Matter Waves

Thornton and Rex, Ch. 5

Matter Waves

EM waves also behave like particles (photons).

1924 - de Broglie asked:

Can particles also behave like waves?

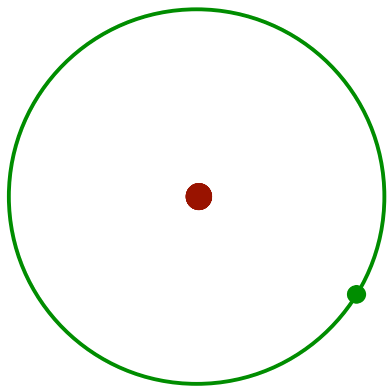
He suggested a relation between wavelength and momentum:

$$\boxed{\lambda = h/p}$$

No experimental evidence for this existed.

But with his matter wave idea, De Broglie could "derive" Bohr's quantization condition.

An electron orbiting an atom at radius r:

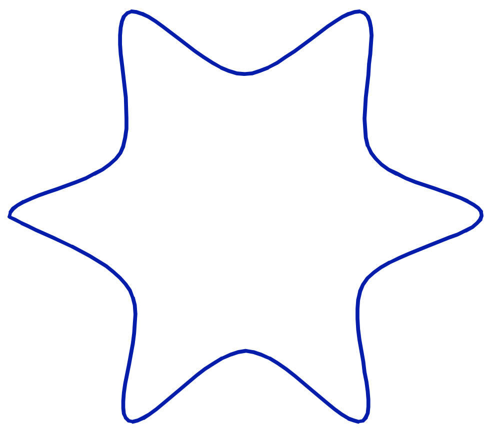


Assume stationary states correspond to standing waves of the electron.
I.e, an integral number of wavelengths must fit into the circumference:

$$2\pi r = n\lambda = nh/p = nh/mv$$

or

$$L = mvr = nh/2\pi = n\hbar$$



n=6 orbit

De Broglie's wave idea fitted naturally with Bohr's atomic model.

Encouraged with this success, de Broglie presented his ideas in his PhD thesis. With no experimental evidence for the idea, de Broglie's professors were skeptical of this radical concept. One sent a copy of the thesis to Albert Einstein. Einstein replied that the ideas certainly appeared crazy, but they were important, and the work was sound.

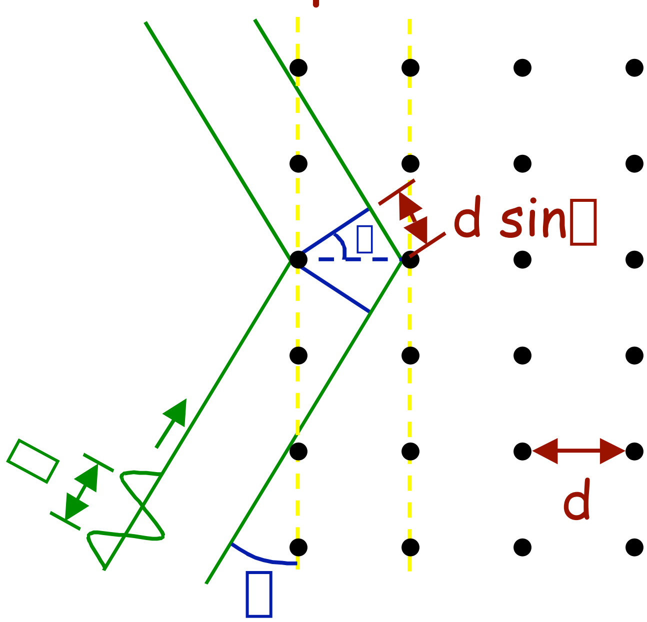
De Broglie received his PhD in 1924.
A few years later the wave nature of electrons was confirmed. In 1929 he was awarded the Nobel Prize.

Discovery of Electron Waves

1925 - Davisson and Germer were scattering electrons from metals.

On scattering electrons off crystallized Nickel, they saw peaks at certain angles.

$\Rightarrow$ Interference spectra!



