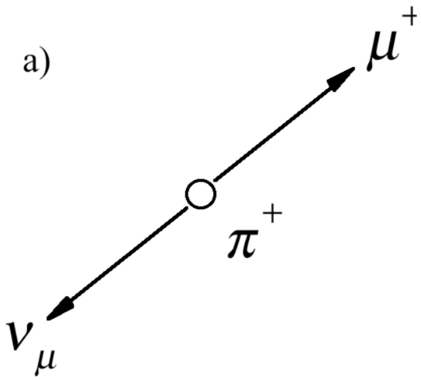
PHY492: Nuclear & Particle Physics

Lecture 24

Exam 2

Particle Detectors

Exam 2



b) Energy and momentum conservation equations:

$$\vec{p}_\mu = -\vec{p}_\nu; \quad \text{note: } p_\mu^2 = p_\nu^2 = p^2 \text{ and } E_\nu = pc$$

$$m_\pi c^2 = E_\mu + E_\nu = \sqrt{p^2c^2 + m_\mu^2 c^4} + pc$$

c) Solve for p_μ

$$\left(m_\pi c^2 - pc\right)^2 = p^2c^2 + m_\mu^2 c^4$$

$$m_\pi^2 c^4 - 2m_\pi c^2 pc + p^2c^2 = p^2c^2 + m_\mu^2 c^4$$

$$2m_\pi c^2 pc = m_\pi^2 c^4 - m_\mu^2 c^4$$

$$p_\mu = p = \tfrac{1}{2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right) m_\pi c$$

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2. Short Answer [4 pts each]

a) To describe the QCD color quantum numbers of quarks *and* gluons, how many colors are involved and give them relevant names.

There are 3 colors and 3 anti-colors, and gluons need both of them

Colors: $red, blue, green$; Anti-colors: $\overline{red}, \overline{blue}, \overline{green}$ or $cyan, yellow, magneta$

b) Consider Electric Charge, Quark Flavor, Baryon #, Isospin, and Lepton #. List those conserved by each of the three interactions other than gravity.

Interaction	Conserved
Strong	electric charge, quark flavor, baryon#, Isospin, lepton#
Weak	electric charge, baryon#, lepton#
Electromagnetic	electric charge, quark flavor, baryon#, lepton#

c) Using the quantum numbers of the strange quark describe why the reaction $K^- p \to \Xi^- + K^+$ can or cannot occur?

Since $K^- = (s,\bar{u})$; $p = (uud)$; $\Xi^- = (ssd)$; $K^+ = (\bar{s},u)$, reaction is allowed

Annihilation of a $\bar{u}u$ pair, and creation of an $s\bar{s}$, generates the given final state.

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d) What are the symbols and names of all three quarks with charge +2/3 .

(*u*) up; (*c*) charm; (*t*) top

e) In the oscillations of the B^0 and $\overline{B}^0$ mesons what quantum characteristic is violated, and what interaction must be involved.

The Bottom quark flavor is not conserved: $B^0 = (b\overline{d})$, while $\overline{B}^0 = (\overline{b}d)$

f) What matrix describes the mixing of neutrino flavor states as linear combinations of the 3 neutrino mass states, ν_1, ν_2, ν_3 ? In what group (e.g., *diagonal* matrices) is the matrix? For what quarks does a similar matrix exist?

i) Neutrino mixing matrix *U*, such that $\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$

ii) The mixing matrix is Unitary. iii) The –1/3 quarks, *d,s,b* mix into *d',s',b'*

g) Which Weak Bosons can be created in a collision of quarks?

All weak bosons can be created in a collision of quarks (Unless this is mentioned)

None, if you were thinking it had to be quark-antiquark or gluon-gluon collision.

