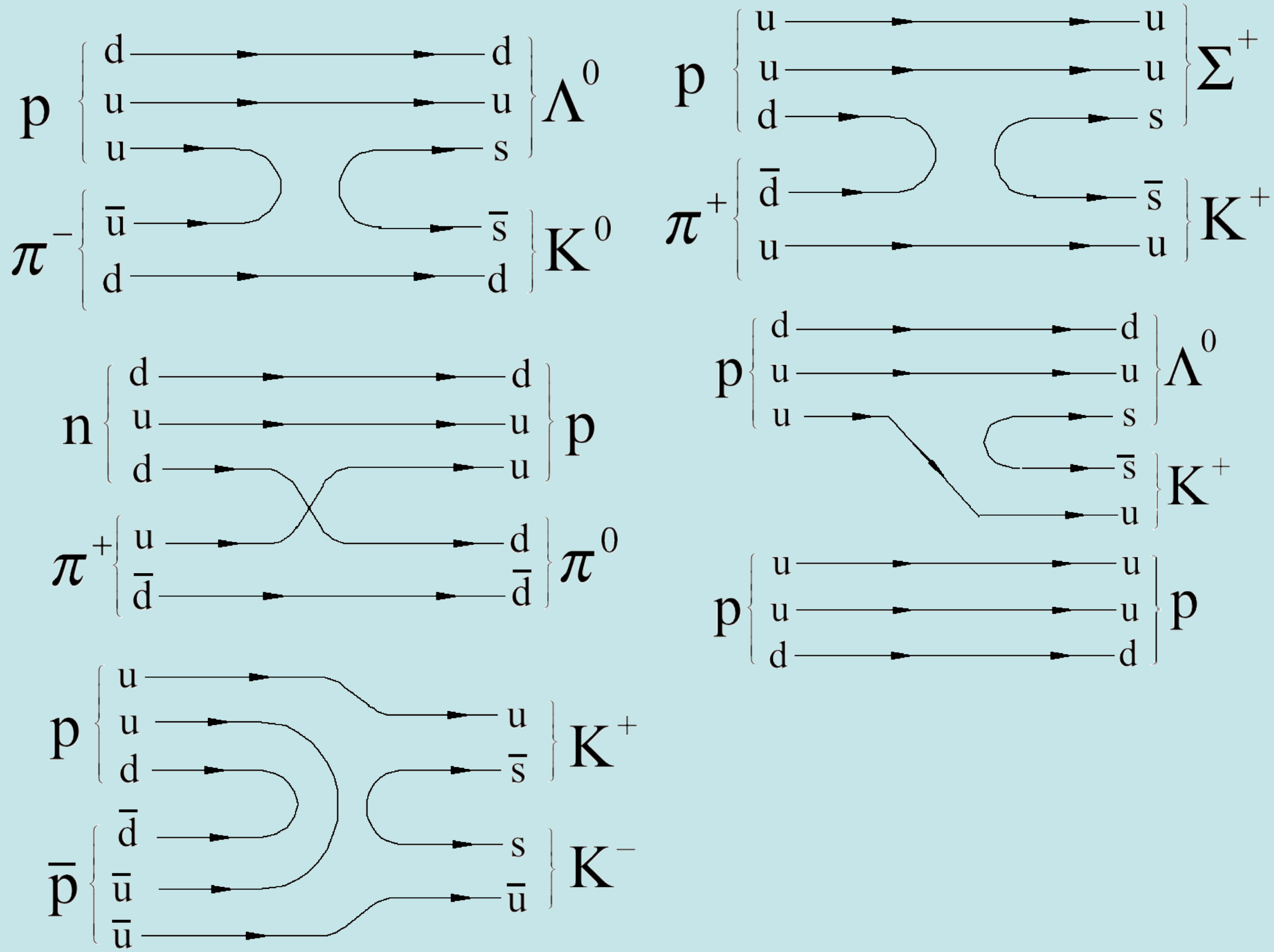
a) $\pi^- + p \rightarrow \Lambda^0 + K^0$

b) $\pi^+ + p \rightarrow \Sigma^+ + K^+$

c) $\pi^+ + n \rightarrow \pi^0 + p$

d) $p + p \rightarrow \Lambda^0 + K^+ + p$

e) $\bar{p} + p \rightarrow K^+ + K^-$



Homework

14.5 Quark mixing matrix

$$\begin{pmatrix} d \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta_C & -\sin\theta_C \\ \sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d' \\ s' \end{pmatrix}$$