Interference is constructive (i.e. peaks) when

$$2\, d \sin \theta = n\, \lambda$$

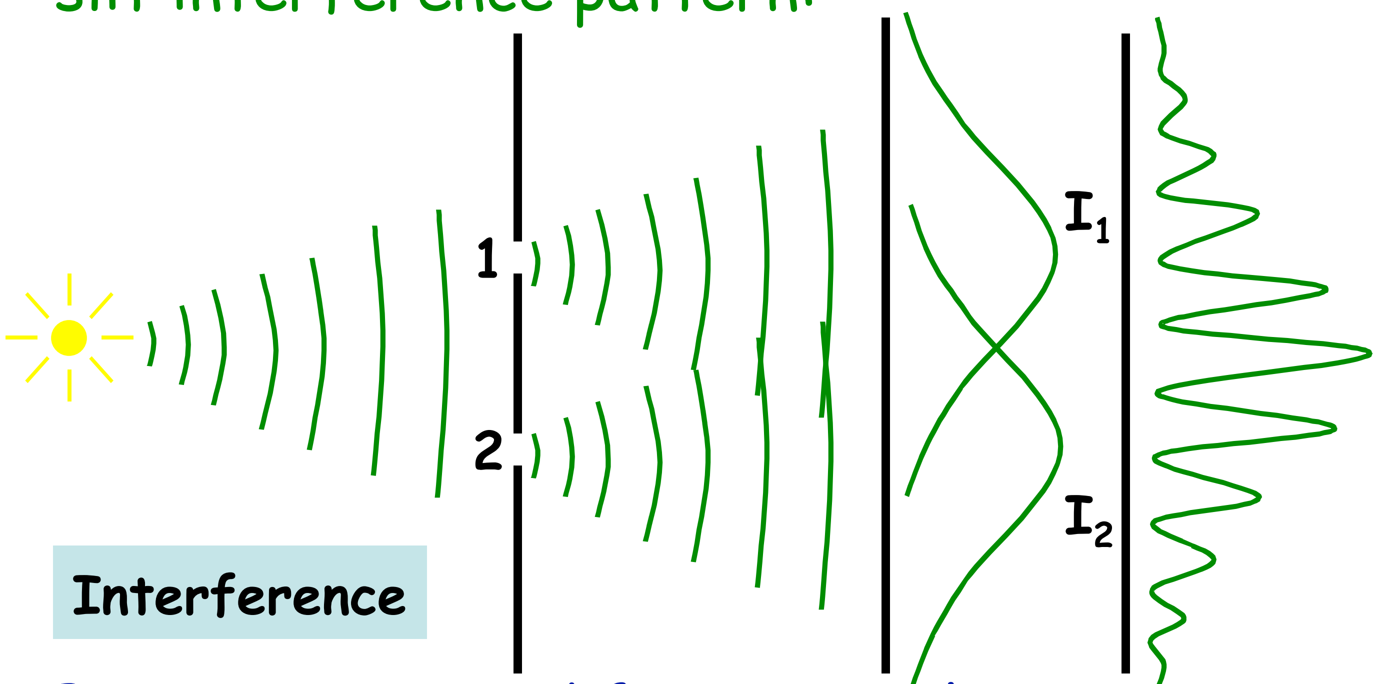
for integer n. (Bragg's law)

□ for the electron agreed with
de Broglie’s formula.

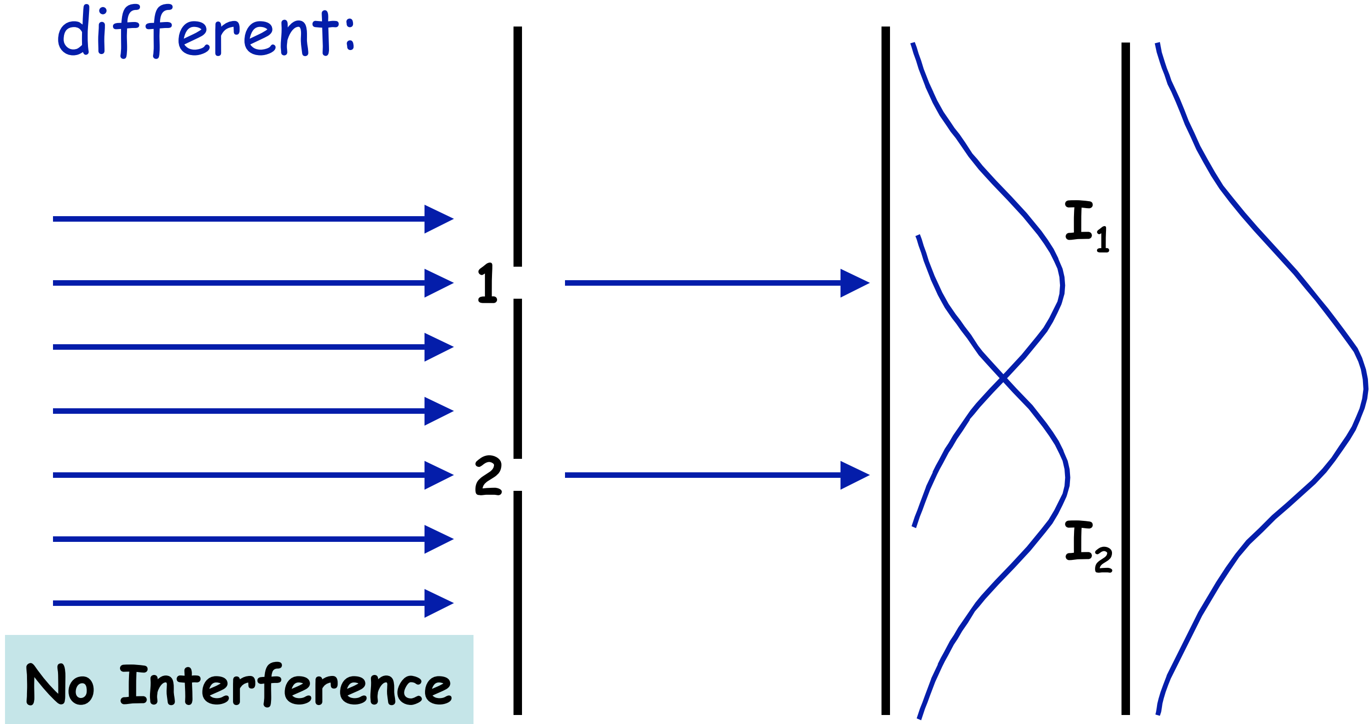
This was the first experiment to reveal
the wave nature of matter.