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h) The Cabibbo angle gives the mixing of which two quark mass states? d and s

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{bmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{bmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

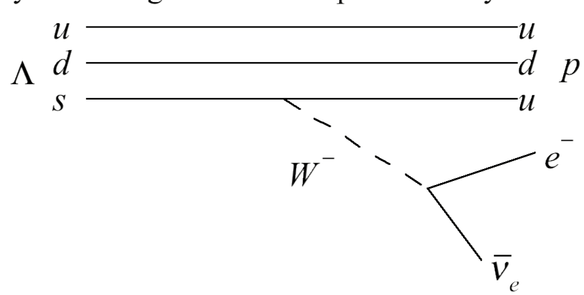
i) What are the symbols and names of all three quarks with charge –1/3 .

(d) down; (s) strange; (b) bottom

j) The W^- boson can be produced in what QCD colors? Same question for W^+?

The weak bosons, $W^{\pm}, Z$, are colorless particles. Their decay into a quark (color) and an anti-quark (anti-color) must proceed where the anti-color corresponds to the color.

3. [40 pts]Draw the Feynman diagram for the leptonic decay of the Λ baryon.



4. [20 pts]Identify the symmetry that is associated with each of these conservation laws: the conservation of energy, the conservation of linear momentum, and the conservation of angular momentum.

conservation of energy $\rightarrow$ time reversal invariance

conservation of momentum $\rightarrow$ translational invariance

conservation of angular momentum $\rightarrow$ rotational invariance

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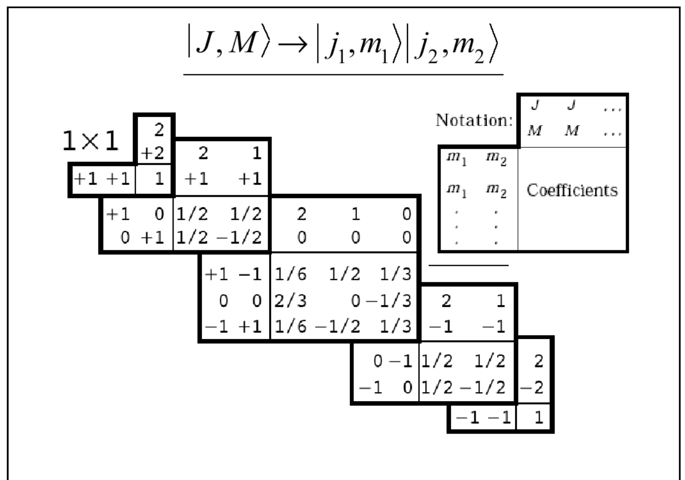
5. [40 pts]The Gell-Mann Nishijima relation is, $Q = I_3 + \dfrac{B+S+C}{2}$, where B is the baryon number, S is the strangeness quantum number, and C is the charm quantum number, is true for all meson and baryon states.
Show that this relationship also works for the quarks with $B = 1/3$, the up and down quarks in an Isospin = 1/2 doublet, and the strange and charm quarks as Isospin = 0 singlets.

Quark	I_3	B	S	C	I_3+(B+S+C)/2	Q	Check
u	+1/2	1/3	0	0	3/6+1/6 = +2/3	+2/3	OK
d	–1/2	1/3	0	0	–3/6+1/6 = –1/3	–1/3	OK
c	0	1/3	0	+1	0 + 2/3 = +2/3	+2/3	OK
s	0	1/3	–1	0	0 –1/3 = –1/3	–1/3	OK

6. [20 pts] Determine the Isospin of particle C that decays, $C \to A + B$, where A and B are Isospin 1 particles, and the decay probabilities for $C^0 \to A^\alpha + B^\beta$ are the same for all possible charges, α and β, of A and B. (show your work)

A	B	(I,I_3)	(I,I_3)	(I,I_3)
m1	m2	(2,0)	(1,0)	(0,0)
+1`	–1	1/6	1/2	1/3
0	0	2/3	0	–1/3
+1`	–1	1/6	–1/2	1/3

Only the (0,0) has equal probability (1/3) for each charge state.
Particle C has Isospin I = 0 .