$K^0 = (d, \bar{s})$ Quark content

$$d = d'\cos\theta_C - s'\sin\theta_C$$
$$\bar{s} = \bar{d}'\sin\theta_C + \bar{s}'\cos\theta_C$$

Weak interaction acts on mixed states

$$\langle d,\bar{s}|Z\rangle = \langle d',\bar{d}'|Z\rangle\cos\theta_C\sin\theta_C - \langle s',\bar{s}'|Z\rangle\cos\theta_C\sin\theta_C$$
$$+\langle d',\bar{s}'|Z\rangle\cos^2\theta_C + \langle s',\bar{d}'|Z\rangle\sin^2\theta_C$$

Z couples only to the same weak flavor quarks

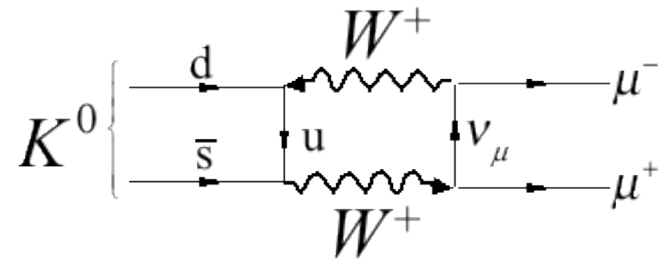
$\langle d',\bar{s}'|Z\rangle$ and $\langle s',\bar{d}'|Z\rangle$ are both ZERO

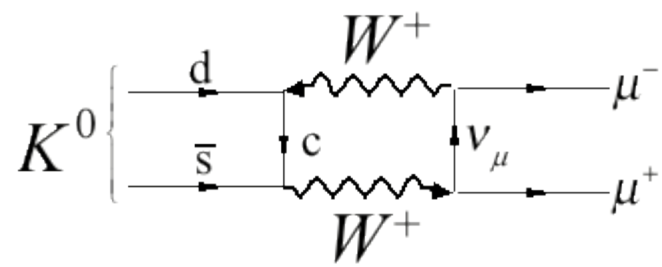
$$\langle d',\bar{d}'|Z\rangle = \langle s',\bar{s}'|Z\rangle \neq 0$$

But 4 quark mixing matrix gives this cancellation

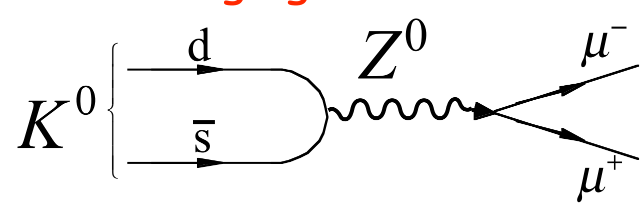
$$\langle d,\bar{s}|Z\rangle = \langle d',\bar{d}'|Z\rangle\cos\theta_C\sin\theta_C - \langle s',\bar{s}'|Z\rangle\cos\theta_C\sin\theta_C = 0$$

2nd order weak decays (u/c)





Not seen: 1st order weak decay flavor changing neutral current



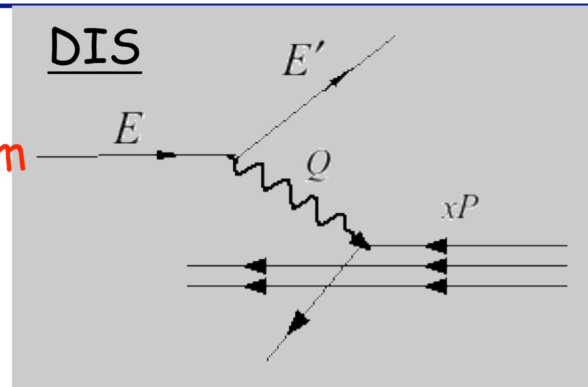
Homework

14.7 a) 4-vector dot product (lab frame)

