Experiment 8

Rotational Motion

Reading and Problems:

Bauer&Westfall, Chapter 10 as needed

Homework 12: turn in as part of your preparation for this experiment.

1. Show that you can rewrite Eq. 6 as Eq. 12. *Hint*: comment on whether any of your steps would require further justification, for example if you equate a value and its time average.
2. Show that you can rewrite Eq. 9 as Eq. 13. *Hint*: same as for HW 12.1 .
3. **Extra Credit**: Substitute Eq. 9 into Eq. 8 to prove that Eq. 9 is a solution of Eq. 8.
4. **Extra Credit:** Show that $\langle \nu \rangle$ is the instantaneous frequency in the middle of timing interval.
5. **Extra Credit:** Why is Eq. 13 true only if the timing interval is short compared to the decay time?

1. Goals

1. To understand the rotational motion of a rigid body.
2. To study different types of frictional losses in a rotating system.
3. To explore the use of least-squares fitting procedures in analyzing a dynamical system.

2. Theoretical Introduction

For a rigid body that rotates about a fixed axis, Newton's second law of motion states that

$$\tau = I\alpha \qquad (1)$$

where τ is the magnitude of the total torque, I is the moment of inertia of the body and α is the angular acceleration, measured in radians/s^2. Let us consider some applications of this equation.

2.1 No frictional torque: Ideal case

Our rigid body is a rotating disk. Suppose there is no friction. Then the total torque, τ, is zero and Eq. 1 predicts that $\alpha = 0$. By definition $\alpha = d\omega/dt$, so that one can readily solve for the angular velocity ω,

$$\omega = \text{constant}. \qquad (2)$$

2.2 Constant frictional torque: realistic support of rotating disk

Now suppose there is a constant frictional torque, τ_f, acting on the disk (via the air bearings supporting it). Then from Eq.1 we have:

$$\tau = -\tau_f = I\alpha = I\frac{d\omega}{dt} \qquad (3)$$

$$-\tau_f = I\frac{d\omega}{dt}. \quad (4)$$

Integrating this equation, we obtain:

$$\int d\omega = -\frac{\tau_f}{I}\int dt \quad (5)$$

$$\omega = \omega_0 - \frac{\tau_f}{I}t \quad (6)$$

where ω_0 is the value of ω at t = 0. The minus sign agrees with the notion that friction causes the angular velocity to decrease with time.

2.3 Frictional torque proportional to ω : magnetic braking

Suppose now that $\tau = - C\,\omega$, where C is a constant. Using Eq. 1 we now obtain:

$$\tau = -C\omega = I\frac{d\omega}{dt} \quad (7)$$

This can be rewritten as

$$\frac{d\omega}{dt} = -\frac{C}{I}\omega. \quad (8)$$

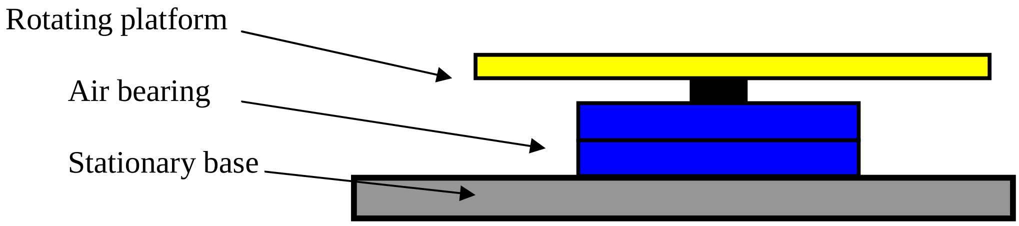
The solution of this differential equation is

$$\omega = \omega_o \exp(-\gamma t) \quad (9)$$

where ω_o is the initial angular frequency and γ is the decay rate (in s^{-1}). We see that the angular frequency now decreases exponentially with time. As we noted in the experiment on damped harmonic motion, 1/γ is the damping time constant, which is the time for the object to decay to 1/e (0.368) of its initial value. Comparing Eqs. (8) and (9), the damping rate is given by γ = C/I.