Charged particle induced ionization

- Moving particle, mass M, ionizes atoms in medium

$$S(T) = -\frac{dT}{dx} = n_{ion}\bar{I}$$

T : kinetic energy of moving particle

n_{ion} : number of electron-ion pairs / unit path length

$\bar{I}$: average energy / electron-ion pair

- n_{ion} is particle velocity and charge dependent (Bethe and Bloch)

Stopping power

$$S(T) = \frac{4\pi Q^2 e^2 nZ}{m_e \beta^2 c^2}\left[\ln\left(\frac{2m_e c^2 \gamma^2 \beta^2}{\bar{I}}\right) - \beta^2\right]$$

$$\gamma = \frac{E}{Mc^2}; \quad \beta = \frac{pc}{E}; \quad \gamma\beta = \frac{p}{Mc}$$

$$S(T) \propto \frac{1}{\beta^2 c^2} = \frac{1}{v^2}$$

$\beta = v/c = < 0.8$

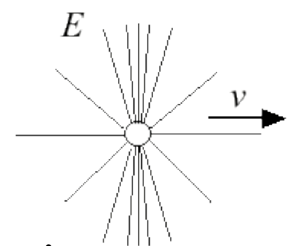
heavy ionization

S minimizes, $\gamma\beta \sim 3$

when $\beta \sim 0.95$

minimum ionization

$S \propto \ln\gamma$

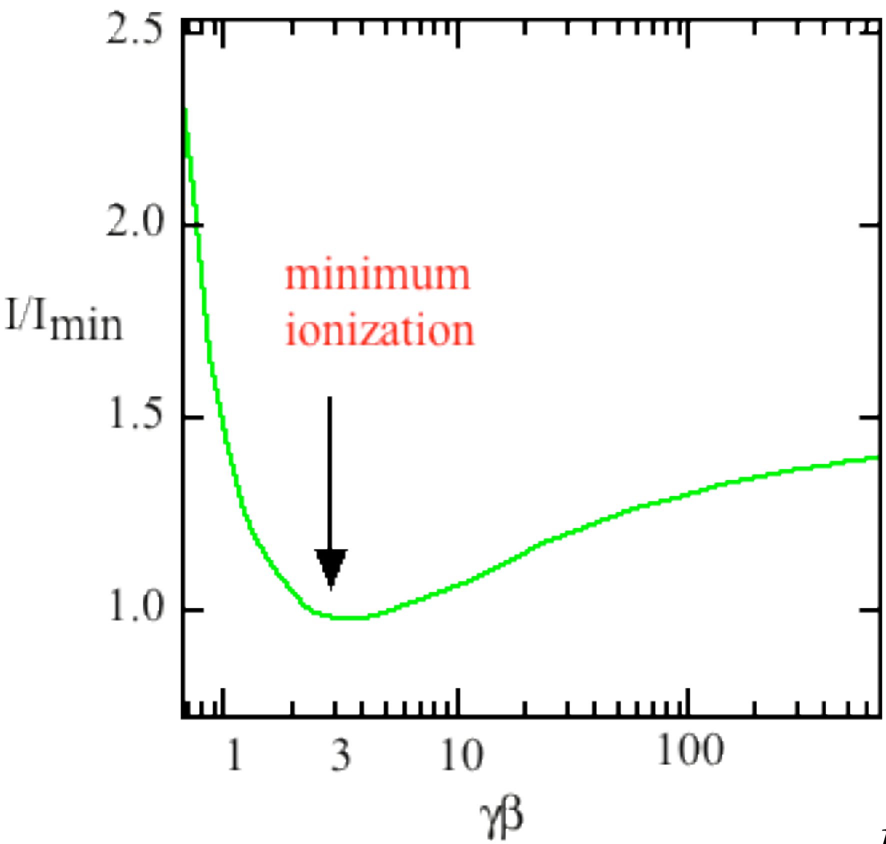


ultra relativistic

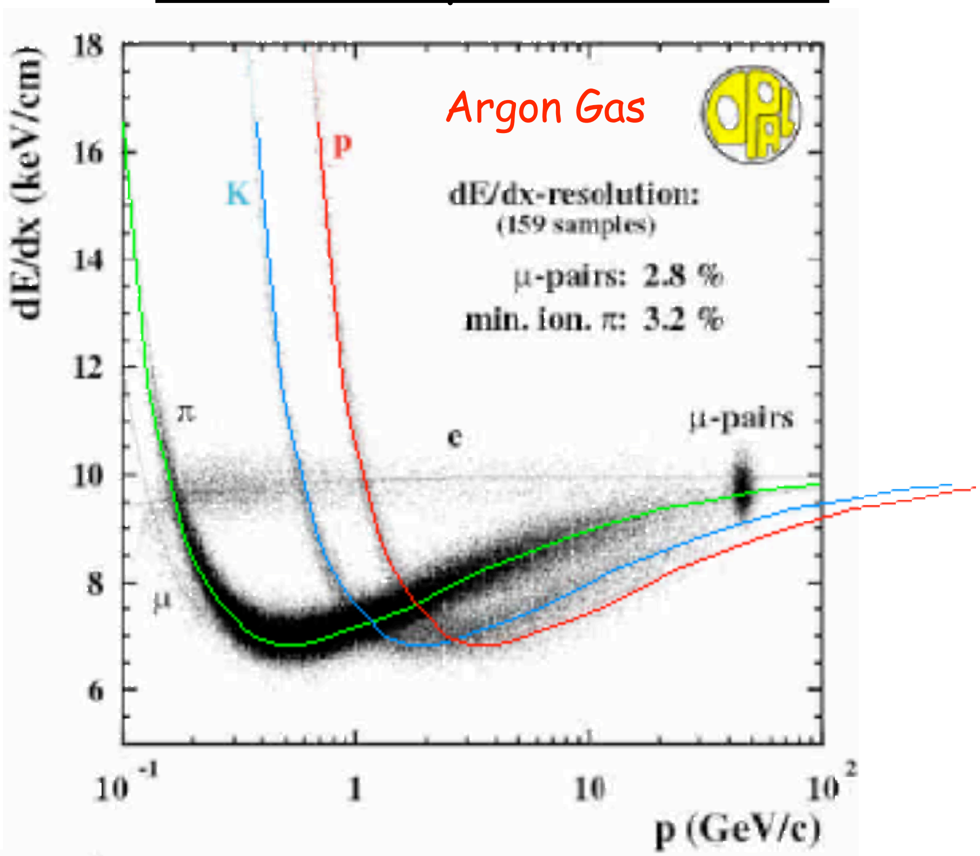
rise of ionization

Ionization in gases (relevant to all particles)

Normalized Ionization vs $\gamma\beta$



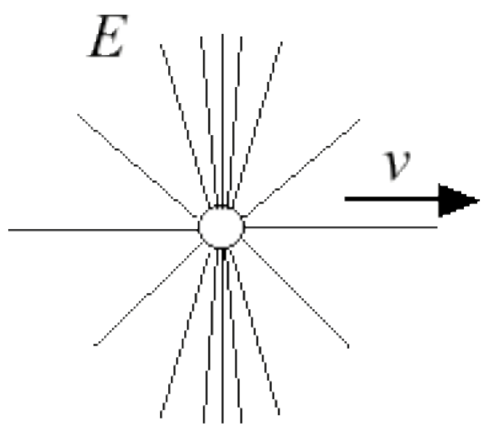
Ionization vs momentum for various particle masses