Wave/Particle Duality

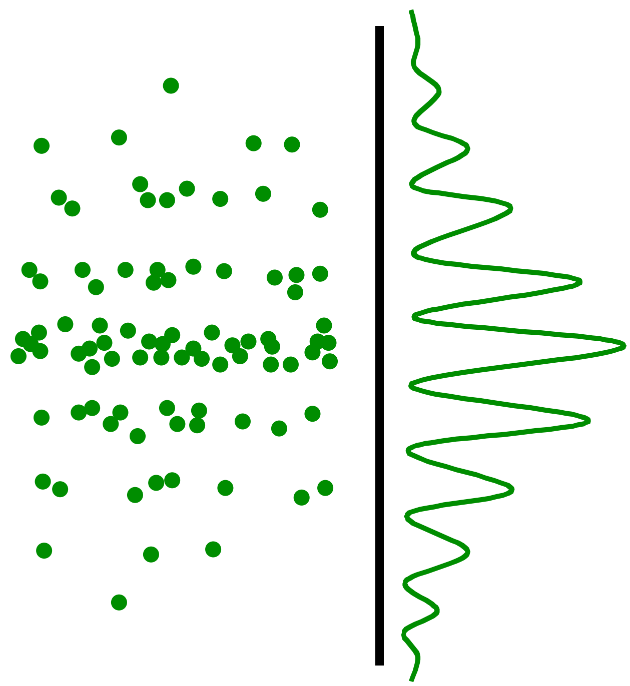
Wave nature of light from the double-slit interference pattern:



Pattern expected from particles is very different:



What happens at very low intensities?

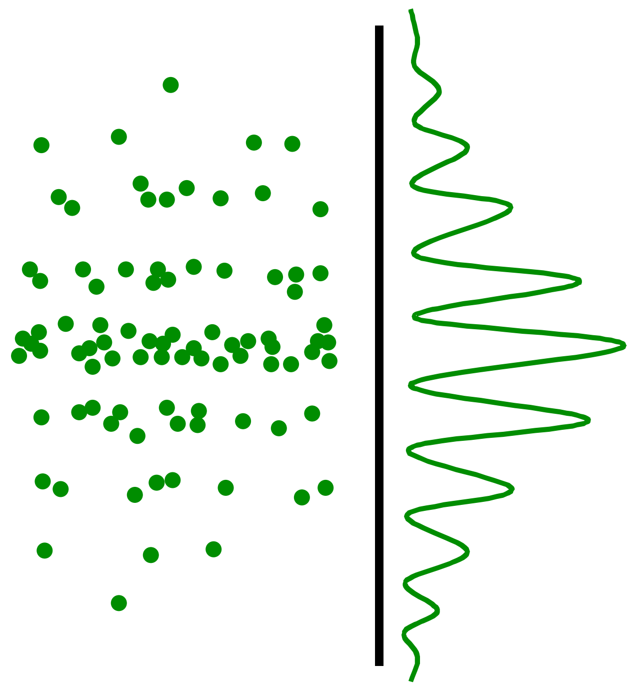


Photons hit at discrete points, gradually building up the interference pattern.

Does the photon go through slit 1 or slit 2?

Neither! (or rather, both!)

What about electrons?



They exhibit the same interference pattern (although at smaller wavelengths than for visible light.)