$$Q = \left[\left(\vec{k}' - \vec{k}\right), \nu\right]; \quad P = \left[0, m_p c^2\right]$$

$$Q \cdot P = m_p c^2 \nu \text{ (invariant)}$$

infinite momentum frame



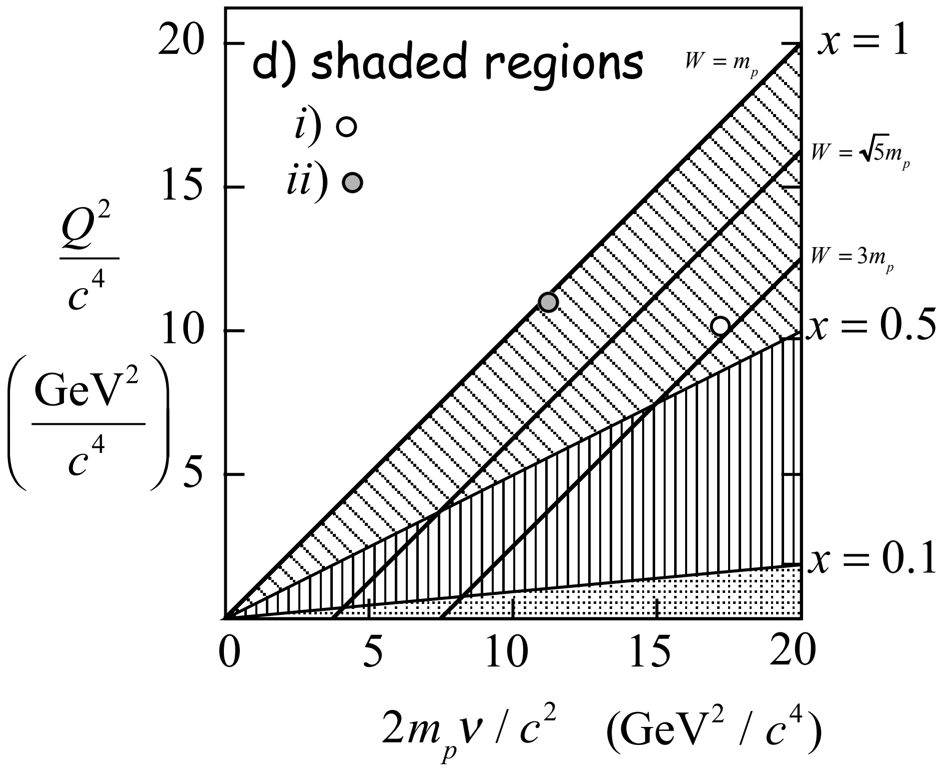
b) parton absorbs Q but remains massless

$$\left(x\vec{P} + \vec{Q}\right)^2 = 0 = x^2 P^2 + 2x\vec{P}\cdot\vec{Q} + Q^2$$

$$x = \frac{Q^2}{2\vec{P}\cdot\vec{Q}} = \frac{Q^2}{2m_p \nu c^2} \quad \text{for } Q^2 >> x^2 P^2 = x^2 m_p^2 c^4$$

c) from 14.6

$$W^2 = m_p^2 + \frac{2m_p c^2 \nu}{c^4} - \frac{Q^2}{c^4}$$



e) two points on plot

$$i)\ \nu = E - E' = 9 \text{ GeV}; Q^2 = 4EE'\sin^2\left(\theta/2\right)$$

$$2m_p \nu = 17 \text{ GeV}^2 / c^2; \quad Q^2 = 10 \text{ GeV}^2$$

$$ii)\ \nu = E - E' = 6 \text{ GeV}; Q^2 = 4EE'\sin^2\left(\theta/2\right)$$

$$2m_p \nu = 11.3 \text{ GeV}^2 / c^2; \quad Q^2 = 10.7 \text{ GeV}^2$$

How to detect particles

- Particles are detected by making them ionize atoms!
- Detecting charged particles
  - The electric field of a moving charged particle can ionize the atoms of the material in which it moves.
  - Ionization electrons are small and low mass, and can be collected by an electric field. Positive ions are big and heavy, and sluggish.
  - A charged particle accelerated by a magnetic or electric field radiates photons that can ionize atoms and release electrons
- Detecting neutral particles
  - Interact the neutral particle with matter and in the process release ionization electrons.
  - Sometimes you must completely destroy the neutral particle, but its energy has been used to create ionization.
- Must study Ionization to understand detectors