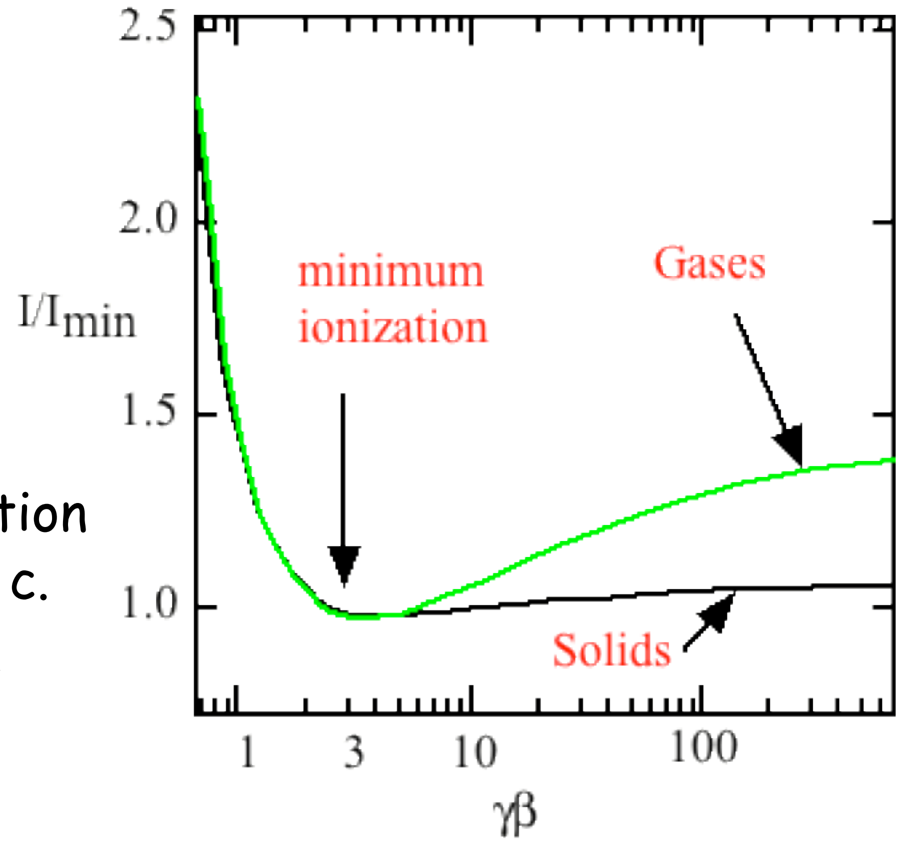


$$\gamma\beta = \frac{pc}{Mc^2}$$

Saturation of ionization in solids

- Relativistic rise of ionization is due to electric field concentration perpendicular to direction of motion





- Atoms along the line of motion see a stronger field as v -> c.
- Effect is largest in large Z gases, e.g., Xe
- Solids polarize and shield far electrons from field → effect saturates at only a few % above the minimum

Minimum ionization in *thin* solids

$$S(T) = -\frac{dT}{dx} \propto nZ = \rho A_0 \frac{Z}{A}$$

Z : atomic number of medium

n : number of atoms/unit volume

$n = \frac{\rho A_0}{A}$; A : atomic number of medium

- Units for energy loss
  - Z/A ~ 0.4 at large A, energy loss proportional to density S~ρ,
  - Divided by the density -> value nearly independent of material.
- $(dE/dx)_{min}$ tabularized for various materials in $MeV/(g/cm^2)$
  - Polystyrene scintillator: 1.95

$$\rho_{\text{scintillator}} = 1.03\ \text{g/cm}^3$$
$$-\left.\frac{dT}{dx}\right|_{\min} = 1.95\left(\frac{\text{MeV}}{\text{g/cm}^2}\right)\rho_{\text{Scintillator}} = 2.0\ \text{MeV/cm}$$

  - Iron (steel) : 1.45

$$\rho_{\text{iron}} = 7.87\ \text{g/cm}^3$$
$$-\left.\frac{dT}{dx}\right|_{\min} = 1.45\left(\frac{\text{MeV}}{\text{g/cm}^2}\right)\rho_{\text{iron}} = 11.4\ \text{MeV/cm}$$