Bohr's Principle of Complementarity

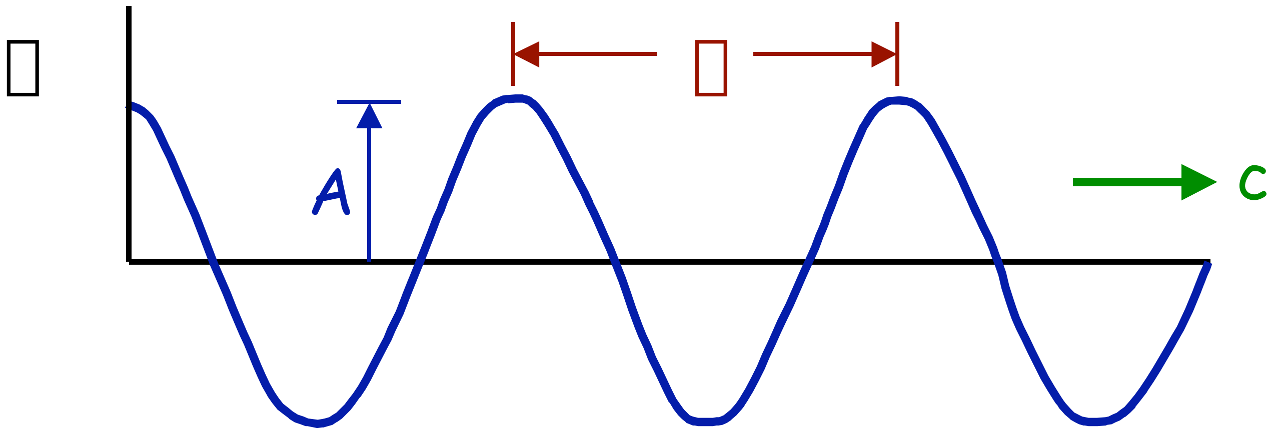
It is not possible to simultaneously describe physical observables in terms of both particles and waves.

Bohr called the fact that all objects (light, electrons, etc.) have both wave-like and particle-like properties

complementarity.

Generalities about light waves

A plane wave:



$\square(x,t) = A \cos[2\square(x-ct)/\square]$

Amplitude: A

Wavelength: $\square$

Speed: c

Frequency: $\square = c/\square$

It is convenient to rewrite:

$\square(x,t) = A \cos(kx - \square t)$

Wave number: $k = 2\square/\square$

Angular frequency: $\square = 2\square\,\square$

Wave relation: $c = \lambda\nu = \omega/k$

All light waves have same speed c in vacuum, independent of wave number k.

$\Rightarrow$ Not true for matter waves.

Planck: $E = h\nu = h\omega/2\pi$

$= \hbar\omega$

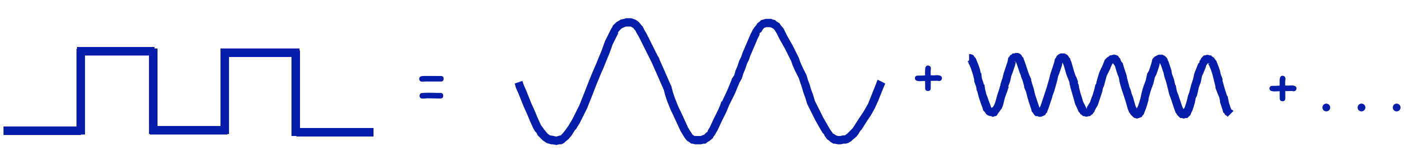
Einstein/de Broglie: $p = E/c = h/\lambda$

$= \hbar k$

A periodic wave can be constructed from a sum of plane waves:

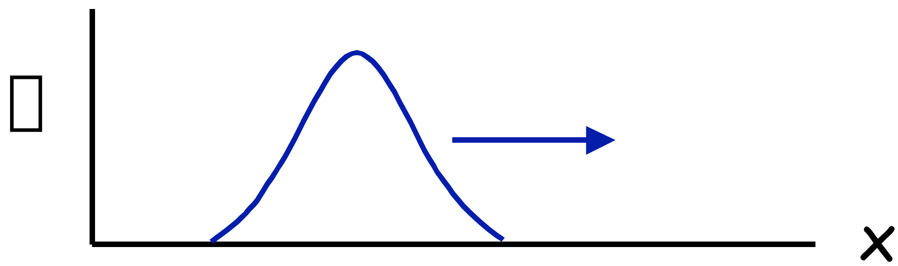
$\Rightarrow$ Fourier Series

$$\psi(x,t) = \sum A_i \cos(k_i x - \omega_i t)$$



A <u>wave packet</u> can be constructed as a continuous sum (integral) of plane waves.

