Physics 471 – Fall 2008

Homework #8, due Friday, October 31

(Point values are in brackets for each problem.)

1. [4] Griffiths problem 3.5, parts (a) and (b) only. To find the adjoint of d/dx, integrate by parts.

2. [3] Griffiths problem 3.7.

3. [4] Griffiths problem 3.8, part (a) only. The first part is essentially done in Example 3.1, but do it anyway. Then show that the different eigenfunctions are orthogonal.

   b) Now imagine that we can confine an electron to move on a ring of radius a. If we call x the distance along the ring, then $x = a\phi$. From this, you can see that the momentum operator is just
   $$\hat{P} = \frac{\hbar}{i}\frac{\partial}{\partial x} = \frac{\hbar}{ia}\frac{\partial}{\partial \phi} = \frac{-\hbar}{a}\hat{Q},$$ hence eigenstates of $\hat{Q}$ are also eigenstates of $\hat{P}$. Let $a = 100$ nm.
   What is the smallest positive value of the momentum of the eigenstates you found in part (a)? What is the corresponding electron velocity, assuming we can use the relation $p = mv$? What is the energy difference between the ground state and first excited state of the system? At what absolute temperature is the thermal energy, k_BT, equal to that energy difference? Give all answers in SI units.

4. [3] Griffiths problem 3.13. For parts (b) and (c), apply the commutator to a test function g(x).

5. [6] Griffiths problem 3.11. Use all the Gaussian integral tricks you learned in Chapter 2,

   including this useful result we derived in class: $\int_{-\infty}^{\infty} e^{-\alpha x^2} e^{\beta x} dx = \sqrt{\frac{\pi}{\alpha}} e^{\frac{\beta^2}{4\alpha}}$.

   By the "classical range" of p, Griffiths means the range of momenta a classical particle undergoing simple harmonic motion would have if its energy were equal to $\hbar\omega/2$. (Remember that momentum can be both positive or negative.)

   After you are done with all the calculations, draw the following two pictures (these can be qualitative rather than quantitative):
   i) A graph of the potential $V(x)$ vs. x, with labels at the classical turning points. Just under it, draw the probability density in real space, $|\Psi(x,t)|^2$. Notice that there is a finite probability for the particle to be found (by a position measurement) outside of the classical turning points.
   ii) A graph of the probability density in momentum space, $|\Phi(p,t)|^2$ vs. p. Again draw the classical limits of p, and notice that there is a finite probability for the particle's momentum to have a value (by a momentum measurement) outside of the classical region, as you just showed.