Relativistic muon loses ~2 MeV/cm in plastic, ~11.4 MeV/cm in Iron

High energy particles in matter

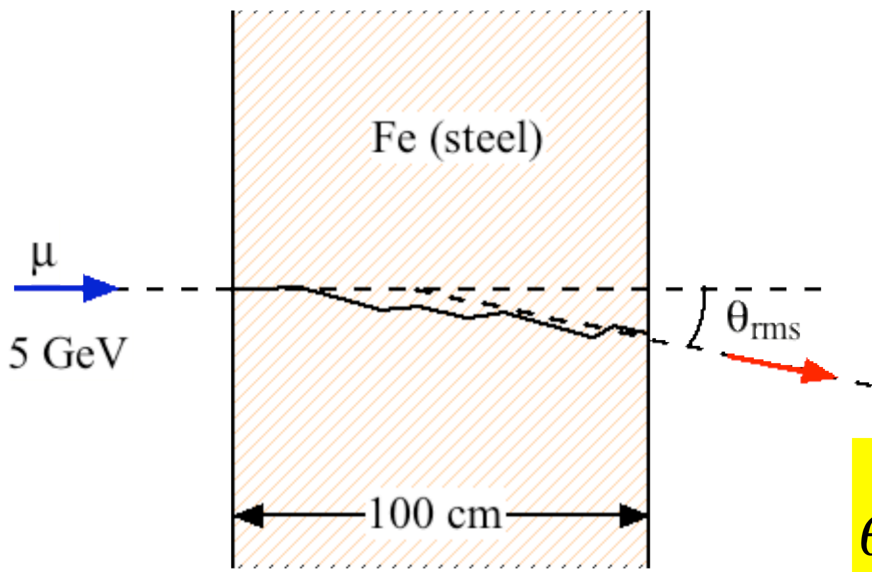
- Particles can be deflected, degraded or absorbed
- Characteristic length is particle, energy, and material dependent

Long lived particles ($\tau > 10^{-10}$ s)

- Muons (mass $m_\mu \sim 200\ m_e$)
  - lose energy mostly by ionization -> energy determines range
  - rare energy loss by photon radiation in the EM field of nucleus
  - very rare EM interaction on nuclear charges, nuclear disintegration
  - deflection by multiple scatterings on atomic electrons
- Electrons and photons at high energy (T > 1 GeV)
  - Electron radiates a photon: X_0 is the "radiation length"
  - Photon converts to e^+e^- pair: $9X_0/7$ is the "pair length"
  - both only rarely collide with a nucleus
- Hadrons (proton, neutron, charged pi-meson, K-meson, ...)
  - nuclear interactions; absorption length, X_{abs} proportional to density ρ
  - additional hadrons often created

Muon multiple scattering

- 5 GeV muon (m=105.6 MeV/c^2) through 1 m of steel looses about 1.1 GeV by ionization. Some atomic electrons are kicked hard.
- Muon will be deflected (either direction) with a probability distribution that peaks at θ = 0 but spread by θ_{rms}.



$$\theta_{\text{rms}} \approx \frac{20\ \text{MeV}}{\beta pc}\sqrt{\frac{L}{X_0}}$$

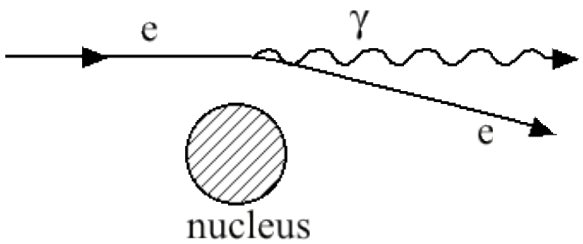
$$\text{Iron } X_0 = 1.76\,\text{cm}$$

$$\theta_{\text{rms}} = \frac{20}{5000}\sqrt{\frac{100}{1.76}} = 30\ \text{mr} = 2^{\text{o}}$$