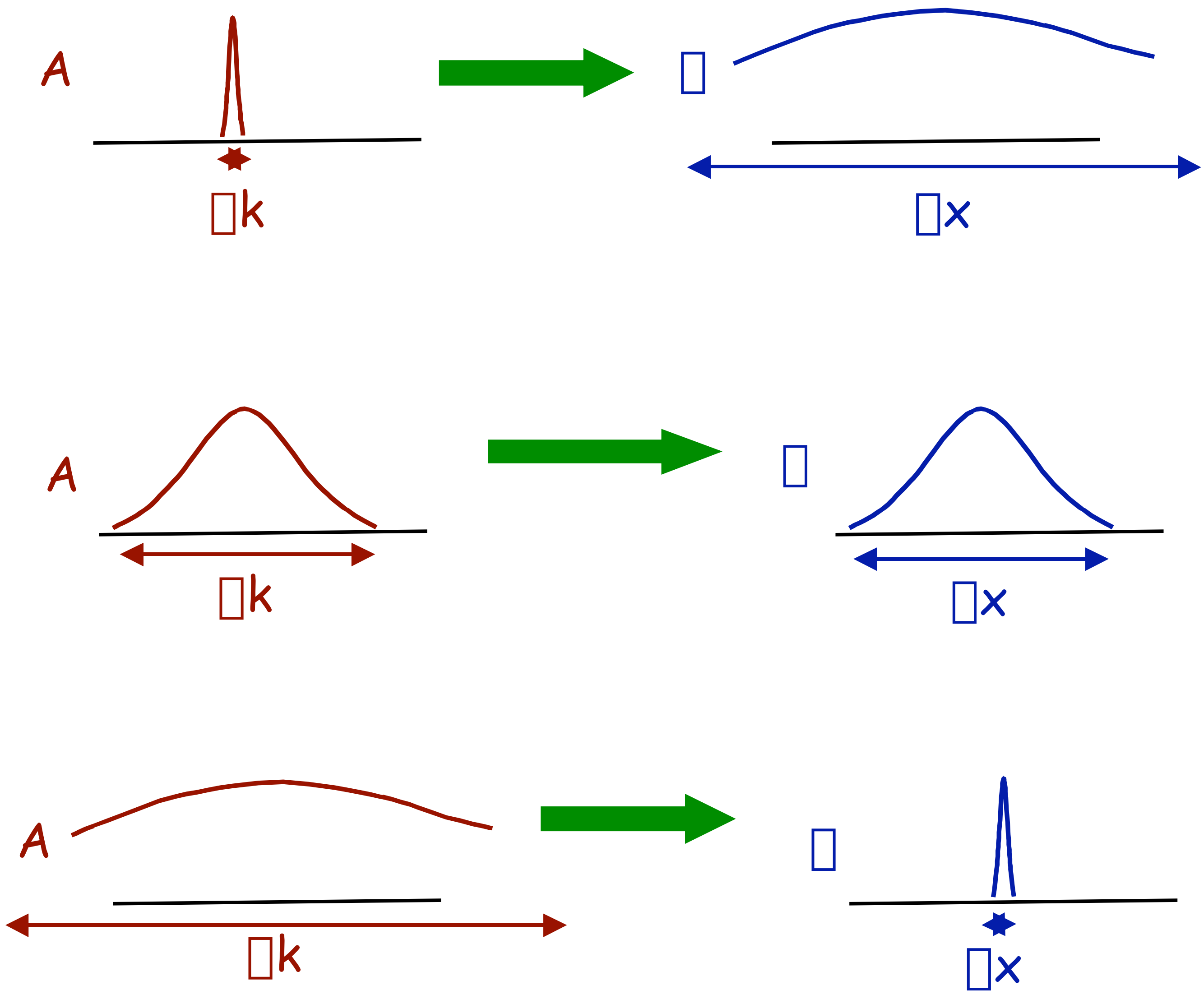
□ Fourier Transform



$\square(x,t) = \int A(k) \cos(kx-\square t)\, dk$

General fact about Fourier Transforms:

The extent □x of the wave □ is <u>inversely</u> related to the extent □k of its Fourier Transform A.


A
k
x
A
k
x
A
k
x

We can write this as

$$\Delta x \, \Delta k \geq 1/2$$

Multiplying by $\hbar$ and using $p = \hbar k$ gives:

$$\Delta x \, \Delta p \geq \hbar/2$$

<u>Heisenberg uncertainty principle</u>

It is impossible to know precisely the position and the momentum of an object at the same time.