

## RLC Circuits

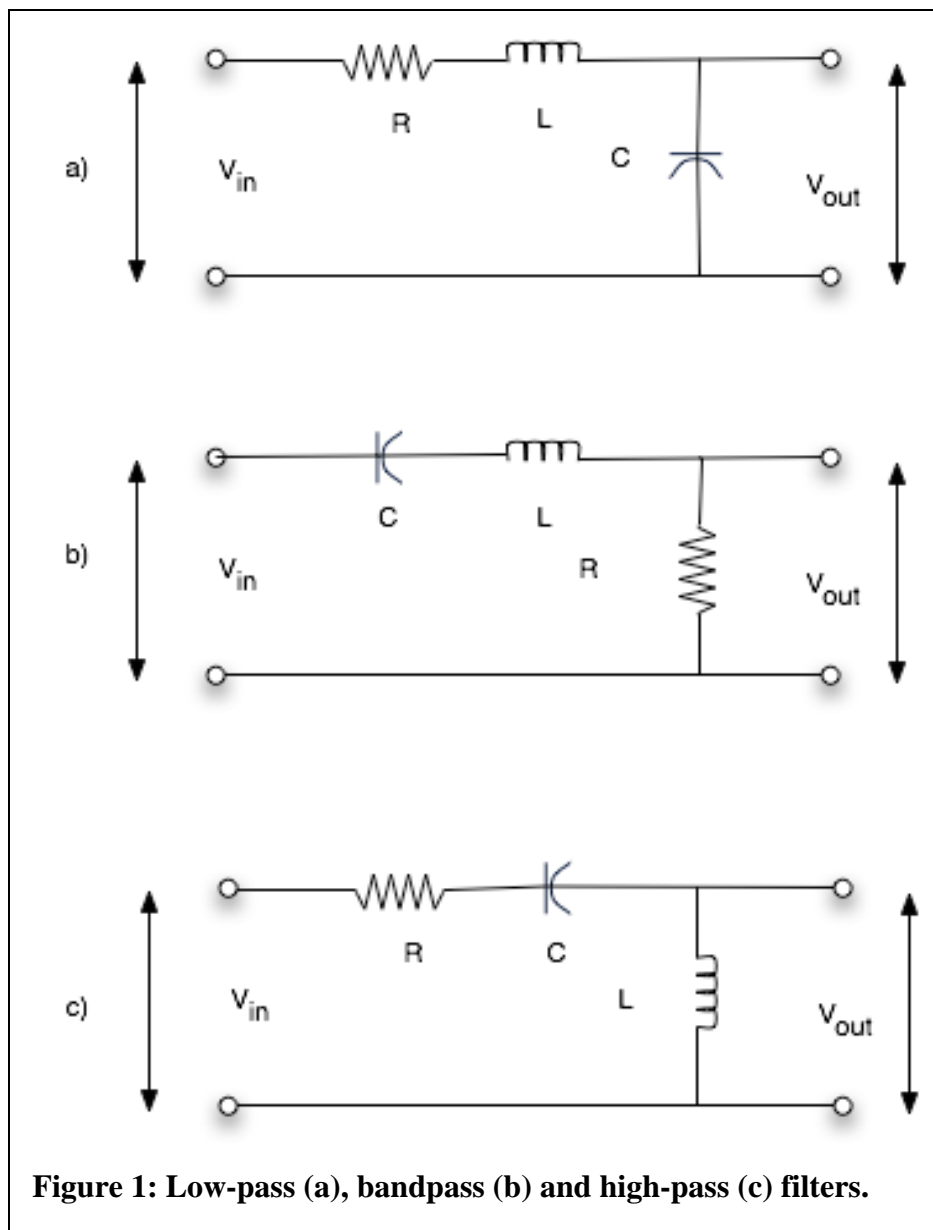
**Note:** Parts marked with \* include calculations that you should do before coming to lab.

In this lab you will work with an inductor, a capacitor, and a resistor to demonstrate concepts of low-pass, bandpass, and high-pass filters, amplitude response, phase response, power response, Bode plot, resonance and Q.

### Series RLC Circuits

#### \*1. Simple filters:

Figures 1 (a), (b), and (c) show low-pass, bandpass, and high-pass filters. Write the transfer function  $H(\omega)$  for each of these filters, showing the ratio  $V_{out}/V_{in}$  as a



**Figure 1: Low-pass (a), bandpass (b) and high-pass (c) filters.**

function of the angular frequency  $\omega$  of the input voltage.