Ionization

- Ionization potential minimum energy to ionize (outer e shell)
  - Hydrogen 13.5 eV
  - Helium 25 eV
  - Lithium 5 eV
  - Neon 22 eV

- Average ionization potential, includes inner shells
  - reaction dependent
  - for charged particles (e.g., electrons)
    - ~$16Z^{0.9}$ (eV) for Z>1
    - Low Z Nobel Gases (He, Ne, Ar, a little higher)

Photon induced ionization

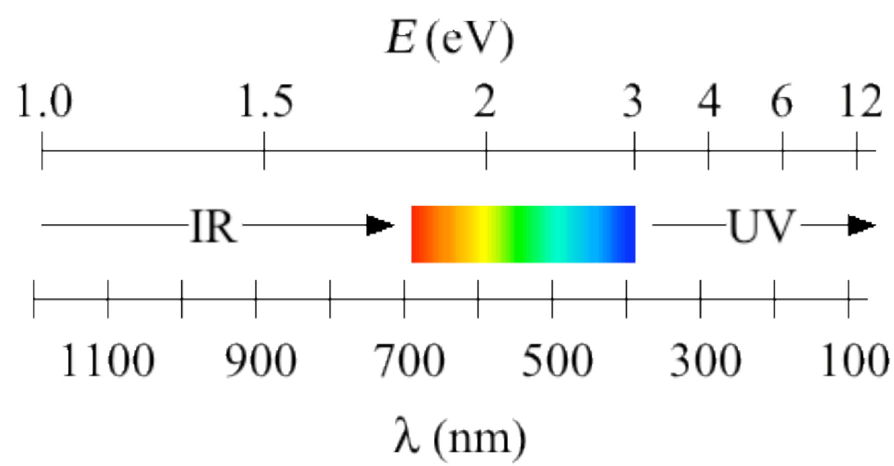
- Photon (<20 eV) induced ionization
  - Only valence electrons (a few)
  - Non-penetrating
  - Gases & surfaces
    - high temperature
      - thermionic emission
    - high electric fields
    - ultraviolet light
      - photoelectric effect
      - ozone
    - photo-cathodes (Cs)
    - silicon photodiodes
- X-ray (<1 MeV) induced
  - All electrons (Z)
  - Penetrating
  - Gases & solid interiors

- Useful conversion

$$\lambda - \text{wavelength},\ \nu - \text{frequency}$$
$$hc = 2\pi(\hbar c) = 2\pi(200\ \text{MeV}\cdot\text{fm})$$
$$= 1.2\times10^{3}(\text{eV}\cdot\text{nm})$$

- Photon energies

$$E = h\nu = \frac{hc}{\lambda} = \frac{1200(\text{eV}\cdot\text{nm})}{\lambda}$$



Particle Physics Booklet

- Particle Data Group - http://pdg.lbl.gov/
- Products
  - Order booklet http://pdg.lbl.gov/receive_our_products.html

http://pdg.lbl.gov/2005/reviews/contents_sports.html#expmethetc

Categories:

- Constants, Units, Atomic and Nuclear Properties
- Standard Model and Related Topics
- Particle Properties
- Hypothetical Particles
- Astrophysics and Cosmology
- Experimental Methods and Colliders
- Mathematical Tools
- Kinematics, Cross-Section Formulae, and Plots
- Authors, Introductory Text, History plots

Experimental Methods and Colliders

Accelerator physics of colliders (Rev.)