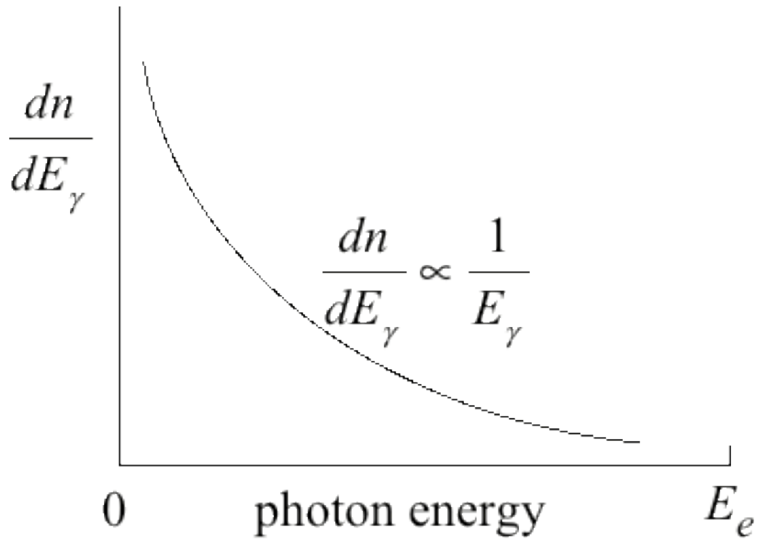
Bremsstrahlung

- High energy electrons loose energy primarily by radiating photons
- The characteristic length, X_0 is material dependent
- Kinetic energy (on average) will drop exponentially.

$$\left.\frac{dT}{dx}\right|_{\text{Brem}} = -\frac{T}{X_0} \longrightarrow T = T_0 e^{-T/X_0}$$



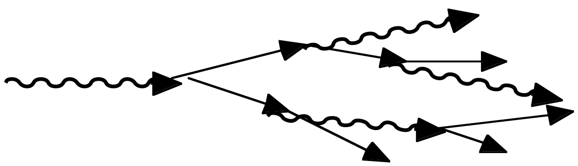
- Photon energies are discrete with a $1/E_\gamma$ distribution
- Many low energy photons (even IR) and a few high energy photons



Photons in matter

- E < 1 MeV, photoelectric absorption and Compton scattering dominate the interactions of photons
- E = 1 - 10 MeV, Compton scattering dominates but pair production is rising
- E > 10 MeV, pair production dominates the interactions

Cascading interactions



1. 1 GeV photon enters a block of lead (X_0 = 0.56 cm)
2. After 5 mm the photon produces a e+e- pair (0.4 and 0.6 GeV)
3. After 3 mm the e+ “brems” a 100 MeV photon. After 6 mm the e- “brems” a 300 MeV photon (e+ and e- are both 300 MeV).
4. After a few more mm each, the 100 MeV and 300 MeV photons pair produce, the 300 MeV e+ and e- both brems
5. Repeats until photons and electrons drop below 1 MeV.

Measuring a particle's momentum, energy, and mass

- Charged particle tracking
  - Gas: MWPC, Drift Chamber, GEM
  - Solid state: Silicon, diamond

MWPC = Multi-Wire Proportional Chamber

GEM = Gas Electron Multiplier

- Scintillators
  - scintillation and conversion -> electronic signals
  - Organic: Plastic, liquid hydrocarbon, fibers
  - Inorganic: Crystals, liquid noble gas
- Calorimeters
  - total absorption
  - sampling
- Particle identification (ID)
  - time of flight
  - ionization
  - Cerenkov light
  - transition radiation

Momentum measurements

- Charge Q bending in a magnetic field

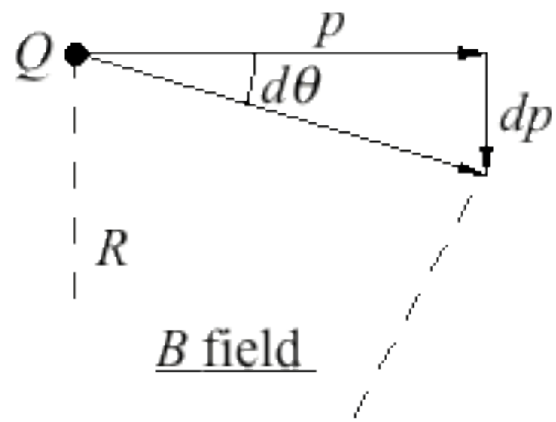
<u>Relativistic Derivation</u>

$$p = \gamma m v$$

$$dp = p d\theta = p \frac{v dt}{R}$$

$$\frac{dp}{dt} = p \frac{v}{R} = QvB$$

$$\underline{p = QBR}$$



<u>Units transformation</u>

$$1\,\mathrm{eV} = 1.6 \times 10^{-19}\ \mathrm{J} \qquad q = Q / \left(1.6 \times 10^{-19}\,\mathrm{C}\right)$$