3. Apparatus

A cross-sectional view of the apparatus is shown on the next page. It consists of a stationary base on which is mounted a rotating platform supported by nearly frictionless air bearings. This rotating disk has alternating black and white bars on its circumference. They allow us to observe how fast the disk is turning by watching the rate at which bars pass. As the disk rotates, the bars sweep by a photo-diode detector whose output is amplified to yield standard (TTL) logic levels corresponding to the presence of a white (TRUE) or black (FALSE) bar. When you are aligned with a white bar, the red LED (light emitting diode) on the display is lit. The output is connected to the data acquisition card attached to your PC. Once in the computer, these signals will be analyzed by various programs written in the LabVIEW language.



The theoretical relationships discussed at the beginning were derived in terms of the instantaneous angular velocity, ω, but the apparatus used in this lab displays $\langle \nu \rangle$ on its readout LCD screen, and produces an output voltage proportional to $\langle \nu \rangle$, which is defined as

$$\langle \nu \rangle = n / T \tag{10}$$

Here n is the number of bars which pass during a timing interval of size *T*, and $\langle \nu \rangle$ is the *average rate* (or *frequency*) at which the white bars pass the detector. The brackets indicate averaging over the time interval T.

However, the angular velocity ω and $\langle \nu \rangle$ are related by the equation

$$\langle \omega \rangle = 2\pi \langle \nu \rangle / N, \tag{11}$$

where N is the number of black bars on the circumference of the rotating platform (and thus represents the number of bars in a single revolution of 2π radians). Knowing N allows us to calibrate the device readout and change $\langle \nu \rangle$ into ω. As in the free fall experiment, we can't measure instantaneous quantities such as ω or ν directly, but only their averages $\langle \omega \rangle$ or $\langle \nu \rangle$ over some time interval. Still, we can use the device readout more directly, and by the linear relationship (11) we can rewrite (6) as

$$\langle \nu \rangle = \langle \nu \rangle_o - \frac{N}{2\pi} \alpha\, t. \tag{12}$$

If the acceleration is constant then velocity changes linearly with time and, as in the gravitational free fall experiment, the average velocity in a timing interval is also the instantaneous velocity at the center of the interval.

If the frictional torque and hence the angular acceleration depend linearly on the angular frequency then it can be shown that $\langle \nu \rangle$ is the instantaneous frequency in the middle of the timing interval, but only if the timing interval is short compared to the decay time I/C. Thus, measuring the time dependence of $\langle \nu \rangle$ yields the time dependence of ω. We can use (11) again to rewrite (9) as

$$\langle \nu \rangle = \langle \nu \rangle_0 \exp(-\gamma t). \tag{13}$$

4. Questions for preliminary discussion

1. How you will assess the uncertainty of N? (You may change your mind after working with the apparatus.)
2. How you will quantitatively determine the uncertainty of $\langle \nu \rangle$?

Experimental Procedures

5. Determination of N

The objective here is to determine N, the number of black bars on the circumference of the disk. Note that N is an integer.

0. Record your table number!
1. **Make sure the air is flowing through the bearing before you spin the aluminum disk!** Failure to do so could result in irreversible damage to the apparatus. Also, be sure the red magnet is far away from the disk, and has the iron "keeper" across its poles.
2. A computer program is provided that counts the number of black bars which pass by the photocell. Open the program in the LabVIEW:

   R:\exp8_ref\counter.llb\simplecounter.vi Set **device**=1; **counter**=0.

   Clicking the Arrow button on its menu resets the counter and starts the count.
3. Position the platform so the little red light is on and reset the counter to zero. Carefully mark this starting angular position of the platform. Now run the program, and smoothly rotate the platform through exactly 5 (or 10) revolutions, being sure to stop at a point where the red light is on. **Do not let the platform reverse direction: edges count in either direction!** Get the edges counted from the program. If M is the number of revolutions and N_c the total number of counts then $N = N_c / M$. Explain your estimate for the uncertainty in N.

6. Undamped Rotation

The objective of this part of the lab is to determine if there is a significant frictional torque in this apparatus.