High-energy collider parameters (2004v)

Passage of particles through matter (Rev.)

Particle detectors (Rev.)

Radioactivity and radiation protection (Rev.)

Commonly used radioactive sources (2004v)

- Detector Lectures for Students/Teachers

http://teachers.web.cern.ch/teachers/archiv/HST2002/

Charged particle induced ionization

- Moving particle, mass M, ionizes atoms in medium

$$S(T) = -\frac{dT}{dx} = n_{ion}\bar{I}$$

T : kinetic energy of moving particle

n_{ion} : number of electron-ion pairs /unit path length

$\bar{I}$: average energy/electron-ion pair

- n_{ion} is particle velocity and charge dependent (Bethe and Bloch)

Stopping power

$$S(T) = \frac{4\pi Q^2 e^2 nZ}{m_e \beta^2 c^2}\left[\ln\left(\frac{2m_e c^2 \gamma^2 \beta^2}{\bar{I}}\right) - \beta^2\right]$$

$$\gamma = \frac{E}{Mc^2}; \quad \beta = \frac{pc}{E}; \quad \gamma\beta = \frac{p}{Mc}$$

$$S(T) \propto \frac{1}{\beta^2 c^2} = \frac{1}{v^2}$$

$\beta = v/c = < 0.8$

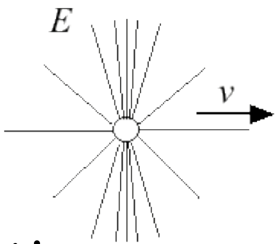
heavy ionization

S minimizes, $\gamma\beta \sim 3$

when $\beta \sim 0.95$

minimum ionization

$S \propto \ln\gamma$

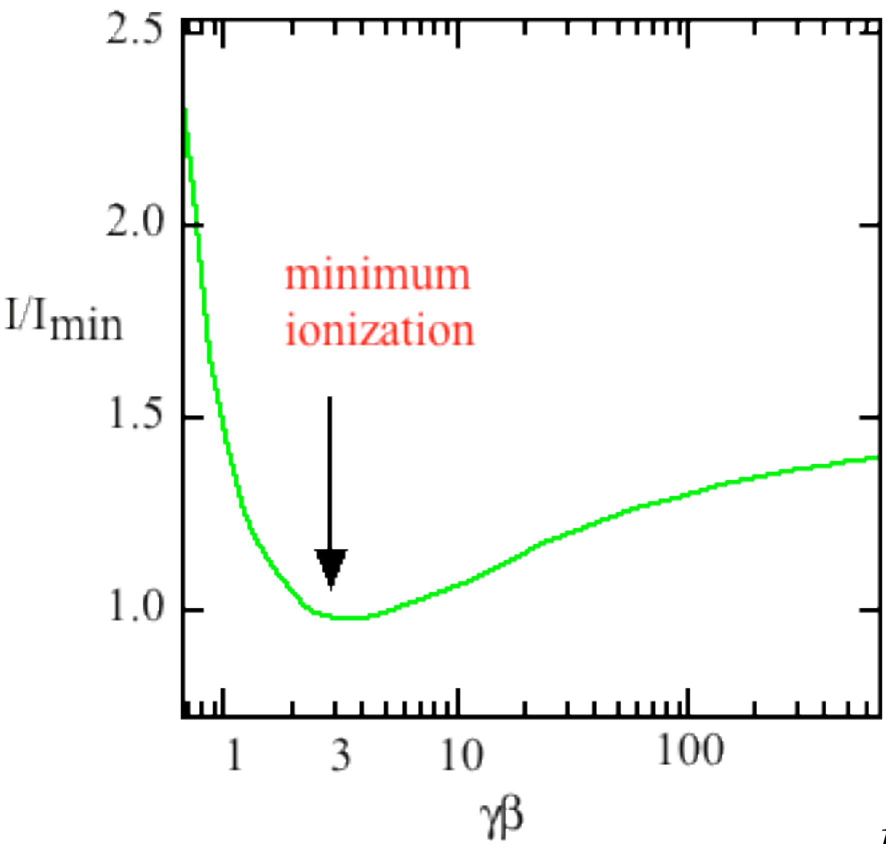


ultra relativistic

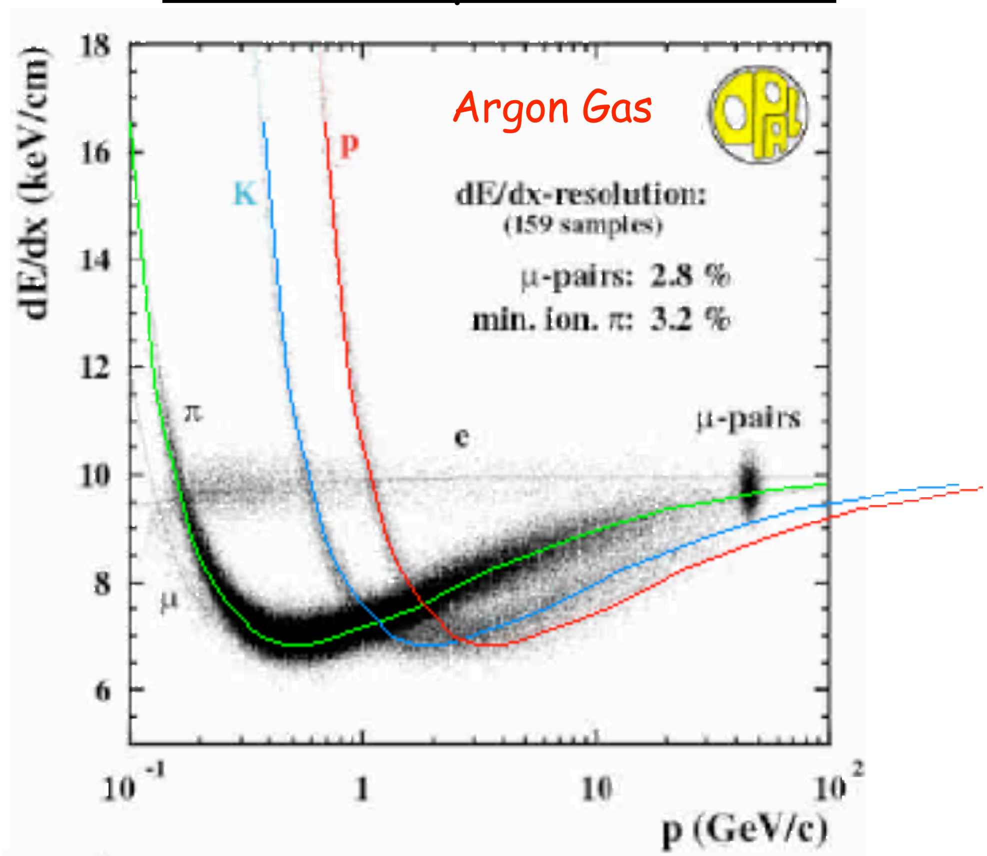
rise of ionization

Ionization in gases (relevant to all particles)

Normalized Ionization vs $\gamma\beta$



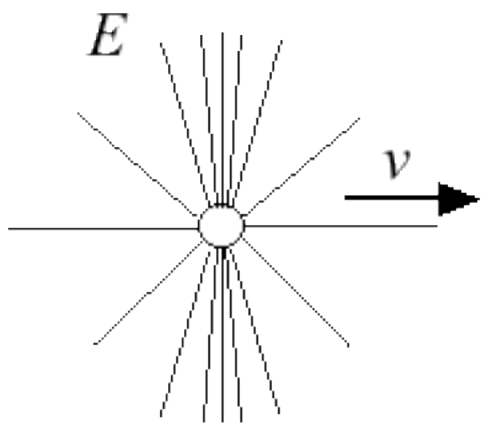
Ionization vs momentum for various particle masses