$$p = QBR\left(1\,\frac{\mathrm{kg \cdot m \cdot s^{-1}}}{\mathrm{C \cdot T \cdot m}}\right)\left[\left(\frac{c}{c}\right)\left(\frac{e}{e}\right)\right]$$

$$= qBR\left(\frac{3 \times 10^{8}\ \mathrm{eV}/c}{\mathrm{T \cdot m}}\right)$$

$$= 0.3qBR\left(\frac{\mathrm{GeV}/c}{\mathrm{T \cdot m}}\right)$$

- Transform to more useful units

$$p \approx 0.3qBR$$

p in GeV/c, q in # of e's
B in Tesla, R in meters

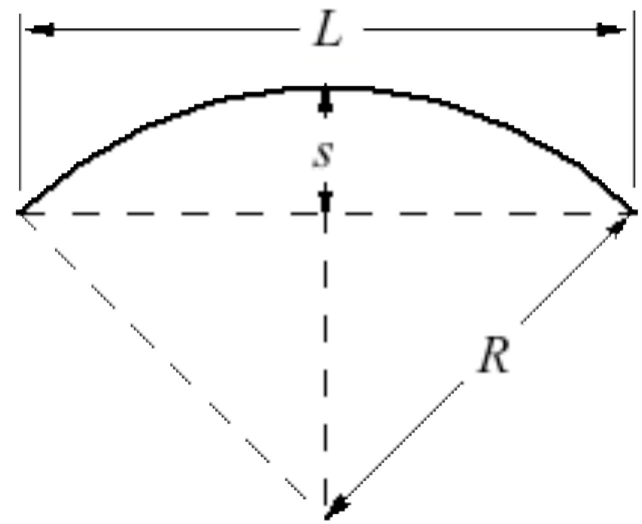
Using the sagitta to find R

- Bending of elementary charge in a magnetic field
  - large radius -> weak bending -> large momentum
  - typically see only a small portion of the circle
  - measurement of momentum is equivalent to a measurement of the sagitta.

<u>easy to show</u>

$$R \approx \frac{L^2}{8s}, \quad s << R$$

s = sagitta



$$p = 0.3qBR$$

$$\frac{\delta p}{p^2} = \left(\frac{8}{0.3qBL^2} \right) \delta s$$

δs is fixed by detectors

make δs as small as possible

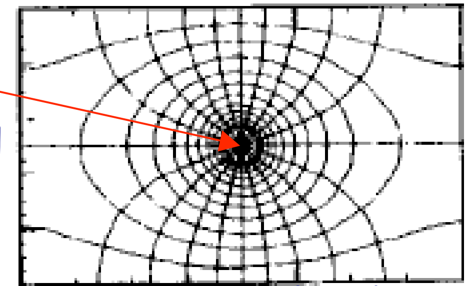
- Momentum errors minimized by big *B*, or even better by big *L*

Basics of wire chamber tracking

- Wire chamber features
  - Isolated gas volume ("chamber")
  - Anode wire, Au plated W, dia. <50μ
  - Cathodes at high voltage
- Gas properties (big subject)
  - Noble gas (Ar) no negative ions
  - UV quencher (hydrocarbon)
  - Cost
    - Cheap (flow & exhaust)
    - Expensive (recirculate & clean)
- Electronics
  - 1 circuit for each wire
  - fast, low noise
  - multi-channel ICs

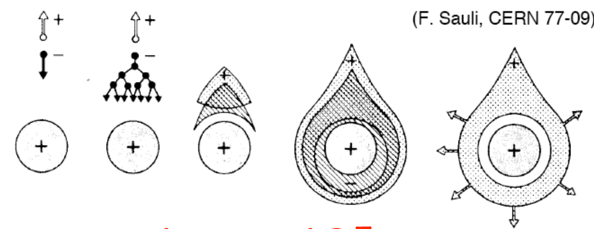
-HV cathode

Anode wire
GND potential



-HV cathode

avalanche ~ 100μ from wire



1 e -> 10^5 e

one electronics "channel"