\*2. *The low-pass – calculations:*

Show that the low-pass filter in (a) above has a power response function:

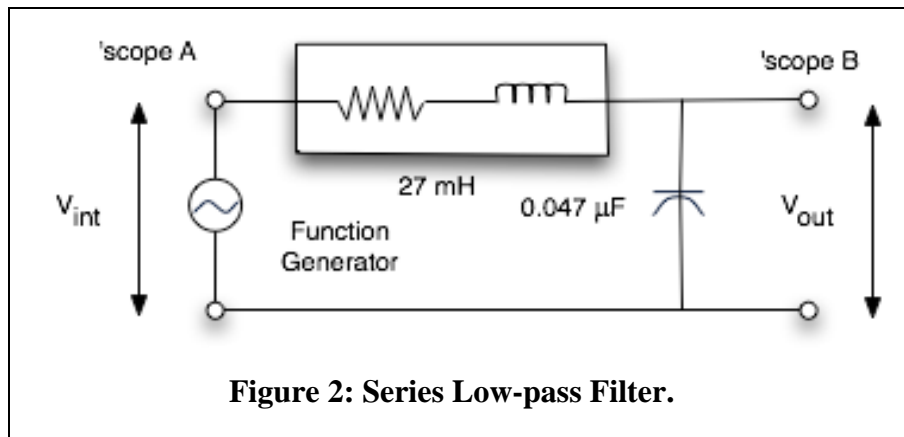
$$|H(\omega)|^2 = \frac{\omega_0^4}{(\omega_0^2 - \omega^2)^2 + \omega^2(R/L)^2}$$

- \* Explain why this is a low-pass filter by finding the limits  $\omega = 0$  and  $\omega = \infty$ .
- \* Explain why we say that resonance occurs when  $\omega = \omega_0$ .
- \* The half-power points are the angular frequencies  $\omega$  where the value of  $|H(\omega)|^2$  is reduced to half the value at resonance. Verify that the half power points are  $\omega_1 = \omega_0 - \frac{R}{2L}$  and  $\omega_2 = \omega_0 + \frac{R}{2L}$ . You may use the approximation that the resonance width is small compared to the central value  $\omega_0$ .
- \* The difference between half-power frequencies is the bandwidth of the resonance. The Q of the resonance is equal to the resonance frequency divided by the bandwidth. Show that  $Q = \omega_0 L/R$ .

3. *The low-pass – experiment*

Set up the series low-pass filter shown below:

Notice that there is no discrete resistor. The resistor in this circuit is the resistance



of the inductor plus any resistance contributed by the Function Generator. Normally the Function Generator has an output impedance of  $50 \Omega$ . Verify that this is the case by measuring the resistance of the function generator output with the generator turned on but the output voltage set to 0 V. Measure the resistance of the inductor, add the value contributed by the Function Generator and use this sum in your calculations below.

Calculate the resonance frequency and measure it by changing the oscillator frequency. You may find it convenient here to attach your DPO probes to both the  $V_{in}$  and  $V_{out}$  and use the “Measure” function to display on the screen both voltage amplitudes, their phase difference and frequency.

Using the measured resonance frequency and resistor value, calculate the Q. Vary the oscillator frequency to find the half-power frequencies and calculate the Q from the measurements. (Note: At the half-power frequencies the output voltage is smaller than the output at resonance by a factor of  $1/\sqrt{2}$ .)

Calculate and measure the ratio of input and output voltages at resonance. You should find that the output voltage is greater than the input! Explain how a passive circuit like this can give a voltage gain.

Measure the ratio of input and output voltages for very low frequency  $\sim 1\%$  of the value at resonance. From the transfer function you expect them to be the same. Are they? What is the phase shift at very low frequency?

### Reduced Q

Reduce the Q of the filter by adding a  $150\ \Omega$  resistor in series with the inductor. Measure the resonance frequency. Do you expect it to be changed? Is it? Calculate and measure the Q for this circuit.