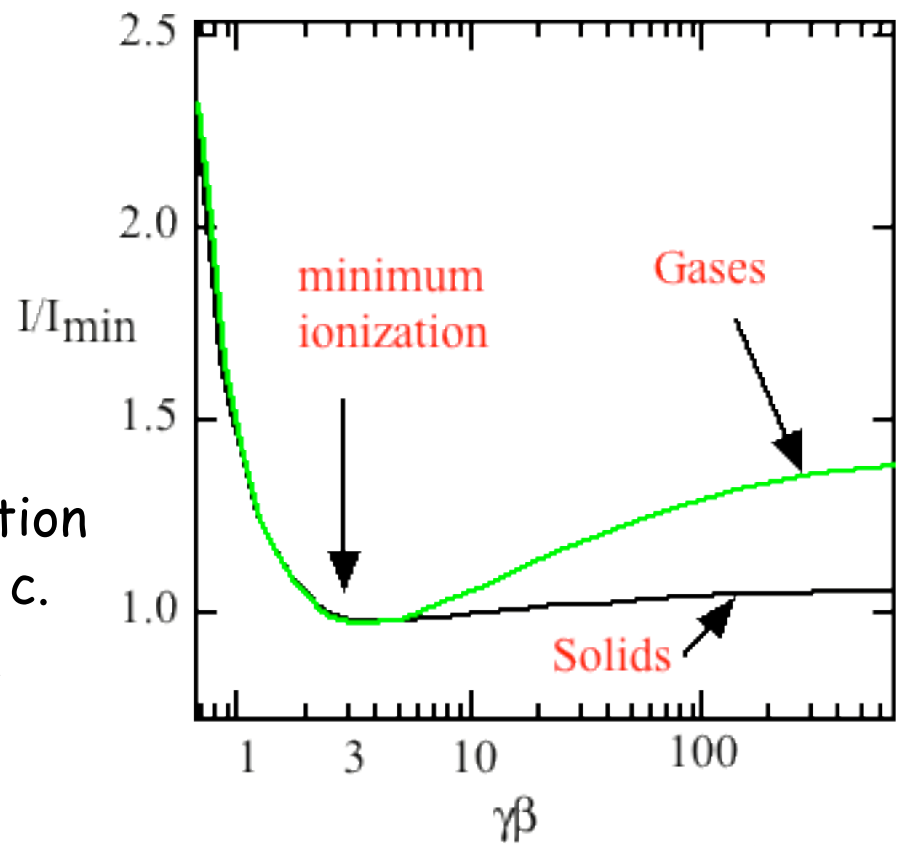


$$\gamma\beta = \frac{pc}{Mc^2}$$

Saturation of ionization in solids

- Relativistic rise of ionization is due to electric field concentration perpendicular to direction of motion





- Atoms along the line of motion see a stronger field as v -> c.
- Effect is largest in large Z gases, e.g., Xe
- Solids polarize and shield far electrons from field

→ effect saturates at only a few % above the minimum

Minimum ionization in *thin* solids

$$S(T) = -\frac{dT}{dx} \propto nZ = \rho A_0 \frac{Z}{A}$$

Z : atomic number of medium

n : number of atoms/unit volume

$n = \frac{\rho A_0}{A}$; A : atomic number of medium

- Units for energy loss
  - Z/A ~ 0.4 at large A, energy loss proportional to density S~ρ,
  - Divided by the density -> value nearly independent of material.
- $(dE/dx)_{min}$ tabularized for various materials in MeV/(g/cm²)
  - Polystyrene scintillator: 1.95

$$\rho_{\text{scintillator}} = 1.03 \text{ g/cm}^3$$

$$-\left.\frac{dT}{dx}\right|_{\min} = 1.95 \left(\frac{\text{MeV}}{\text{g/cm}^2}\right) \rho_{\text{Scintillator}} = 2.0 \text{ MeV/cm}$$

  - Iron (steel) : 1.45

$$\rho_{\text{iron}} = 7.87 \text{ g/cm}^3$$

$$-\left.\frac{dT}{dx}\right|_{\min} = 1.45 \left(\frac{\text{MeV}}{\text{g/cm}^2}\right) \rho_{\text{iron}} = 11.4 \text{ MeV/cm}$$

Relativistic muon loses ~2 MeV/cm in plastic, ~11.4 MeV/cm in Iron