1. Open a program in the directory R:exp8_ref\rotationalvelocity.vi. Set **Device**=1, **Total Time**=100 sec. The program counts edges passing during 1 second periods.
2. Give the disk enough angular momentum so that $\langle \nu \rangle_0$ reads between 250 Hz and 300 Hz on the LCD display of the apparatus. Run the program by clicking the arrow.
3. Exit the program. Choose **No** to the question "Save changes to rotationalvelocity-.vi?"
4. Use Kgraph to open the data file as done in Exp 7. The data file contains **Average Time** and **Frequency**. Make a plot of $\langle \nu \rangle$ vs. time. Estimate the uncertainty for $\langle \nu \rangle$ (and explain how you got it!) and show error bars on the plot.
5. Fit the data to Equation (12) to determine values and uncertainties for $\langle \nu \rangle_0$ and α.
6. **Extra Credit:** Check your uncertainty estimate by examining the residuals to the fit. What uncertainty would have given about 2/3 of the points within the error bars? *Hint*: You can make a data column with the residuals after you have done a general curve fit by Curve Fit | (select your fit name) | View | Copy Residuals to Data Window.

7. **Extra Credit:** Predict from Eq (12) the behavior at long times. Then let the wheel run a long time and see whether Eq(12) correctly describes the long-time behavior.
8. **Extra Credit:** See whether Eq (13) does a better job of describing the data from § 6.4 .

7. Damped Rotation

The objective of this part of the lab is to determine if a magnetic brake causes a frictional torque that depends linearly on ω.

1. Remove the "keeper" bar from the poles of the red magnet and place the magnet near the air bearing and underneath the top rotating aluminum disk. Use the same program as you did in part 6, set **Total Time**=100 sec. Spin the disk so that its initial $\langle \nu \rangle$ is between 250 and 300 Hz, then run the program. When done, take the magnet off the apparatus and replace the "keeper" across its pole faces.

2. Make plots of $\langle \nu \rangle$ vs. time. Fit the data to equation (13) and determine the best fit values and uncertainties for parameters $\langle \nu \rangle_0$, γ.

3. Now use another method to estimate the same parameters. Convert Eq. (13) to a linear form by taking the natural logarithm (ln) of both sides (see the Reference Guide, and Taylor §8.6). Calculate $\ln\langle \nu \rangle$ from your data, and make plots of $\ln\langle \nu \rangle$ vs. t. *Predict* whether a fit will give you the same parameters as in part 7.2 . Now fit your data using the general fit editor and find parameters $\ln\langle \nu \rangle_o$ and γ and their uncertainties.
4. Are the values from 7.3 compatible with the values you found in §7.2? Summarize your comparison in a table. If you have trouble finding parameter uncertainties from 7.3, just use the uncertainties from 7.2 .
5. **Extra Credit:** Why do the values you obtain in 7.3 differ from those in section 7.2? After all, it's the same data, and both are claimed to be best fits! *Hint*: see Taylor §8.6 .
6. **Extra Credit:** From your 7.3 results, does damping becomes weaker, or stronger, at long times? Explain how you reach this conclusion.
7. **Extra Credit:** The differential equation with both linear and constant torques is given by ω' = - a ω – b, with b = - τ_f / I and a = γ (from Eq 6 and 9). The solution to this equation is given by ω = - (b/a) + (ωo + b/a) Exp(- a t). Does this fit your data of §7.1 better than Eq (13)? Do the value of ωo and γ match? Does the value of b match the value from §6.5? Which of them would you expect to match?

8. Questions and Analysis

1. From your data in Part 6 deduce if there is a significant frictional torque. If so, do your data imply that this torque is constant? In other words, is Eq. (6) obeyed by your data? Be sure to show clearly how you draw your conclusions.

2. In Part 7.2, how well does Eq. (13) describe your data? Look for systematic deviations in your experimental data records. If you find any, what might be causing this?

3. **Extra Credit:** What is the physical mechanism by which the magnet causes the wheel to slow down? Why, in particular, causes a torque proportional to ω? You can see that the magnet has no tendency to stick to the disk when it is stationary: the aluminum disk is non-magnetic. *Hint*: look for information on "eddy currents" or "magnetic braking".