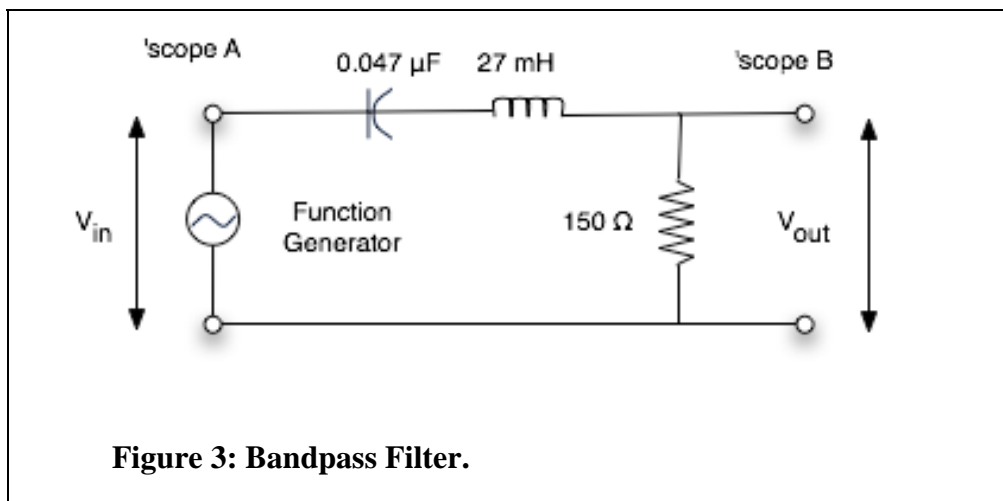
### More advanced topics

Measure the output voltages when the input frequency is 20 kHz and when the input frequency is 40 kHz. Use these measured values to show that the high frequency response of the filter decreases at a rate of  $-12\text{dB}$  per octave.

Use the oscillator and DPO to measure a Bode plot for this filter. The first part of the Bode plot is the magnitude of the response, expressed in dB as a function of frequency, 10 Hz to 50 kHz. The second part is phase (expressed in degrees or radians) as a function of frequency, 10 Hz to 50 kHz.

### Bandpass Filter

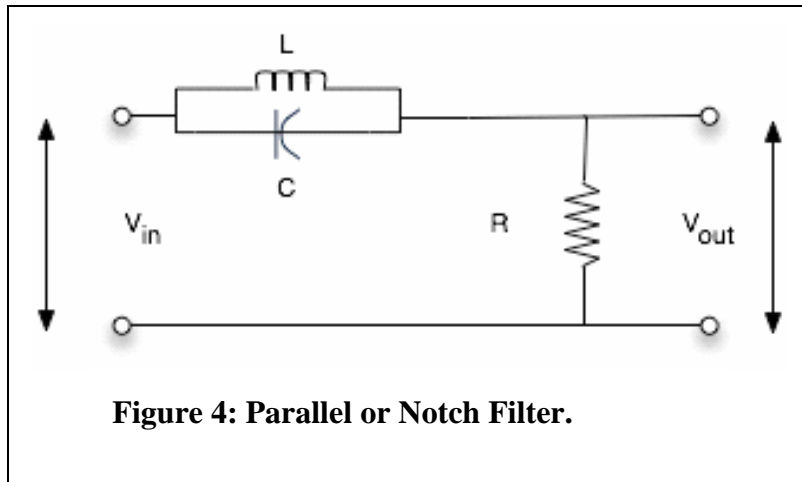
Connect the same components as in Figure 3:



**Figure 3: Bandpass Filter.**

Use the oscillator to show that this is a bandpass filter. Find the resonance frequency. Compare with the resonance frequency of the low-pass filter above.

Find the Q. Compare with the Q of the low-pass filter above.



\* Show from the transfer function that the amplitude response at high frequency is  $-6\text{dB/octave}$ , namely the output decreases by a factor of 2 when the frequency doubles. Measure the amplitude response at 20 kHz and 40 kHz to check for  $-6\text{dB/octave}$ .

### Parallel RLC Circuits

As an example of a parallel circuit, consider the filter Figure 4 and calculate its transfer function.

\*Explain why this is a notch filter. What is the frequency of the notch?

Use  $L = 27\text{ mH}$ ,  $C = 0.047\text{ }\mu\text{F}$  and  $R = 150\text{ }\Omega$ . Measure the depth of the notch by comparing the response at the bottom of the notch with the response at low or high frequency. Why doesn't the response go to zero at the bottom of the notch?

### Asymptotic notation

A filter can be described by its asymptotic frequency dependence. Although the transfer function may be a complicated complex function of frequency, the asymptotic characteristic is simple. For example, a low-pass filter may have a transfer function that is inversely proportional to frequency in the limit of high-frequency.

We say,  $H(\omega) \sim \omega^{-1}$ . In general  $H(\omega) \sim \omega^n$ , where  $n$  is a negative number for a low-pass filter.

In the asymptotic limit, a filter has a gain characteristic of  $20n$  decibels per decade (dB/decade).

Proof: The gain characteristic in dB is

$$L = 20 \log |H(\omega)|.$$

If  $\omega$  increases by a factor of 10 (one decade) then the change in level is

$$\Delta L = 20 \log |H(10\omega)/H(\omega)|,$$

and this is just

$$\Delta L = 20 \log[10^n] = 20n.$$

Similarly, the asymptotic dependence can be given in dB/octave. Whereas a decade is a factor of 10 in frequency, an octave is a factor of 2 in frequency. If the asymptotic frequency dependence is, again,  $H(\omega) \sim \omega^n$ , then this is just

$$\Delta L = 20 \log[2^n] = 6n.$$